HIGHWAY SURVEYING

AND PLANNING

The quality of the materials used in the manufacture of this book is governed by continued postwar shortages.

HIGHWAY SURVEYING AND PLANNING

FORMERLY PUBLISHED UNDER THE TITLE OF
HIGHWAY CURVES AND EARTHWORK

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PREFACE

"Highway Surveying and Planning" is the second edition, revised and enlarged, of "Highway Curves and Earthwork."

As the new title indicates, the second edition is broadened in scope and entirely rewritten to include many progressive features of planning and design not fully treated in the first edition.

Special consideration is given to highway location and design for greater safety and permanence.

Up-to-date surveying operations for the scientific layout of roads, culverts, and bridges, incorporating the best current practice of representative state highway departments, are presented in sufficient detail to be useful whether the book is used as a field and office manual or as a text book.

The treatment of vertical curves (including unsymmetrical curves), transition spirals, and curve superelevation is radically revised and greatly simplified. Whatever material in the first edition has become obsolete has been omitted from the new volume.

Several new tables have been added to the second edition, among which are the following:

Table 2.—Giving deflections, arcs, and chords for even radius curves.

Table 4.—Giving radii for both the arc and chord definitions of degree-of-curve.

Table 10.—Giving deflections locating any point on a spiral; so that, for the first time, full-station points along the curve may be established readily.

Table 12.—Giving functions of the spiral which is compounded with a 1° circular curve or with a unit curve of radius 10,000 ft.

The articles by E. W. James of the U. S. Bureau of Public Roads and others in *Engineering News-Record*, Jan. 17, 1935; and by A. R. Losh in the 1932 *Transactions* of the American Society of Civil Engineers have provided valuable and inspiring reference material for which thanks are hereby expressed. Acknowledgment is also due many of the state highway departments, particularly those of Ohio, Maryland, California, Missouri, and North Carolina, for useful data bearing on current methods of practice.

THOMAS F. HICKERSON.

CHAPEL HILL, N. C.,
January, 1936.

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CHAPTER I

HIGHWAY LOCATION

Preliminary Considerations

1. Importance of Highway Location Surveys

Good roads are intended primarily to supply means of communication and transportation from place to place with a minimum of time and effort. The highest aim of road making, it has been said, is "endeavoring to help make smooth the paths of humanity." One of the fundamental operations in road building is the *location survey*, which becomes, or should become, through construction a *permanent feature*.

Among the characteristic features of a *bad road* may be mentioned *steep grades*, *abrupt curves*, *dangerous exposure* (to intersecting traffic, snowdrifts, etc.), *insufficient drainage*, and *uneven surfaces*. The first four of these obstacles to traffic are of a *permanent nature* due to a *faulty location*, which might have been obviated with an unrestricted application of engineering principles. Too many locations in the past have followed an old road or the line of least resistance; which sooner or later must be abandoned as unsafe and unsatisfactory for the traffic of today and tomorrow. Obviously, future changes in location, unless the old road can be used advantageously for local purposes,

are attendant with losses of right of way, roadbed, surfacing, and structures as well as a depreciation in the values of property developments along the abandoned route. The ideal procedure in any road-building program is to make sure that the location is the best possible, so that what is done is permanent and an integral part of any future improvement.

The engineer and those charged with the layout of highways are faced with the dual responsibility of selecting a location which will become and remain a major route of highway transportation and of satisfying public opinion as to whether the initial expenditure is economically justifiable. Under all circumstances, the engineer should be permitted to so execute his work that it may be successfully defended against future criticism.

The greatest skill and care should be exercised to see that the located center line, with respect to appearance as well as usefulness, will be the most consistent location possible. The newly selected line forms the basis of a high-priced improvement to serve intense modern traffic that may in the future require a pavement two or more times the present contemplated width.

2. General Considerations Governing the Location of Highways

The primary technical considerations in road location, whether it be a revision of an existing road or the layout of an entirely new route, are *alignment* and *grade*. These considerations must be reconciled with many other factors, as follows:

Highway safety;

Funds available and costs;

Purpose of the road in question and its relation to the highway system;

Classification of the road as to standards of width, alignment, and grade, and the type of surface and improvement;

Restrictions on location: strategic points; service to the towns and cities; local sentiment;

General public welfare;

Traffic count; present and probably future traffic conditions;

Right-of-way privileges;

The ultimate plan;

Topographical and geological features of the country;

Best stream and ridge crossings;

Flood areas; drainage;

Climatic conditions—exposure to the sun and snow control in areas of heavy snowfall; and

Aesthetic considerations.

The influence that some of the foregoing factors may exert justifiably in road location will now be treated in more detail.

3. Highway Safety

a. Foreword. During 1934, 34,000 persons were killed and 954,000 injured from automobile accidents on the streets and highways of the United States.

In classifying highway and street accidents, there are mainly three general types of contributing factors, as follows:

1. Incidental to the drivers themselves; such as reckless speeding, general disregard for or ignorance of traffic regulations, drunkenness, bad vision, drowsiness, inattention, overstrain, and general incompetence resulting from no standard licensing law.
2. Incidental to the motor vehicles; such as bad brakes, defective steering gear, bad headlights, broken parts, etc.
3. Incidental to the location, design construction or maintenance of the highways or streets; such as exposure to intersecting traffic, sharp curves with insufficient superelevation, steep grades, high crowns, slippery surfaces, narrow pavements, treacherous shoulders, steep ditch slopes, narrow bridges, holes in the surface, etc.

Conditions contributing to the third class of accidents are primarily the engineer's problem. Although statistics show that the larger proportion of motor vehicle fatalities are attributed to reckless speeding, it is none the less of an obligation for the engineer to aim to effect every reasonable

feature of design that promotes the greatest possible comfort and safety for all those who use the highway.

So long as motor vehicles are being manufactured with greater speed capacities and in greater quantities for both passenger and freight transportation, the highway engineer will be expected to keep pace with these developments and, within limits set by the funds at his disposal, he must not only design and construct highways sufficiently spacious to accommodate present traffic and its normal future increase without undue traffic delay, but he must also safeguard them against every hazard that high-powered, high-speed motor vehicles can create in the hands of incompetent or reckless drivers.

b. Highway Intersections. At an ordinary crossroad, there are twelve possible courses open to traffic—four right and four left turns that may be made, besides four ways to go straight. Each, except the right turns, cuts across four other possible paths. Occasional collisions under such circumstances are inevitable, especially at the high speeds of rural travel.

In the United States, during 1934, 364,260 accidents occurred at the intersection of streets and highways, resulting in 7,770 persons killed and 408,310 persons injured. The remedy must be found either in setting up some form of traffic control or by redesigning the intersection. The traffic signal and the stop sign, when perfectly obeyed, achieve an orderly movement of vehicles and a notable reduction in hazard; but neither solves the left-turn problem with its possibilities of almost head-on collisions. Ample vision ahead and advance warning or stop signs of a standard type are necessary safeguards at rural intersections. Billboards and other nonofficial signs should not be allowed within the vicinity of a crossroad; and it seems desirable that all roadside advertising should be regulated by law, as is done in the state of California.

The majority of highway intersections, even in congested areas, will probably remain at grade; and their treatment for minimizing the hazard is deserving of much study.

Four types of intersections[1] are shown in Fig. 1.

Type A. Two additional traffic lanes extending for a length of 300 to 500 ft. are added on each side of the inter-

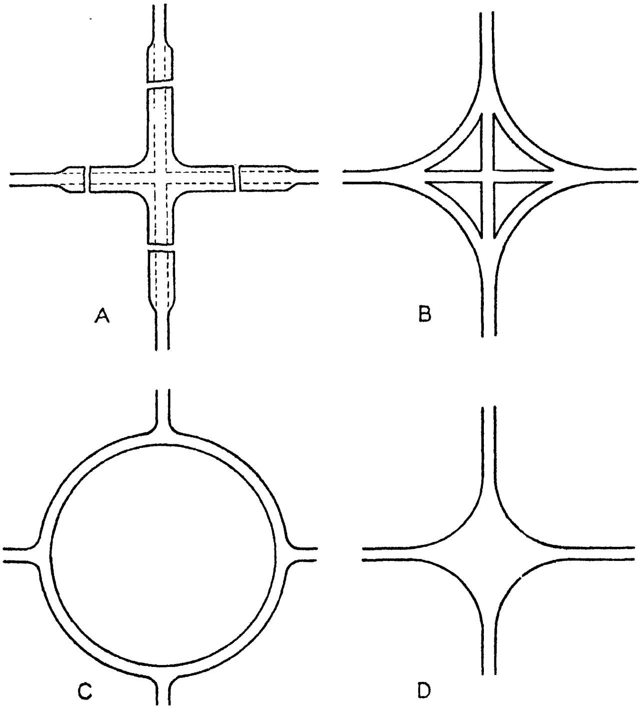


FIG. 1.—Types of intersections.

section, and curves of about 30-ft. radius are introduced at the intersections. Traffic about to make either a right or left turn enters the extra lane and makes the turn at slow speed.

[1] Arthur G. Bruce. "Highway Design and Construction," p. 108, 1934.

Type B. Curves having radii of 300 to 500 ft. are introduced so as to permit a right or left turn before reaching the right-angle intersection.

Type C. The traffic circle, with radius of at least 300 ft., permits the interweaving of traffic moving in essentially parallel lanes. There is no reduction in the number of intersecting paths, but the crossings are accomplished with a minimum likelihood of serious collision. The circle

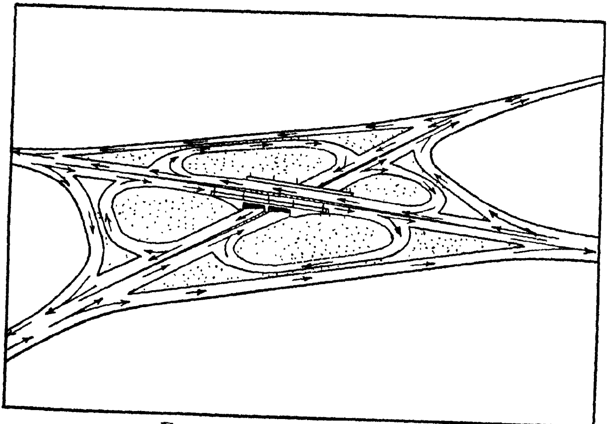


FIG. 2.—"Clover-leaf" design.

also imposes on the traffic a limitation in speed through the intersection danger zone. At a multiple intersection where more than two roads cross, a traffic circle is almost the only means of securing orderly movement.

Type D. This type provides a considerable amount of area for traffic.

If traffic circles or similar grade structures are used, it is *fundamental that the driver be informed in advance of their presence and of the proper course to be pursued.* This means that the islands must be adequately lighted and signed.

Many special types of highway grade separation designs with various arrangements of ramps and underpasses have been built in New Jersey to accommodate traffic conditions

occurring in congested areas. But the so-called "clover-leaf design" (Fig. 2) has the advantage that there are no lanes of traffic which cross each other and that there is no slowing up of the straight traffic. The complete clover-leaf design has a ramp in each quadrant so that when left turns are made in spiral fashion—that is, by crossing over or under the intersecting road and then making two successive right turns—no vehicle need ever cross the path of another, and the only possibility of collision arises when a vehicle turns to the right into a through traffic lane, a minor hazard, especially on multi-laned roads.

Experience in the congested areas of the north has shown that any grade separation on super-highways less effective than the clover-leaf may prove more troublesome than a grade crossing. This is due to the fact that a separation with inadequate ramps exaggerates the confusing features of turning traffic and multiplies the number of left-hand movements across the paths of the straight traffic.

The minor intersections of local or private roads or drive-ways with through highways are a constant source of danger because they are so numerous and, frequently, so obscure. Expensive treatment of each individually by traffic signals or redesign cannot be justified, and stop signs too often fail in their purpose because of nonobservance by drivers.

Accordingly, the concept of the freeway proposed by E. W. James, of the Bureau of Public Roads, has been developed—a highway for express traffic to which abutting property has no right of access. All local traffic must use service roads in the vicinity, and traffic may enter or leave the freeway only at widely spaced intersections where it can be properly safeguarded. Under this plan, a new express highway remains an express highway instead of becoming just another suburban or urban road.

Few areas can now produce enough traffic to justify the most expensive type of construction, but even the less important highways may have some of its features, and *the probability of future development should always be considered in present planning.*

c. Railroad Intersections. The problem of railroad-highway crossings is similar to that of highway intersections, in that only a separation of grades can make safety sure, especially with the development of the new high-speed trains. There is a growing sentiment in favor of closing many existing crossings and providing more adequate protection, or grade separation, at the others on the theory that modern motor-vehicle traffic can well afford the slight expense and inconvenience of detouring a mile or more for the advantage of greater protection. For protection of the remaining crossings where grade separation is not practicable, the standard flashing lights of wigwag signals, put into operation by approaching trains, are becoming more general on the rural highways.

It has been estimated by Mr. Ford of the Rock Island Lines, that there are now 235,800 highway-railway grade crossings in the United States, of which 30,000 should be separated, 100,000 need protection, 45,800 require but minor attention, and 60,000 might be closed by relocation or some form of proper planning.

d. Separated Roadways. Considering the number of vehicles which pass each other in opposite directions on the open road, the statistical chance of any two colliding is infinitesimally small. In the aggregate, however, head-on collisions make up a very serious part of the total; and among the earliest of highway improvements was the widening of the pavements to increase passing clearances. Demands for greater traffic capacity have been met by widening to three lanes, four lanes, and more, but vehicles using the center lane or lanes must still pass close to opposing cars and, in addition, face the added danger of being crowded by cars on their right.

Three-lane highways, due to the questionable right-of-way in the middle lane, are less satisfactory than four-lane highways. In new construction, if funds are insufficient to finance the entire four-lane width, the two outer lanes might be paved for main traffic lanes and the two inner

lanes lightly paved until funds for full pavement become available.

The painted center line, which is now almost universal on curves and on hills, especially where sight distance is impaired, is effective and popular, but it cannot prevent the careless motorist from crossing over to the wrong side at times, with possible serious consequences.

To keep opposing cars safely apart, it is clear that there must be some physical barrier between them. The *safest and most ideal highway* must be built with two separated roadways, each for traffic in one direction only. Each must be at least two lanes in width to accommodate fast and slow traffic in different lanes. But until vastly greater highway funds are available, this design will probably be limited to highways on which the volume of traffic justifies the expense of four-lane construction.

A recent study in Massachusetts showed only 54 per cent as many fatal nonintersection accidents per million vehicle-miles (excluding pedestrian accidents) on *divided highways* as on ordinary *four-lane highways*, and only 37 per cent as many as on two-lane roads. An incidental but important advantage of the dividing center strip is that it may be planted to serve as a screen against opposing headlights, thus removing permanently one of the greatest hazards of night driving. At intersections where it has not been feasible to separate grades, furthermore, it offers a very desirable "dead" area between the opposing traffic streams where a crossing or turning vehicle may remain protected until a favorable opportunity occurs for entering or crossing the traffic on the far side of the roadway.

The construction of "by-pass" routes to carry traffic around congested cities rather than through business sections is a feature of highway work which usually promotes not only safety, but convenience as well.

e. Sight Distance. As an element of safety, ample sight distance or visibility is desirable so that every condition of the highway, whether an intersection, a curve, or any-

thing that requires the driver's attention, should be seen far enough ahead to enable him to take appropriate action. In the design of both horizontal and vertical curves, sight distance should be a controlling factor. The American Association of State Highway Officials recommends a minimum sight distance of 800 ft. for two-lane trunk highways. In cases where a sufficient distance cannot be obtained, extra width of surfacing is desirable; or under some circumstances, the roadway may be separated into one-way roads, thus permitting the required sight distance to be cut in half.

f. Alignment. In establishing the layout of a highway between objective points, *directness* has always been a major consideration subject to the limitations of avoiding excessive grades. With the advent of fast-moving, high-powered motor vehicles, improvements in alignment are increasingly desirable, not only as an element of economy in reducing distance, but as an element of safety against accidents. The alignment should be made as safe as possible considering the funds available, even at the expense of a steeper grade (up to a reasonable maximum). Short curves and abrupt turns are particularly hazardous on steep grades, at the foot of long hills, and between long tangents.

There should be a logical reason for every curve. A combination of maximum curvature and grade should be avoided by reducing the maximum grade around curves according to a standard compensating rule similar to that adopted by the railroads.

If reverse curves are necessary they should be separated by a tangent distance of at least 200 ft., thus allowing space for runoff easements on each curve.

Hidden reverse curvature or a combination of horizontal curve with a summit vertical curve is an unsafe combination, and should be avoided.

Curves in the same direction with a short tangent between them, commonly known as broken-back curves are objectionable from the standpoint of appearance, and they may

be a source of danger. Usually it is feasible to combine them into a compound curve; but in this event, the difference between the two branches of the curve should not exceed 5°.

Curved approaches to a bridge are objectionable. They may often be remedied by placing the structure on a slight skew.

Every effort should be made to secure flatter curves, even down to 30-minute curves, although a sharper degree of curve is adopted as a maximum. In a 60° turn, the actual distance saved by using a 1° curve instead of a 6° curve is 513 ft.

On major highways, it is specified by many states (California, for example), that the minimum radius of curves shall be 2,000 ft., which is equivalent to about a 3° curve. But California practice allows, in difficult mountainous locations, a low radius of 300 to 350 ft. for blind curves and 250 to 300 ft. for open curves, with liberal requirements for extra width in both cases. Allowing sharp curvature at one place, however, does not justify its use at other points.

An unexpected change from one standard of alignment to a lower standard, as, for example, a few isolated sharp curves within a zone of good alignment or an unexpected reversed curve at the end of a long tangent, is a source of danger and responsible for many accidents. Whenever such special conditions unexpectedly reduce the vision of motorists at isolated locations, much time and money should be given to eliminate or improve them, particularly on main routes. If the elimination of these hazards is impossible, as in mountainous country, then unmistakable warning signs should be on display far enough away for the motorist to gradually reduce his speed limit.

To avoid the appearance of a kink in alignment, curves with small angles should not be less than 800 ft. in length. All horizontal curves are greatly improved in appearance and in comfort and safety as well, when connected to tangents with long *transition spirals* which should apply to both the outer and inner edge of roadway; and all curves

sharper than $\frac{1}{2}$ to 1° should be *superelevated.* Pavements on curves sharper than 3° should be *widened.*

Properly spiraled and superelevated curves of 1,000-ft. radius can be comfortably negotiated by an automobile at speeds up to 55 to 60 miles per hour.

The statement is often made, and correctly so, that more accidents occur on straight roads than on curves. But since there are many more miles of straight road than there are of curved, this statement is misleading.

The accidents on highway curves are relatively more fatal than those on any other part of the location except railroad grade crossings.

Unfortunately for the motorist, there are altogether too many curves on which the surfaces are flat or even worse, have negative banking—that is, sloping downward toward the outside of the curve. This creates a tendency to encroach on the wrong side of the road by making a left turn on the inside of the curve. Proper superelevating and spiraling and the painted middle strip will help remedy this condition.

g. Grades. As a general rule, grades in excess of 5 per cent are undesirable for two reasons: (1) Difficulty of ascent and increased danger of descent; and (2) possible erosion of the road surface, shoulders, or ditches. But it is better to use a steeper grade (up to 7 or 8 per cent), for a short distance, if bad alignment is obviated thereby.

From the standpoint of ascent, the modern high-powered car can climb readily short grades as steep as 8 per cent; but even so, continuous grades of more than 5 to 6 per cent are objectionable on trunk highways. To descend grades steeper than 7 per cent may at times subject the motorist to great hazards, particularly when the road surface is coated with ice.

For grades of 6 per cent, or more, a common rule is to compensate on curves up to 1,000-ft. radius according to the formula:

$$\text{Compensation in per cent} = \frac{125}{\text{radius of curve}}.$$

The compensation provided by the foregoing formula does not include or contemplate compensation which may be necessary to furnish slack grade for future shortening of the line. Where subsequent improvement in alignment is anticipated on sharp curves, additional compensation should be introduced to the full extent that careful grade design and location will permit.

An effort should be made to obtain the least possible grade consistent with a reasonable expenditure of funds. By the use of lower maximum grades, more uniform speed of car operation can be maintained and greater reserve power will be available for retarding and accelerating when passing slow vehicles in emergency. Heavy commercial vehicles find easy grades better adapted to their use.

In New York state, the maximum grade is 5 per cent on primary routes and 7 per cent on secondary routes.

Rolling grades may be justified by construction economy, and they often present no appreciable economic loss in operation. Their objectionable feature lies in hazards at vertical curves and in roadway appearance. For this reason, grade rates should be held to a minimum and vertical curves should have a long sight distance.

h. Shoulders and Roadside Safeguards. Essentially uniform construction features and clear and unmistakable *warning signs* are necessary for the quick perception and reaction of motorists at high speeds.

To keep the traveled lanes safely clear of stopped vehicles, every highway must be designed with *shoulders* that are wide, smooth and substantial enough to serve as an emergency traffic lane. Generous shoulders clear of all obstructions and gentle ditch slopes will often make a minor accident out of what otherwise would be very serious.

Guide and guard rails serve a twofold purpose: (1) as valuable safeguards for warning motorists of the existence of curves, embankments, side hills, or other hazardous conditions; and (2) to prevent a vehicle from leaving the pavement and shoulders of the highway.

Either the guide-rail posts or the rail itself, or both, should possess a limited amount of resiliency for greatest effectiveness.

Separate pathways for pedestrians should be constructed on one or both sides of the main highways. Separations for pedestrians are advisable in congested school zones.

i. Widths of Pavements and Bridges. The American Association of State Highway Officials recommends for safety at least 10 ft. of width for each traffic lane of the highway, and a shoulder width of 10 ft. Engineers are beginning to realize that the present narrow traffic-lane widths are among the most hazardous features of highway design. Accordingly, there is a growing opinion for travel lanes on main roads with widths of 11 or 12 ft., whether for speed traffic or trucking. A bold stand by highway engineers for greater road widths would be timely. Width deserves preference over the additional mileage built to the current narrow standards.

Irregularities in alignment and grade are more definitely localized and noticeable at *bridges* than elsewhere. The effect is made more acute by the fact that most bridges are narrower than the roadbed, and therefore dangerously narrow. Good practice as to the width of bridges is to use multiples of traffic lanes 10 ft. wide plus 2 ft. on each side. The widths are thus 24 ft., 34 ft., 44 ft., etc., except where special conditions require them to be otherwise. Highway bridges in suburban areas should include sidewalks.

j. The Driver and the Vehicle. The fitness of the driver and the condition of the vehicle, particularly as to brakes and steering gear, are vitally important phases of traffic safety; both of which present problems more perplexing than the condition of the highway itself.

A measure of control of the few drivers prone to be reckless on the highway may be expected from policing and from licensing laws; and good results should come from educational programs aimed to create a greater sense of responsibility among all those who use the highway. Instead of establishing maximum speed limits, it seems best to adopt

the basic code rule that "*No person shall drive a vehicle on a highway at a speed greater than is reasonable or prudent under the conditions then existing.*"

4. Funds Available and Costs

Highways are usually financed either by (1) bond issues or (2) "pay-as-you-go" plan.

The bonding plan requires a definite program in advance. This plan is best when the program is just beginning and before any great part of the system has been completed. It permits quick construction of projects large enough to secure the advantage of big contracts at low prices; but in carrying out a mass production plan, there has been a tendency to cut short the time needed for a thorough study of each location problem.

The "pay-as-you-go" plan should be used when the construction of the highway system is more advanced or is nearing completion. It may be successfully used if annual revenues are plentiful and it should be used in any case where the present bonded indebtedness is so heavy as to have affected adversely the public credit of the state or county.

If $100,000 is to be spent by the "pay-as-you-go" plan on 10 miles of construction and the annual highway fund is $10,000, the cost is $10,000 per mile and it will take 10 years to build. With a $100,000 bond issue, serial type, 4 per cent maturing in a 20-year period, the cost of the bond issue will add $42,000 to the cost of the highway, so there must be added $4,200 to the cost of each mile of the roadway, but the entire 10 miles of newly constructed highway would be immediately available.

If this immediate construction of the entire mileage accommodates enough traffic to save or earn the difference in cost, the bond issue is always justifiable.

Bonds. The forms of bonds in general use are the *term*, *serial* and *annuity*. Term bonds mature after definite periods and require a sinking fund. Serial bonds are retired annually or at other periods, a fixed portion of the issue being retired at each maturity date. The annuity is similar

to the serial, but the retirement plus interest is maintained at a constant figure.

The serial bond is the cheapest and is the most popular for highway financing. This type is particularly adapted for financing operations which by their very nature involve a depreciation of property. A highway is in part a depreciating property.

In order to minimize the possibility of wasting funds, the following recommendations represent good current practice:

a. State highway bonds should be serial in form and should mature over a period not exceeding 30 years. The annual charge decreases very slowly from this point whereas the total cost increases rapidly.

b. County bond issues should be serial in form and should mature over a period not exceeding 20 years. Interest rates on county bonds are as a rule higher than those on state bonds. The use of the serial bond eliminates the hazard of a large sinking fund which is in constant danger of violation.

c. Maturities should be arranged so that the annual requirements of principal and interest will be as nearly uniform as practicable. This also avoids the danger of accumulation of large sinking funds and a fluctuating tax rate.

Comparative Costs. Alternate locations may be studied in terms of their comparative capitalized cost (or annual cost). The capitalized cost of a highway is that sum which is sufficient to pay the initial cost of construction and provide for future maintenance and operation and renewals forever for those elements of the investment which are not permanent.

Let X = capitalized cost;

C = first cost = $C' + C''$,

in which C' is the permanent part and C'' is the temporary part;

M = annual maintenance and operating expense;

r = annual rate of interest;

R = the capitalized depreciation;

it being the amount which, when placed at compound interest during the estimated useful life of the parts which wear out, will yield a sum equal to the first cost, C'', plus a perpetuating sum R for future renewals. If the amount, R, is placed at compound interest as stated above, we have

$$R(1 + r)^n = C'' + R,$$

whence
$$R = \frac{C''}{(1 + r)^n - 1}.$$

Therefore the total capitalized cost is

$$X = C + \frac{M}{r} + \frac{C''}{(1 + r)^n - 1}.$$

The *annual cost* is the interest on the capitalized sum. It includes annual interest on the first cost plus maintenance and operating costs plus a replacement fund. Expressed as a formula, we have

$$\text{Annual cost} = Xr = Cr + M + \frac{C''r}{(1 + r)^n - 1}.$$

In the foregoing expressions, maintenance costs are for keeping the roadway in proper condition; while operating costs refer to the expenses encountered by the traveling public. The latter item may be only a rough estimate in any one case, but for purposes of comparison with alternative routes, it is more definite. The costs of future replacements are not accurately known, but they may not exceed the first cost if the old parts have scrap value.

Relatively speaking, the capitalized cost serves as a guide to the judgment in considering the merits of two or more routes.

5. Purpose of the Road

The primary use of a modern highway, generally speaking, is to provide means for vehicular transportation of the various classes of travel and commercial haulage from one common center to another, either local, state, or national. They should be established over the most direct route

possible with the least gradient. The most direct route may be defined as a road with alignment free from curves that have dangerously short sight-distances based on the importance of the road and the speed limit involved; and the least gradient should be such that it will not retard materially the reasonable speed of present-day vehicles.

The foregoing requirements for line and grade may be dispensed with advantageously, to some extent, in cases of scenic development, when locating forest and park roads or other lanes of traffic promoted solely for pleasant landscape effect, recreation or educational travel.

From the standpoint of transportation service, public roads are classified into three major groups: (1) *primary* (roads comprising the federal and state systems), (2) *secondary* (principal county trunk highways), (3) *third-class roads* (purely local township roads). In addition, a special classification or subdivision is needed for the so-called express, arterial, boulevard, or superhighway (for which no standard name has been adopted). This type has become an essential factor in the vicinity of large cities and between near-by cities where the traffic is exceptionally heavy.

6. Restrictions on Location

The general location of a highway project, or the route of which it is a part, is sometimes fixed by legislation to the extent that it must pass through one or more points. Within the limits established as controls, the engineer should determine, as far as practicable, upon an economic location, considering at their relative weights both through and local traffic. Local sentiment may call for the diversion of the main line from its direct course in order to serve an intervening town or community. The engineer must wrestle with this problem and give due weight to all the factors involved. The relative cost of the direct line with a lateral spur and the indirect line through the town should be considered from the standpoint of construction, maintenance, and operation.

The location and design of trunk-line highways into, through, and around cities are a problem for the engineer

that requires extensive study and investigation. A by-pass and local route serving both local and through traffic offer the ideal solution to be attained. Belt-line highways with local connections may be the best plan for still larger cities. Each situation is a problem in itself, but the solution should always have the same object—the greatest possible service to all the users of the highways.

A by-pass location, or belt line, is desirable for through traffic to avoid the delays and hazards incident to city congestion. A direct through connection, even if it be congested by city traffic, is also desirable in order to accommodate traffic making a temporary stop in the city.

In Bulletin 18 of the American Road Builders Association, we have the following statement expressing the changing attitude of cities relative to through highway traffic:

Spur roads represent a recent development in highway location having the several objects of efficient service to local communities, relief of traffic congestion in municipalities, together with safety and efficiency for through traffic.

Up until recent years there was a strong sentiment in cities and villages for trunk-line highways to pass through such municipalities on their main streets. In some cases this sentiment was prompted by a hope for early improvements, in other cases by the thought that the municipality would gain recognition and a certain amount of advertising by routing all traffic through the business streets whereby travelers could not fail to notice the busy aspect of the community, the public buildings, residences, and other features of local pride. In still other cases the motive behind this sentiment was a hope for increased business. It was probably reasoned that if a certain number of people in a locality produce a certain amount of business then the logical way to increase business is to get more people to the locality. In general, these several factors were combined to form the sentiment for trunk-line locations through municipalities.

It is far from correct to state that this sentiment is entirely changed, but it is true that it has changed in many municipalities and is changing in others. This change has taken place first in the larger cities and is spreading through the lesser

cities and even among many villages. It has been found that through traffic desires to keep moving, and moving traffic does not help business, but may even hinder it by interfering with those who have local business to transact. Those cities with a problem of congested traffic are rapidly developing a sentiment for routing through traffic out of the congested business districts. The present desire for through trunk-line traffic on important local streets appears to be somewhat in inverse ratio to the size of the community and the volume of traffic, although the character of local business and traffic also have a considerable bearing on the matter.

After a careful traffic study and analysis relative to arterial routes for traffic to and from the city and belt lines for traffic distribution, the city of Cleveland, Ohio, and its contiguous territory, with the cooperation of state and county authorities, have developed a plan for major road and street improvements for ten years ahead. They may therefore proceed with individual projects, knowing their final position in the broader plan.

7. Right-of-way Privileges

The width of right-of-way needed depends on the future requirements of the road and also the means for its protection from encroachment and exploitation. Few states have adequate laws regarding right-of-way acquisition and roadside protection. A Texas statute permits the state to obtain 100 ft. for a state highway, which ordinarily is ample.

The minimum standard widths of the various states on new locations vary from 80 to 100 ft. for main trunk highways, and from 60 to 80 ft. for county or secondary roads, with widths varying from 100 to 200 ft. for superhighways. These adopted minimum standard widths appear adequate for present or future traffic requirements, except near a few industrial sections which have an enormous traffic over short stretches. Several states have wisely adopted the plan of widening the right-of-way at railroad crossings and road intersections. The methods of acquiring right-of-way are rather varied and include the following:

1. County or municipality acquires the right-of-way at no expense to the state.

2. Purchase by the state, payment being based on assessed valuation.

3. Purchase by the state by agreement or condemnation suit.

The American Road Builders Association suggests the following as a means of obtaining necessary rights-of-way:

1. That the right-of-way be acquired by the State Highway Departments, the county in which the rights-of-way lie to be required to pay into the State Road Fund the necessary amount of money, to cover the purchase price, or certain taxes now accruing to the county be diverted to the State Road Fund to cover right-of-way costs.

2. That the value placed on each piece of property bear some direct relation to the assessed value.

3. That the State Highway Departments include as a bid item the removal of fences and obstructions from the limits of the right-of-way, and the rebuilding of the fences from salvaged material, and the construction of a standard type of right-of-way fence where necessary. If the property owner desires a better type of fence than that constructed by the Highway Department, then the property owner may construct his own fence at his entire expense.

4. That the Highway Department provide suitable passage onto the new road for the property owners at least as good as that existing before the improvement, and if the highway is on entirely new location suitable passage shall be provided.

5. That the Highway Department will so handle the construction as to cause no damage to the property owner, and will construct such retaining walls, etc., as may be necessary for protection of his property.

8. The Ultimate Plan

A splendid mode of procedure for all cases is to adopt a policy that a major location plan be worked out for future improvement, and that this be utilized on individual sections as the opportunity arises. Proceeding in this way, the ultimate plan, when finally completed, may really be considered a permanent improvement.

9. Climatic Conditions; Snow Control

In regions subject to heavy snowfall, the highway should be located with reference to preventing or reducing snow drifting. Quoting Mr. H. K. Bishop of the Bureau of Public Roads, the following procedure will tend to attain this result:

> Northern slopes should be avoided; in open areas the highway should follow ridges where possible or should be placed on slopes against the prevailing winds; long shallow cuts should be avoided, but where this is impracticable the location and construction should be such that they will not cause drifting; in flat country, drifting is reduced by elevating the grade 2 or 3 ft. above the surrounding land and with the construction of 1 on 4 slopes; in mountainous or sparsely settled areas where winter traffic is not sufficient to justify the expense of keeping the roads open during the entire winter, proper exposure is highly essential in order that the snow may melt and free the road for traffic as early in the spring as possible.
>
> In the plains country fine dry snow carried by a winter gale will drift into every cut and depression. Until the snow has settled and partly packed, it is difficult to keep an east and west road open.

10. Drainage

It is desirable to avoid locations which encounter unstable foundations, such as swamps and soft ground, where construction is expensive and drainage difficult. But if the road must be built across wet land, its grade must be raised to a height well above standing water.

All points of the road should be above the level of flood water; and sufficient drainage structures must be provided to take away quickly the surface and underground waters. The position and size of culverts and bridges (see page 52) must be determined carefully.

In most cases, the roadbed drainage may be taken care of by one or more of the following methods; tile drainage, open ditches or gutters, French drains (trenches filled with coarse stone for the purpose of intercepting and carrying off ground water), well maintained subgrade with frequent

rock-filled berm drains, and insulation courses composed of sand or screenings.

Ohio practice as to *when tile drains should be used* is given below:

After a complete soil survey has been made, it is generally found that different types of drainage are necessary to meet various soil conditions. The following classification should be made for the use of *tile drainage:*

1. Flat roads where water collects along roadbeds from areas adjoining. Ponds, marshes or swamps, and springs come within this class.

2. On roads where new open drainage ditches are objectionable on account of their excessive depth and cost.

3. Through cuts that have seeping banks and slopes. Tile drains when properly constructed through seeping banks and slopes, are generally located under the gutter ditch and backfilled with a porous material. These drains cut off many sources of water which lie deep under the surface.

4. Roads underlaid with retentive subsoils. This class of soil will show an unaerated, water-logged condition for the greater part of the year.

5. Roads having gutters, shoulders and slopes that erode excessively. Drainage of this class of roads can often be accomplished by diverting surface water into catch basins and outletting these catch basins through sub-drains. This type of subdrainage also equalizes the runoff.

6. Roads that require earth shoulders of maximum width, and shallow gutters. This class of roads are the main highways, also highways adjacent to populous centers which are liable to become city streets in the near future.

11. Aesthetic Considerations

The general appearance of any highway, whether it is attractive or not, depends upon the design of the road itself as well as the treatment of the roadside areas.

Straight lines and graceful curves are pleasing to the eye; while the reverse is true of broken lines, unsightly kinks, or ill-fitting curves. It has been suggested that a location containing, within one view, three tangents and two curves, or two complete changes of direction, whether horizontal

or vertical, is not pleasing. Transition spirals improve the appearance of horizontal curves, aside from adding to the ease and safety of travel.

Quoting W. W. Crosby, the following axiom seems appropriate here: "What looks right may be wrong, but what looks wrong cannot be right."

Bridges and other structures along the right-of-way when designed with attractive architectural proportions may be made an ornament to the highway.

More attention should be given to roadside improvement, beautification, and scenic effects. Unsightly objects should be removed, and ugly banks or slopes should be sodded or covered with evergreen vines, such as wild honeysuckle. Large and rare trees and shrubs found in their natural state on the right-of-way should be preserved; and along barren stretches of the road, trees should be planted. All shrubbery plantings along the inside edge of the highway should be limited to the dwarf varieties in order not to obstruct safe visibility.

Until such time as adequate legislation is enacted placing roadside development under the supervision of highway authorities, the engineer should use such means as are available to protect and preserve the highway for its complete utilization. To this end, Mr. A. R. Losh[1] offers five suggestions:

1. Acquire sufficient right-of-way for future development of the highway.
2. Obtain extra widths at all road intersections and other strategic points where it is likely that roadside enterprises would be established.
3. In making acquisitions it will usually be possible to retain in the right-of-way the narrow strips, gores, and other parcels of land cut off from the original tract. These parcels may be utilized for highway purposes in the future or for roadside planting and beautification. In any event they are not available as sites for billboards and roadside establishments.
4. Through areas of natural beauty, either obtain in fee or protect with long-term leases, several hundred feet extra width on each side of the right-of-way.

[1] *Trans. Am. Soc. C. E.* 1932, p. 475.

5. The interest of civic organizations and public-spirited citizens should be encouraged and their aid secured to obtain the desired ends.

12. Comparison of Highway and Railroad Location

The engineering operations establishing the line and grade of highways are essentially the same as those for railroads. But the highway engineer has greater latitude in which to satisfy alignment and profile requirements. Steeper grades are permissible on highways; but with advent of fast-moving streamlined cars, greater refinement in alignment is required of both the highways and railroads. Safe and pleasing vision ahead along highways for easy steerage of motorcars is relatively more important than in the case of railroad location. The problems of drainage structures are alike, except that subsoil and its drainage under and near the highway require special attention.

13. Surveying Operations

The surveys which ordinarily precede the construction of a properly located highway may be classified as (1) *the reconnaissance;* (2) *the preliminary survey;* and (3) *the final location.* In addition to the foregoing, there may be *special surveys* for various investigative purposes; and immediately before construction starts, it is usual to make a *construction survey.*

The nature of the project, whether new construction or reconstruction, heavy or light work, mountainous or valley location, brushy and heavily timbered or open, will determine the methods and steps necessary to secure the final location. The operations may involve all the investigative steps including reconnaissance, grade line and preliminary surveys or may eliminate some of the preliminary steps.

The Reconnaissance

14. Definitions

The reconnaissance is a general survey and investigation of the territory through which the road is to be built; with a view of selecting a route that is worthy of a more detailed

instrumental survey. The reconnaissance should be of an area rather than a line.

15. Procedure

Strategic points of the probable route should be noted: such as favorable stream crossings, low ridge crossings and other places where the road should most advantageously be located. Ground subject to slips or landslides should, so far as possible, be avoided. A *topographic map* of the area is exceedingly helpful in studying the territory and projecting a tentative line.

The instruments used in the reconnaissance need rarely be more than a pocket compass and Abney hand level. Frequently, a competent person with the eye alone can pick favorable routing and judge conditions as he proceeds, coordinating controls en route and by a process of elimination reducing to a minimum the extent of the territory that should be studied further.

If there is an existing road near the proposed improvement, it should be surveyed with transit and tape (or with stadia) and *mapped* to serve as a base line for aid in establishing a new line. In some cases the actual location under consideration is predetermined by the location of the old road except for minor changes in line and grade; thus dispensing with much of the reconnaissance work required of an entirely new route.

The reconnaissance survey should include information as to the general topography of the territory; classification of materials; drainage; character of clearing; development of the country; service to existing communities; right-of-way privileges; purpose of the road in question and its relation to the highway system; classification of the road as to standards of width, alignment, and grade and the type of surface and improvement; etc.

The *reconnaissance report* (according to California recommendations) should be a complete record, with maps and photographs appended, of all the collected information useful in administrative circles and in the conduct of future surveys. Also, it should conclude with a

discussion of economic factors and a brief résumé of comparisons, deductions and recommendations.

THE PRELIMINARY SURVEY

16. Definitions

The preliminary survey is supposedly an instrumental survey of the line tentatively established by the reconnaissance. This line is an accurate traverse which serves as a base line for obtaining topographic information of a strip of territory. It may be on an entirely new route or it may be along an existing road which is to be improved with perhaps minor adjustments here and there in line and grade. Frequently, an old road is surveyed merely for the purpose of mapping a convenient base line from which a new and more desirable location can be projected and established by subsequent surveys.

The preliminary survey need not be staked out as comprehensively as the final location survey, but it should be referenced so that it may be accurately retraced. Angles and distances should be measured with extreme care and elevations for percentage of grades should be determined. Topography should be fully noted, together with any details that would affect the final location.

It is desirable of course to place the so-called preliminary as close as possible to the line that will become the final location.

17. Organization and Duties of Surveying Parties

The three surveying parties for either a preliminary or location survey consist usually of: (1) *the transit party* of four or more men—chief of party, transitman, and two or more helpers—who are chainmen, rodmen, stakemen and axmen; (2) *the level party* of two or three men—levelman, rodman and perhaps note keeper; and (3) *the topography* or *cross-section party* of three or four men—levelman, rodman, chainman and perhaps note keeper.

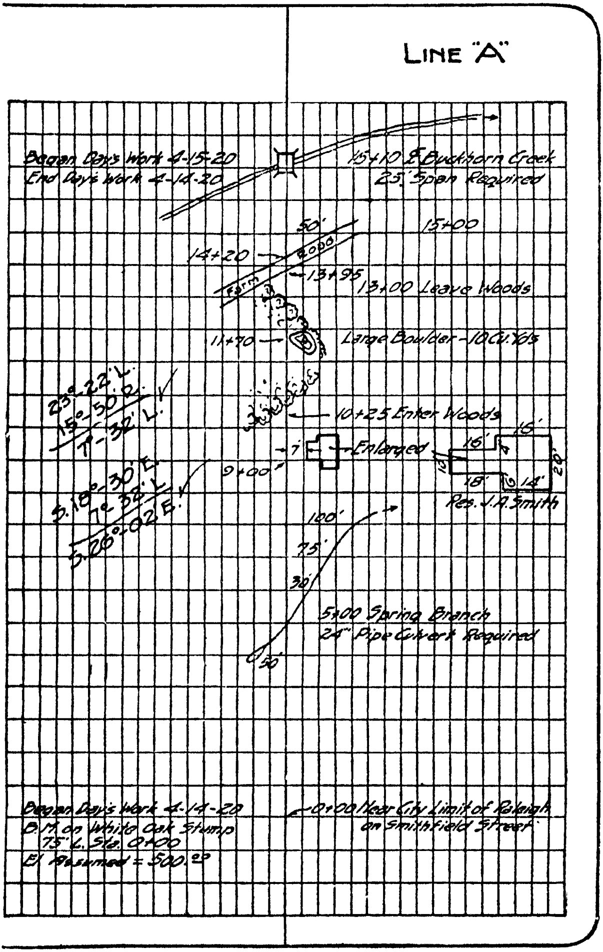
The *chief of party* must be experienced in general surveying work, and he should have a good eye for choosing the most

FORM OF NOTES

PRELIMINARY SURVEY OF ROAD

Station	Angle, L.	Angle, R.	M.C.	C.C.	Distance
16					
15+01.5		15°-18'	S.26°-00'E.	S.26°-02'E.	451.5'
15					
14					
13					
12					
11					
10+50.9		0°-32'	S.41°-15'E.	S.41°-20'E.	379.9'
10					
9					
8					
7					
6+70.3	18°-19'		S.42°-00'E.	S.41°-52'E.	460.3'
6					
5					
4					
3					
2+10.9	5°-03'		S.23°-30'E.	S.23°-33'E.	210.9'
2					
1					
0+00			S.18°-30'E.		
	23°-22'	15°-50'			

PLATE


LINE "A"
Began Day's Work 4-15-20
End Day's Work 4-14-20
15+10 Ⓔ Buckhorn Creek
25' Span Required
50'
15+00
14+20
Farm Road
13+95
13+00 Leave Woods
11+70
Large Boulder - 10 Cu. Yds
23°-22' L.
15°-50' R.
7°-32' L.
10+25 Enter Woods
16'
16'
7
Enlarged
13'
20'
18'
14'
9+00
S.18°-30' E.
7°-30' L.
S.26°-02' E.
Res. J.A. Smith
100'
75'
30'
5+00 Spring Branch
24" Pipe Culvert Required
50'
Began Day's Work 4-14-20
B.M. on White Oak Stump
75' L. Sta. 0+00
El. Assumed = 500.00
0+00 Near City Limit of Raleigh
on Smithfield Street

I

favorable ground. He must possess sound judgment, executive ability, and diplomacy. In general, his duties are to handle the reconnaissance, do the planning for the party, designate the duties of each man, and accept general responsibility for the whole procedure. As a rule, he works ahead of the party; determines the location of the center line, the angle points, character of the curves, size of drainage structures; and makes such notations as are necessary to furnish complete information for future use. He interviews farmers regarding general drainage conditions in the vicinity, and property owners as to their land lines, and makes all the general arrangements for the party. On construction, the chief of party usually becomes *resident engineer.*

The *transitman* reads and records all angles, bearings, and distances; and he sees that the axmen and chainmen are in line. He should be able to make sketches and take field notes neatly, speedily and in clear and concise form. It is his duty to see that the instrument is *constantly in good adjustment.* The transitman frequently acts as chief of party.

A typical form for keeping transit notes on preliminary surveys is shown in Plate I. The summation of angles to the left and to the right is given at the foot of their respective columns, and the difference of these, added to or subtracted from the first calculated course on each page, must check the last course on that page. Both magnetic and calculated bearings should be shown for each course, the needle reading being used as a check; and the instrumentman should see that these properly agree with each other before moving his instrument forward.

The courses of all lines should be referred to the *true meridian,* determined directly or indirectly by a Polaris or solar observation.

Unless a special topography party is organized for the purpose, the transitman should record neatly and clearly on the right-hand page any *topography notes* that may have influence in determining the final location, such as—the

position of streams, drainage structures, property lines, buildings, fences, trees, pole lines, power lines, rock outcrops, railroad tracks, intersecting roads, gas and water mains, etc., within or near the proposed right-of-way.

The *head chainman* carries a rod (or "flag") and the zero end of the tape, which should be held firm with one hand, while the rod is moved into line with the other. He should always put himself nearly in line before receiving a signal from the transitman; and he must be able to manipulate the tape and rod accurately and rapidly, using plumb bobs when advisable. He should exercise good judgment in selecting transit points and taking pluses. It is his duty to direct the axmen in clearing the line ahead and to see that the stakes are marked and driven correctly. A proper understanding should be had with the transitman as to signals.

The head chainman should be the most energetic man available for the place, as he regulates the speed of the party during much of the time.

The *rear chainman* holds his end of the tape over the stake last set after seeing that the tape is taut and free of kinks and that the stake is not accidentally loosened in any way. He must be careful not to obstruct the view of the transitman. All pluses to fences, roads, streams, property lines, etc., are recorded by him in a notebook and reported to the transitman at the earliest convenient opportunity. He takes care of the tape and is held responsible for its condition.

The *back rodman*, when signaled by the transitman, holds the rod plumb over the backsight station for orientation. He then comes forward immediately and brings whatever excess equipment may be on hand.

The *stakeman* carries the stakes, which should be plainly labeled with the correct station number, and driven as directed by the head chainman.

The stationing is continuous from the beginning to the end of the survey; thus Sta. 12 + 47.2 means 1,247.2 ft. from the beginning and not from the preceding turning

point. At each turning point, a hub should be driven nearly flush with the ground, and within about a foot of it, there should be a sloping guard stake on which is written the correct station number. In case the point is on a traveled road, a nail through a metal washer may serve instead of the hub, and the guard stake is set at a certain offset distance on the side of the road.

The *axmen* are supposed to clear the line of brush and trees so that the transit and level parties can get along successfully, but no trees should be destroyed that are actually not in the line of vision. When not otherwise occupied, they can make stakes.

The *levelman* following the transit party operates the level and keeps it constantly in good adjustment. Aside from running profile levels over the line, he establishes bench marks at about every 1,000 ft., and usually he keeps his own notes.

A typical set of level notes is shown in the table below, and it should be seen that the difference in the sum of the backsights (plus) and front sights (minus) equals the difference in elevation of the turning points. The right-hand page is for remarks and descriptions of turning points and bench marks.

A few engineers prefer level notes running *up* the page, in order that any sketches or cross-section notes on the right-hand page may appear to be in a better sequence.

The *rodman* carries the rod and holds it vertical on the ground at all stations and at breaks along the profile. He should not overlook readings for the elevation of near-by streams, visible high-water marks, intersecting roads, etc. The rodman carries a small notebook in which he enters the readings on all turning points.

The *topographer* should be one of the most capable and experienced men in the party organization. He will have one or two assistants. Frequently the *transitman* or *levelman* will do the work of the topographer.

The *topography* of the narrow strip or belt embracing the preliminary center line is understood to include cross

Form of Level Notes

Sta.	B.S.	H.I.	Rod	F.S.	Elev.	Remarks
B.M.	(+) 4.32	504.32		(−)	500.00	B.M. on W. Oak Stump 75′ L. Sta. 0 + 00.
0 + 00			1.3		503.0	
0 + 50			2.5		501.8	
1			6.7		497.6	
2			10.1		494.2	
2 + 10			12.01		492.31	
⊙	1.64	493.97		11.99	492.33	
3			2.0		492.0	
3 + 50			6.1		487.9	
4			11.4		482.6	
⊙	0.14	483.52		10.59	483.38	
5			0.5		483.0	Spring Branch
6 + 00 ⊙				2.31	481.21	
Totals .	+6.10		{	−24.89 + 6.10		500.00 481.21 }
Total difference			=	18.79	=	18.79

sections (or contours) and also ties to property lines and to all objects such as buildings, fences, pole lines, bridges, culverts, etc., and any other features which may in any manner influence the design of the road. Topography is advantageously taken by a party organized primarily for the purpose unless it is so scattered that it can be obtained without too much delay when running the center line. If the survey has been made as heretofore outlined, much

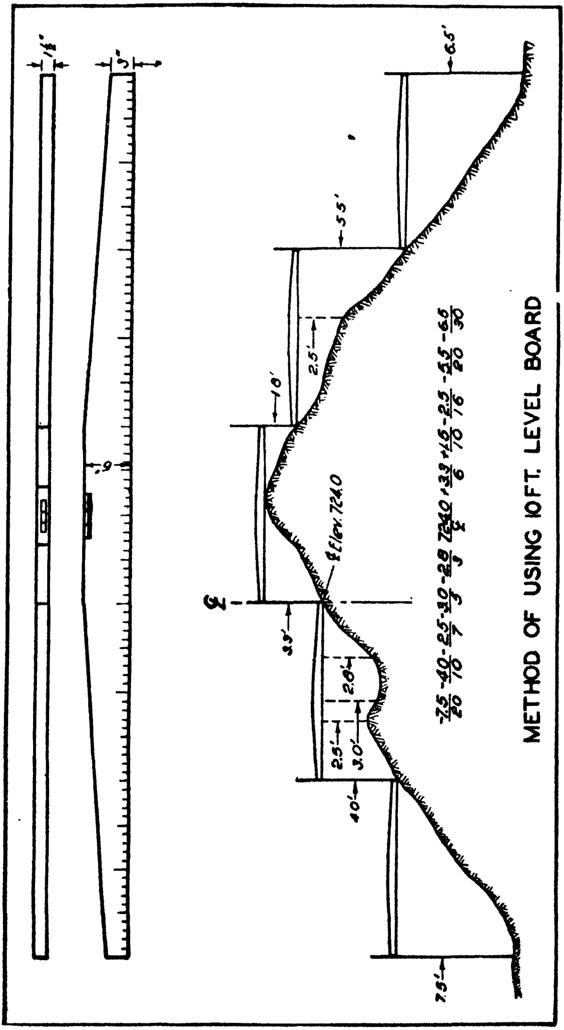

METHOD OF USING 10 FT. LEVEL BOARD
-7.5/20 -4.0/10 -2.5/7 -3.0/3 -2.8/3 724.0/2 +3.3/6 +4.5/10 -2.5/16 -5.5/20 -6.5/30
℄ Elev. 724.0

FIG. 3.

of this will have been accomplished, and the *cross-section* party can in many cases complete it.

The so-called *cross-section party* includes at least a *level-man* and *rodman*. The instruments used are ordinarily a *hand level*, *rod*, and *tape*. If the ground is not very abrupt, a *wye* or *dumpy* level is sometimes substituted for the hand level. A *slope board* and rod (see Fig. 3) has been used successfully on steep ground.

The cross sections are taken at right angles to the center line.

A typical form of *cross-section notes* is shown below. The results are given in the form of a fraction in which the numerator is the difference in elevation of a point on the ground relative to the center-line station, and the denominator is the distance from the center line. Care should be taken to use the proper sign—*plus* to indicate points above, and *minus* to indicate points below the line. Thus, observing that the notes run up the page, we see that at Sta. 28, a point 30 ft. to the left of the center line is 4.8 ft. *above* Sta. 28, and a point 35 ft. to the right is 0.8 ft. *below* this station. The intermediate points represent breaks in the ground.

Typical Form of Cross-section Notes

Sta.	Elev.		Left	℄	Right
30	115.6		$\frac{+6.5}{28}\ \frac{+4.6}{18}\ \frac{+0.9}{10}$	$\frac{0}{0}$	$\frac{-0.5}{8}\ \frac{-0.6}{16}\ \frac{-1.2}{28}$
29	118.4		$\frac{+6.1}{26}\ \frac{+4.1}{16}\ \frac{+1.0}{12}$	$\frac{0}{0}$	$\frac{-0.4}{10}\ \frac{-0.6}{18}\ \frac{-1.0}{30}$
28	120.1		$\frac{+4.8}{30}\ \frac{+2.4}{16}\ \frac{+0.5}{10}$	$\frac{0}{0}$	$\frac{0.0}{12}\ \frac{-1.0}{20}\ \frac{-0.8}{35}$
27	120.8		$\frac{+2.8}{25}\ \frac{+1.2}{12}\ \frac{0.0}{9}$	$\frac{0}{0}$	$\frac{-2.9}{10}\ \frac{-5.0}{15}\ \frac{-5.0}{32}$
26	121.2		$\frac{+1.7}{32}\ \frac{+1.2}{23}\ \frac{+1.1}{15}\ \frac{0.0}{8}$	$\frac{0}{0}$	$\frac{-3.1}{12}\ \frac{-6.1}{16}\ \frac{-5.7}{20}\ \frac{-4.4}{30}$

Some engineers have found it better to use a wye or dumpy level and record all rod readings giving actual

(and not relative) elevations of points along the cross sections.

Many State Highway Departments have a *central locating department* which directs highway surveying operations in the state through the medium of the *chief locating engineer*, whose duties are as follows:

1. To make reconnaissance of each survey either in advance of or with the party chief, although the ideal condition would be for the locating engineer to make a thorough reconnaissance before the party reports on the job, thus definitely establishing the general routing of the road.
2. To visit parties under their supervision as frequently as possible, to organize the party so thoroughly that all members may be kept busy and to see that work progresses smoothly and harmoniously.
3. To decide when the taking of contour levels is necessary to the proper prosecution of the work.
4. Where paper locations are made, to study the contour map with a view to suggesting changes of alignment, involving saving in cost and also with a view to ascertaining whether standards have been followed.
5. To suggest short cuts and revisions and to pass on difficulties encountered.
6. To investigate, as occasion arises, the objections of citizens, or groups of citizens, to the proposed location.
7. To visit proposed over- and undergrade crossing sites with representatives of the railroad company, making reports to general headquarters.

The party chiefs report directly to the locating engineers under whose direction survey is made; they also report progress to general headquarters.

18. Contour Maps

In hilly or difficult country, a contour map—which is a picture of the ground on a reduced scale—permits a comprehensive study of the area (or narrow strip embracing the line) as a guide for adjusting and projecting the final center line.

An experienced topographer will see the situations where much care is required in securing controlling contour features for projection.

In easy, rolling country, the preliminary profile and a few cross-section notes often suffice for final projection if the engineer has a good mental picture of the lay of the land.

Several methods may be used for determining contour maps, among which are: (*a*) *field location and sketching*, (*b*) *plotting from slope or cross-section notes*, (*c*) by means of *transit and stadia or plane table*, and (*d*) by means of *airplane surveys*.

a. Field Location and Sketching. Knowing the actual elevation of each station as previously determined by the level party, the position of the contours is fixed on the ground at the points where they cross the surveyed center line and where they cross a line at right angles thereto at each station; and between these points, the contours are sketched in by eye.

To illustrate the foregoing procedure, assume the "height of eye" of the levelman to be 5.4 ft. (as would be obtained, say, by actual measurement), then if he stands on the station whose elevation is 592.4 ft. (above sea level), his H. I. will be at elevation 597.8 ft. Hence to locate the 590-ft. contour, the rodman should continue moving until the rod reads 7.8 ft., and to locate the 595-ft. contour, he moves until the rod reads 2.8 ft. It will therefore be seen that the rod reading for fixing a point on the contour is found by subtracting its elevation from that of the H. I., and the rod is to be moved vertically on the ground in the direction of the center line (or at right angles thereto) until this reading is obtained. The horizontal distance to this point is then measured and recorded as part of a set of notes running up the page in the form of fractions of which the numerator denotes the proper contour and the denominator is the distance from the center-line station.

The levelman may continue this operation in any direction by moving to an established point on the contour and proceeding with a new H. I. to locate the next contour. It may expedite the work at times if the levelman himself moves forward and locates the next contour (at his feet)

by continually sighting to the rod held fast by the rodman on an established contour until the desired reading is obtained.

The contour interval is usually five feet, but it may be *more* for very abrupt ground or *less* for rather flat country.

b. Plotting Contours from Slope or Cross-section Notes. Interpolating contours from cross-section notes may be done in an office, but in irregular country, this method is unreliable, and the finished contour map should be taken into the field and visually checked.

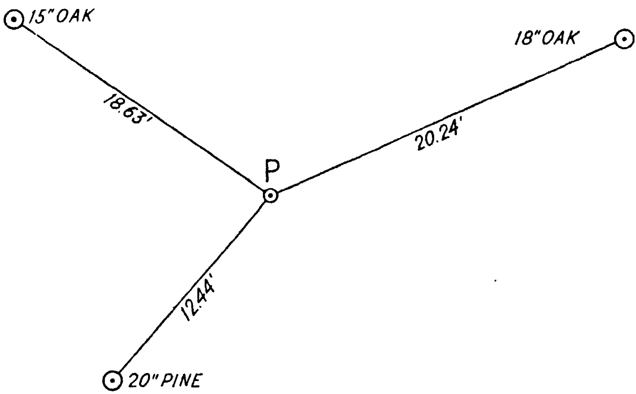


FIG. 4.

c. Contour Maps by Means of Transit and Stadia or Plane Table. This method is particularly adaptable to rather extensive areas such as those embracing bridge sites, underpasses, and intersections.

d. Airplane Surveys. Topographic maps by means of aerial photography are particularly advantageous where the country is covered with swamps, dense brush, or forests of an even height, and where a proper triangulation system exists. Such maps are unique in that they represent in detail every topographic feature, such as extent of wooded areas, streams, roads, etc., and the individual prints may be combined into a mosaic map covering an area as large as

desired. Actual contours may be obtained by the use of an aerocartograph; in this case, ground elevations must be determined at points which can be recognized on the photographs.

19. Referencing

Two ways of referencing points in order that they may be re-established, if lost, are shown below.

Method A. Measure carefully the distance from the given point (P) to three different objects, say a nail at the root of a tree, as shown in Fig. 4. It is important to choose

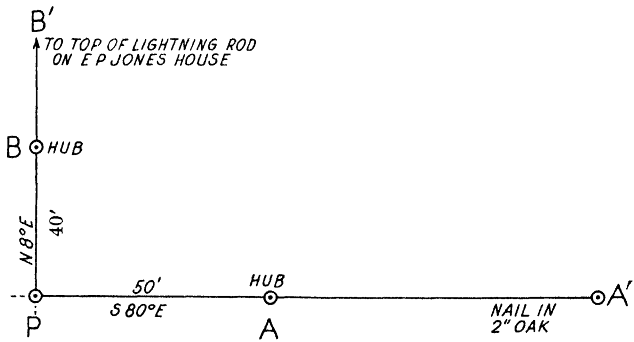


FIG. 5.

convenient objects, less than 100 ft. away, that can be found later, and that do not make acute angles with the given point (P).

This method is simple and rapid, but it should not be used in the case of final location when the points will fall within the limits of construction.

Method B. Two "tacked" hubs (A and B in Fig. 5) are set at convenient distances from point P, and each exactly in line with distant points A' and B', which may be nails in trees or preferably distant objects, such as church steeples. These hubs should be placed where they will not be disturbed during construction, and the angle APB should approximate a right angle for the best intersection.

Point P can be relocated by setting the transit at points A and B, in turn, and finding the intersection of lines AA' and BB'. Thus, supposing the transit is set at hub A; fix the line of sight on point A' and then locate two points so that a string stretched from one to the other would cover point P. Next, set the transit at hub B, fix the line of sight on B' and then determine the intersection with the string stretched in the direction AA'; which should locate point P.

20. Bench Marks and Check Levels

Bench marks should be established at intervals of about 1,000 ft., and at, or near, all drainage or underpass structures. All levels should, if possible, be based on U. S. Geological Survey or U. S. Coast and Geodetic Survey datum. Check levels must be run over the entire line, and the allowable error in profile levels between consecutive bench marks is the square root of the distance (in miles) times 0.05 ft.

21. Preliminary Maps

Preliminary maps should be drawn on a good grade of paper that will not shrink or distort. Working maps should be in sizes convenient for handling in field and office.

Some drafting during the progress of the preliminary survey may be done advantageously by field draftsmen in the headquarters of the survey party.

The map scales may be 1″ = 200′, 1″ = 100′, or 1″ = 50′, depending upon the difficulty of the country.

It is best to draw the maps with the stationing running from *left* to *right* and the lettering should be so placed that it can be read with the map right side up.

To avoid cumulative errors in plotting, draw a standard bearing across the sheet, and lay off in turn the calculated courses by the method of tangent offsets; or if there is a great deal of curvature with short tangents, plot by calculated latitudes and departures.

A study of the preliminary line and the topography adjacent to it, plotted on the map, enables the engineer to make a tentative *paper location* of the proposed route.

When this is projected on the ground, further adjustments are usually necessary to establish finally the best and most economical line.

22. Methods of Staking the Projection

The California practice is as follows:

The projected line shall not be staked by offsets from the preliminary line but by a continuous transit line based on traverses of calculated and scaled ties to the preliminary line. By this means the staked line will pass through or fall at the desired distance from the control points noted on the paper projection. It should accordingly fit the ground as expected except where inaccuracies in preliminary data and cross sections are discovered. At such places it will have to be adjusted by revision, and as these revisions become necessary, they should be made by the location party. Minor inequalities in fitting the ground may be made by grade changes, unless the grade line would thereby be distorted. It is not to be expected that a projection can be made which would not be subject to some advantageous changes during location for to attain that accuracy would involve more expense on preliminary survey than on revision of reasonably accurate projection. The method of running in the projection and reaching the final adjustment to the ground will depend upon the individual practice of the locating engineer, upon the accuracy of the base line and preliminary data, and upon the care used in projection.

Method A. The projection is run in under the personal direction of a skilled chief of party who supervises the operation of applying the calculated projection notes and traverses in the field. He must follow the accuracy with which the center line fits the ground and revise or adjust it in the field as the survey proceeds and projection errors are discovered. Advantages apparent as the line develops are incorporated into revisions. It is necessary that profile levels be carried close on the heels of the transit party to check elevation and cross-section controls. The complete alignment notes and curve data are made up as the survey progresses. In order to obviate frequent backing up and rerunning courses or curves the chief of party must also investigate the accuracy of the projection ahead of the party and recalculate same in the field.

Method B. The locating engineer depends upon the accuracy of the preliminary data to the extent that his projection can be

run in by a qualified instrumentman and survey party and not be subject to numerous readjustments requiring his constant attendance with the party. To do this he should, before considering his projection completed, review his designed alignment by a "projection in hand" inspection of the ground and, if necessary, have important control point data or cross sections checked from the preliminary line. The projection is completely traversed, and exact ties to the preliminary line indicate where they can most easily be made. Traverses of preliminary and of final line, being on a common coordinate system, are easily calculated and manipulated. Sketches of the traverse and precalculated location alignment notes can be furnished the transit party. At each field tie the calculated relation of the located line to the preliminary line is checked and proper change substituted in the traverse notes of the succeeding course to bring it back to the calculated position.

When using Method (B) it is assumed that preliminary contours, cross sections or side notes have been checked when the projection develops need for greater accuracy in tight controls, this being done before location traverses are calculated. In Method (A) the check at controls must be done in the field and projection corrected as the line is placed. The locating engineer will find it profitable to familiarize himself with the advantages and possibilities in both methods and not confine himself to one method when requisite accuracy, speed, and economy could be secured by occasionally using the other method.

23. Road Intersections

Intersections with state or county roads must be given careful study. Frequently they will materially affect the location.

The traverses, profiles, and topography on these lateral roads should be obtained for a distance of several hundred feet from the intersection.

The approaches should be sufficiently long to avoid any abruptness in grade and to provide proper curves for meeting the angle of the intersecting roads.

Special designs are usually needed for the intersection of trunk highways. Grade-separation structures should

be used where considerations of safety and economic studies involving traffic intensities will justify the expenditure.

24. Railroad Grade Separation Projects

In order that delays incident to the carrying out of a grade separation project may be cut to a minimum, it is important that those who make the preliminary surveys and prepare the subsequent plans, be guided by uniform rules which promote the progress of the improvement.

Before any hearing is held in connection with a grade separation project, a general survey is made and a "situation plan" prepared which will accurately portray the general situation and supply all the data that must be taken into consideration in the preliminary hearings and conferences dealing with the project.

Grade separation projects will be one of three types as follows:

Overhead (highway carried above railroad), subway (highway carried under railroad) and relocation (the location of the highway changed so as to avoid crossing the railroad tracks).

Aside from this last class, the use of the overhead or subway type may involve a relocation of the highway so as to effect a practical crossing.

If the highway goes *under* a railroad, the clear distance from bottom of overhead structure to crown surface of roadway should be at least 14 ft., if it goes *over* a railroad, the minimum clear opening vertically above the top of rail should be 22 ft.

All plans for under- or overhead crossings, which should in all cases be based on complete field surveys, must be approved by representatives of both the highway department and the railroad company.

The following procedure is the practice of the Ohio State Department of Highways:

a. Highway Information. Station 0 + 00 of the survey shall be located in the center of the system of tracks crossing the highway and the stationing shall be carried each way along the highway from this point, designating each station by its

appropriate direction from the crossing. For example, Station 6 + 50 North indicates a point in one direction from the center of the track system and station 6 + 50 South indicates the same distance in the other direction on the same survey. By the center of the system of tracks is meant the center line between tracks for double track, the center line of the middle track in the case of three tracks, etc.

The survey shall determine the following information as affecting the highway:

1. The location of the existing center line of the highway affected by the improvement.

2. The angle of intersection between the highway center line and that of each track if same are not parallel.

3. Pluses to the center line of each track crossing the highway, railroad right-of-way lines, property lines, side roads and drives, center line of bridges and culverts, manholes, or other similar structures, houses, buildings, and any other features affected by the improvement.

4. Stadia location of trees, pole lines, transmission towers and other general topography, together with contours carried over an area sufficiently large to show the nature of the terrain within the scopes of the improvement.

5. The ownership and location of abutting property or other property likely to be affected and the location of all houses, buildings, platforms, or other permanent structures together with the elevation of porches or lower floors and a note on the type and condition of the building especially as to foundations and basements.

6. The width and type of the highway crossing of the tracks with the top of rail elevation of each rail.

7. The type, width, and thickness of existing pavement and the width of berms and ditches, within the probable limits of the project.

8. The location of all existing safety signs and signals, drainage lines, drainage structures, gas lines, water lines, sewers, telegraph, telephone or power lines, traction lines, guard rails, etc.

9. Record of any franchise, permit or agreement which affects the occupancy of the highway by any private corporation or individual.

10. Location of streams and size of openings thereon, together with the drainage area, and approximate rate of fall per mile,

and high and low water information of any stream desirable as an outlet for subway drainage.

11. A profile of the existing highway and any relocation thereof, profiles of side roads and driveways and the profile of the site of a temporary detour if known.

b. Railroad Information. 1. The location of railroad tracks, right-of-way lines, switches, stations, buildings, interlocking towers, and other facilities likely to be affected by the improvement.

Stationing along the railroad shall be run each way from the zero station of the highway survey and shall be designated in the same manner.

2. A profile of the tracks to the extent required for the improvement, including industrial tracks.

3. The location, type, and profile of any road crossing the tracks in the vicinity, with information on any overhead structure or subway that might restrict a change of grade of the tracks.

The extent of the information gathered on the survey must be governed somewhat by the type of the improvement contemplated.

If, when the survey is made, it is positively known that a particular type of separation is the only one which can be built, the judgment of the man in charge of the survey must be relied upon to gather adequate information on pertinent factors and omit those which do not apply.

If the type is indeterminate at the time of the survey, then it is the duty of the field party to secure as much information as possible over the territory which would be involved in any type that might be considered.

Situation Plan. The survey information outlined above shall be shown on a situation plan, which, for convenience, may consist of two or more sheets. The scale should be 1″ = 50′ if practicable; if not, a scale of 1″ = 100′ will be acceptable. A vertical scale of 1″ = 10′ should be used on profiles. In order that there may be available space for adding the profile of the proposed improvement, the profile should be so located on the drawing as to leave space for 30 ft. of elevation above or below that of the tracks at the proposed crossing. In case the proposed grade crossing elimination involves a relocation of a substantial length of highway, the scale of the drawing should be reduced so as to permit the entire affected relocation to be

shown on one sheet. This should be done even if a scale of 1″ = 1,000′ would be required. In such cases, however, an additional sheet or sheets should be prepared at the usual scale showing the detailed conditions surrounding any crossing or crossings that are proposed to be eliminated.

This situation plan should be platted with the north point generally toward the top of the sheet, modified so as to permit the alignment of the highway to be approximately parallel with the top and bottom border line. The crossing which is to be eliminated should be placed near the center of the drawing. It will generally be found desirable to devote one sheet to the highway alignment profile and pertinent information and another to showing information pertaining to the railroad. As our survey will usually develop less information to be shown in connection with the railroad than with the highway, it is advisable to include on this second sheet, profiles and other pertinent information as to auxiliary highways that may be affected by the proposed improvement.

The survey should not only be platted accurately but all information such as angles, essential distances, curvature, etc., should be set forth in detail. The name of the railroad company or companies should be shown with directional indication toward the nearest important cities to the crossing.

A title for the plan is to be placed in the lower, right-hand corner as follows:

State of Ohio
DEPARTMENT OF HIGHWAYS
Situation Plan
Crossing No.......
—With—
The..............................Railroad
Located.....................................
S. H............Sec..................County
Scale As Shown..............Date...........
Sheet 1 of..........................Sheets
Surveyed By..........Platted By.............
Checked By.....................

When the situation plan has been completed as outlined above, consultation should next be had with the Bureau of Bridges and Railroad Crossings before any proposed features of

the contemplated improvement are added to this drawing. As the plan finally adopted in connection with these projects must be agreeable both to the State Highway Department and to the affected railroads, it is frequently a waste of time to develop details in advance of a hearing, at which the thought of the railroad company is developed.

When the general type of separation has been agreed upon and the limits of the project have been determined, further plans will be prepared in detail. These latter plans should conform, so far as any highway work is concerned, with the general requirements for a Federal Aid Highway Project.

In making surveys and preparing the situation plan for the reconstruction of an existing separated railroad crossing, all pertinent information should be secured and shown in the same manner as for grade separation projects and in addition thereto the following information shall be secured:

a. A complete description of the existing structure, including vertical and lateral clearances, width of roadway and sidewalks and structural defects.

b. A record of the ownership of the structure and by whom it was built.

c. A record of any legal proceedings, agreements or other documents under which the construction was carried out.

As all negotiations with the railroad company are carried on by the Bureau of Bridges and Railroad Crossings, it is necessary that during the development of these plans, the division and resident engineers' offices keep in close touch with this Bureau, and should avoid the giving or receiving of information from affected railroad companies, traction lines, or public utilities involved in the project.

25. Railroad Grade Crossings

Ohio practice is as follows:

The Bureau of Bridges and Railroad Crossings has been designated to coordinate various phases of grade crossing matters between the several railroad companies and this Department, since this Bureau has constant contact with the various companies and is in position to handle negotiations with them expeditiously.

Its object is to arrive at agreements with the railroad companies in establishing a fixed grade at all existing crossings on

new construction projects, or on other projects involving a material change in grade of rigid or semi-rigid type paved approaches to crossings, or involving any change in elevation of railroad tracks. The purpose of such agreement is to arrive at a mutually satisfactory grade for such crossings in order to avoid future changes in track elevations to the detriment of the highway.

Agreements for the establishment of new grade crossings or relocation of existing crossings are similarly handled in conjunction with the legal proceedings necessary for such projects. The General Code of Ohio requires the consent of the Court of Common Pleas, of the particular county, to such new or relocated crossing, and any plans or work completed prior to such consent are presumptive of the court's action and may result in scrapping of such plans or work, or at least result in delay through injunction.

It is not the purpose of this Bureau to establish fixed grades for crossings on trafficbound roads, unless the relative positions of rails require adjusting to provide a satisfactory crossing profile, or unless built up conditions adjacent justify such action. Likewise, this Bureau has no jurisdiction over the maintenance of approaches, and when a project adjacent to a crossing is completed, routine maintenance matters should proceed as usual.

It is presumed that the division engineers or their local representatives can work out with local officials of the railroad companies, matters pertaining to the extension or routine maintenance of crossings and approaches, although the Bureau of Bridges and Railroad Crossings is always willing and ready to assist in such matters. In this connection, it should be remembered that the maintenance and construction of crossing pavements between the outside ends of ties are obligations of the railroad companies, and no authority exists for the expenditure of public funds for such work within the highway limits.

In order that a standard practice in regard to grade-crossing matters may be formulated, the Bureau of Bridges and Railroad Crossings has adopted the following rules on projects coming under its supervision: (1) Crossing pavements (between outside ends of ties, and where there are no sidewalk requirements) should be two feet wider on each side than the adjacent pavement, to afford leeway for the driver off of the edge of pavement. (2) Where practicable, approach grades to crossings shall not

exceed 2 per cent within 50 ft. of the near rail. (3) Highway grades shall meet rail elevations without a break in grade, care being taken that the same condition applies to pavement edge grades where a skewed crossing exists. (4) Where possible, curved alignment should be avoided across tracks unless the superelevation of the tracks and the proposed highway superelevation can be made to coincide. (5) Flasher lights, wig-wag signals, or other warning devices, unless suspended over the highway, should be set a minimum of two feet clear of the pavement (the maximum clearance being left to the railroad officials as such signals are their obligation).

To expedite the reaching of agreements with the railroad companies, a site plan showing in detail, present and proposed construction should be submitted to the Bureau of Bridges and Railroad Crossings about a month prior to completion of construction plans, or as soon as possible after it is determined that a project will proceed. Such site plan should be on form GC-10, transparent paper—copies of which will be furnished on request.

As blocking a crossing for crossing repair work, or raising rail levels to the detriment of the highway without the permission of the Director, is prohibited by law, all employees of the Department should promptly report to the respective Division Engineers all such violations, with details of the work being done. Division Engineers, unless authorization has been made for such blockade of highway or raise of track, shall promptly report same to the Bureau of Bridges and Railroad Crossings, and secure the service of the local police (in case of municipalities), or the sheriff or State Highway Patrol to see that such unauthorized blockade or track raise is stopped and corrected.

Details of any work done by railroad companies on their crossings, or details of any work affecting material changes in approaches to such crossings should be furnished the Bureau of Bridges and Railroad Crossings promptly upon completion, so that grade crossing survey records may be kept complete and up to date.

In general, any project likely to lead to extended negotiations with a railroad company, should be referred to the Bureau of Bridges and Railroad Crossings before being initiated, in order that the proper procedure for such negotiations may be worked out, and a consistent policy maintained.

26. Area of Waterway for Culverts and Bridges

The fundamental factors involved in estimating the *area of waterway* for culvert and bridge openings sufficient to take care of flood conditions are: (1) *rainfall* (or runoff from melting snow); (2) *size, shape, and slope of the drainage area;* (3) *nature of the ground,* whether rocky or porous; and (4) *the design of the waterway itself.*

Studies should be made of local conditions, including maximum flood height, frequency of its occurrence, size and performance of other openings in the vicinity carrying the same stream, characteristics of the channel and of the drainage area, climatic conditions, available rainfall records, and any other information pertinent to the problem and likely to affect the safety and economy of the structure.

Empirical formulas taking some of the foregoing factors into account may be used, but they should not displace *careful field observations* and the exercise of *intelligent judgment.* Talbot's formula (see Table 17) and the Burkli-Ziegler formula have been widely applied. The watershed boundary may be traced on topographic maps, if such are available; or it may be run out in the field, and the area computed.

The Burkli-Ziegler formula takes into account rainfall in addition to the size and condition of the drainage area:

$$Q = CR\sqrt[4]{\frac{S}{A}}, \tag{1}$$

in which Q = the quantity of water that reaches the culvert in cubic feet per second per acre;

R = the average rainfall during the heaviest fall, in cubic feet per second per acre (1 in. of rainfall per hour is closely equal to 1 cu. ft. per second per acre), and the value of R depends upon rainfall conditions and varies in practice from 2 to 10, with 3 to 4 as a general average;

S = the average slope in feet per 1,000 ft. of the ground surface within the drainage boundary;

A = drainage area, in acres;
C = a constant which depends upon the topographic and climatic conditions, the average values being as follows:
C = 0.20 for rural areas;
C = 0.30 for municipal areas with macadam streets;
C = 0.75 for municipal areas with paved streets.

The Burkli-Ziegler formula is not so widely used for culvert design as the Talbot formula, but it is more appropriate for use in connection with the design of sewers which receive the drainage from municipal areas.

A comparison with *existing* culverts and bridges will give valuable information. It is not sufficient to know merely the size of opening provided by an existing structure, but there should be information secured as to whether that waterway is *deficient* or *excessive;* whether it carries a part or all of the flood waters; how the drainage area at an existing bridge compares with that at the proposed site; and whether the stream bed offers equal resistance to scour at the two locations.

Considering an *ordinary* flood to be the largest in an average three to five year period, and an *extreme* flood to be the largest on record; Ohio practice is as follows:

> In the case of a bridge at a rural location, it will generally be sufficient to provide a waterway area adequate for *ordinary* floods, if reasonable provisions are made to safeguard the structure against damage due to the erosive velocity and high water level incident to extreme flood; but for a bridge in or very near a municipality or at other locations where a restricted waterway opening may result in heavy damage to persons or property, provision for *extreme* flood conditions shall be given serious consideration.

The foregoing shows the lack of definiteness which characterizes waterway determinations and should emphasize the importance of complete and reliable data, so that all information necessary will be at hand for applying a

given method of computation, and further, that several methods may be compared.

27. Culvert Surveys

The survey for a culvert or bridge of 20-ft. span, or less, should be made with sufficient thoroughness to provide all information that is needed for the preparation of the plans. The dimensions, elevations, and angles should be taken carefully and recorded so that the structure will accurately fit both the road and the stream, and that the quantities can be correctly computed. The subsoil should be investigated in order to determine if any special foundation provisions must be made. The adequacy of the existing structure (or a portion of it) should be fully investigated as to its strength, roadway widths, skew, waterway opening, and suitability for use as a whole or in part.

In case of bridges with spans *beyond* 20 ft., or a difficult smaller structure, the foregoing information should be obtained in even greater detail as would be required in the preparation of a site plan (see Art. 28).

28. Bridge Surveys and Investigations

The object of a bridge survey is to secure information on which to base decisions on certain fundamental points which determine the present efficiency of an existing bridge, or which govern the design of a new bridge. A survey should always contain sufficient data to fix definitely the location, elevation, and design of all parts of the structure. The suitability of the design that is prepared will depend upon the correctness and the completeness of the notes which are supplied by the survey.

The following procedure as applied to (*a*) *existing bridges* or (*b*) *proposed bridges* is the practice of the Maryland State Roads Commission:

a. Existing Bridges. When details of an existing bridge are wanted and no plans of it are available, it will be necessary to measure the structure in the field and simultaneously to draw it legibly in a notebook, to approximate scale, as a means of avoiding omissions and errors of measurements. Complete details of an existing bridge will be required as well as exact

information regarding its present condition; so that from the field data submitted an analysis of the stresses in the structure under a given loading can be made in the office.

In giving a measurement, always show clearly the exact point to which it was taken. Dimensions and spacing of floor beams and floor stringers are particularly important. In addition to the detail dimension sketches of superstructure and substructure (the latter so far as possible), *information* should be secured as to when the bridge was built and by whom; present condition of details of construction and materials used; evidence of scour, backing up or overflow; direction of flow; complete information regarding high water stages, and, if possible, dates of their occurrence; the effect of natural forces, such as salt air or salt water, teredo, temperature and settlement on the action and life of the bridge; weights and kinds of loads passing over the structure; and elevations of top of floor, bottom of stringers, water surface, and stream bed.

Alignment and grade of the stream, of the structure, and of the roadway approaches should be determined. The angle of skew to be recorded is the smaller angle between the highway center line and the face of the abutment.

Every effort should be made to obtain depth of footings, character of foundations and thickness of abutment walls, pick and shovel or earth auger being used where necessary. Careful note should be made of the general *condition of the structure*, such as settlement, movement or cracks in foundations, abutments or piers; loss in area of steel from rusting; amount of rot, shakes and other defects in timber; texture of concrete, spalling or other signs of deterioration; condition of floor; the type and action of expansion joints, expansion rollers or shoes; buckled or loose steel members or connections; and kind and condition of paint.

Where an existing bridge is to be *retained* by being repaired, widened, or lengthened, the masonry and other parts should be measured very carefully and with much detail so that it can be duplicated from the notes. Measurements should be accurate and dependable to the nearest inch.

Photographs should be taken to show the location and construction of the bridge as a whole, as well as the most important details.

Finally, careful and complete *recommendations* should be included, from a study of field conditions, as to the manner or

method of improving, reconstructing, or otherwise modifying the existing structure; or, when so thought advisable, the reasons for abandoning the present structure and its replacement by a new one.

b. Proposed Bridges. Having determined the necessity for a new bridge, exact and detailed site and construction data must be obtained for the proper selection of the type, span, roadway width, location, skew, grade elevation, etc., of the proposed bridge, and for the purpose of preparing final designs and specifications.

Unless the bridge alignment is fixed by an existing road or by the location of a road survey, it is necessary to secure a proper *site* for the proposed bridge. Before starting the survey, a reconnaisance of the stream should be made for a distance above and below the tentative center line. The best location for the bridge is at a point where the stream is narrowest and where foundations can be secured at highest elevation. Skewed crossings should usually be avoided. A slight shifting of the line may often secure a crossing at approximately right angles to the stream.

In general, the location should be such that sharp curves in the alignment will not be introduced near the ends of the bridge tangent. A location that would require a heavy grade on the structure should be avoided whenever possible. When grades are being laid, allowance should be made for clearance between the bottom of the trusses or girders and the floor, and also for clearance above high water. The clearance from high water to top of floor should be 5 ft. or more, dependent upon the character of bridge and size of stream. In general, the larger streams require more clearance to allow free passage of driftwood.

The center line should be carefully referenced so that it may be readily found when the bridge is staked out. Bench marks should be definitely located and accurately described and should be located in such places that they will not be easily displaced during construction of the bridge and the adjacent highway. The datum of the bench marks should always be given, and also whether it is that of the U. S. Coast and Geodetic Survey, the U. S. Geological-Survey, or assumed.

Profiles should be taken of the center line and on a line parallel thereto and offset 20 ft. each side. If there is much

irregularity, sufficient cross sections should be taken so that an accurate contour map can be drawn of the proposed site.

In case the grade of a bridge is to be determined by the roadway grade, it will be necessary to secure the roadway profile for several hundred feet each side of the bridge site.

A traverse of the stream for a distance of 500 ft. each side of the center line of location should be made so that its general course can be determined. Where a channel change can be made to advantage, this should be noted and described. The topography should include all structures, islands, shoals, boundaries of wooded lands, buildings, fences, and all works of man. The north point, as well as the direction of flow of the stream, should always be shown.

Elevation of normal water and extreme high and low water should be determined and given. The dates of extreme high water and sources of information are also required. For navigable streams, the mean or average high and low tide should be determined and care taken to see that these levels check the datum of the U. S. Geological Survey or the U. S. Coast and Geodetic Survey. The datum of the Geological Survey is average tide while that of the Coast and Geodetic Survey is mean low tide.

An estimate of the velocity of current, in feet per second, at high water and normal water should be made. In mountain streams this is not always possible. In navigable streams, the velocity and direction of the current at mid flood and mid ebb tide should be obtained by the use of rods or floats. The character of the soil at the bed of the stream is often an index of the velocity during maximum flood conditions. This should always be noted.

Any evidence of erosion by the stream, particularly along the banks, should be recorded as such evidence affects the layout of the structure.

The amount and character of driftwood and ice should be determined and noted. Drift may be classified as heavy, medium or light. Heavy drift may be considered such as requires a horizontal opening greater than 25 ft.; medium drift such as will pass an opening of 12 to 25 ft.; and light drift, such as will pass an opening less than 12 ft.

At locations over tidal streams, it is important to know whether or not the water is brackish. Nearby existing struc-

tures should be examined to see if there is any evidence of the presence of the teredo.

A preliminary investigation should be made to determine the depth and suitability of *foundation material.* If rock foundation is present, and not at a considerable distance below the bed of the stream, a profile of the rock surface can be easily determined and will be sufficient. If rock is not present, sounding rods or augers should be used to determine the character of the foundations; and all of the area which is likely to be included by a bridge structure should be explored. Other material besides rock, such as a deep bed of gravel or firm clay, may be considered adequate for foundation. When taking *wash borings*, the vertical distance occupied by each stratum of material should be determined, elevations given where changes occur, and samples taken and put in small bottles for this purpose.

Soundings should be carried to a depth sufficient to insure against incorrect footing design. For larger structures, this is more important than for smaller ones; and where an arch bridge is contemplated, it is of utmost importance. The selection of the type of structure may depend largely upon the information submitted concerning the foundation soil.

The depth to which soundings should be taken will be governed by the nature of the material encountered. Where the type of bridge depends upon a rock foundation, sufficient soundings should be taken to determine accurately the elevation of suitable rock and to prove the existence of sufficient depth of rock below the proposed foundation.

When the use of piling is anticipated on account of soft foundation material, or as protection against failure from scour, the depth of soundings should be governed by the nature of the material encountered. As it is not desirable to use piling of a length less than 10 or 12 ft., it is important that soundings should establish this depth of penetration as a minimum. They should be carried as much deeper as the softness of the material requires.

The availability and quantity of *local material* suitable for concrete aggregate should be noted. The location of near-by quarries or deposits of stone suitable for rip-rap and masonry is important. It is also desirable to know whether lumber may be obtained locally. Where bridges are to be constructed

as separate projects a determination should be made as to the availability of material for the approach fills.

A standard form has been prepared which provides blanks for stream information, as well as construction data. Every item called for on the form must be filled in with the best information obtainable.

The Location Survey

29. Definitions

The location survey consists of laying out on the ground the final center line which has been projected on paper or otherwise predetermined. The tangents, curves, and drainage structures are established by means of a continuous transit survey, and complete data are obtained for making plans, profiles, and cross sections, from which the yardage of excavation and embankment and various other quantities can be determined.

30. Field Notes

The field notes must be clear and concise, yet comprehensive enough for making accurate and complete plans of the project.

It is important that the levelman must check every item of his notes, and that check levels must be run over the entire line. Slight deviations, within permissible limits, may be adjusted over the line.

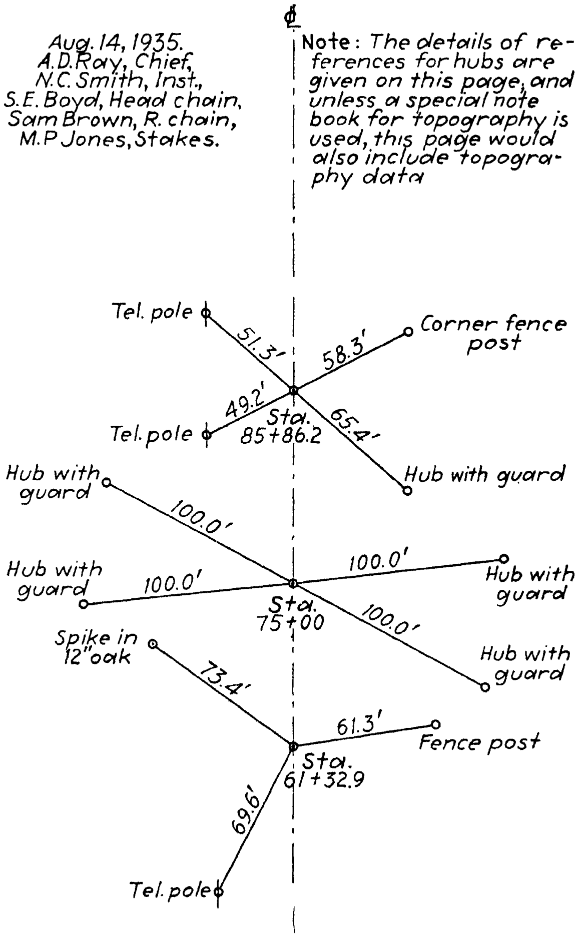
The transit notes are kept in special field notebooks. Many engineers prefer to use duplicate loose-leaf notebooks. Referring to Plate II (page 58), the left page is used for recording stations, angles, curve deflections, curve data, and magnetic and calculated bearings; the right page is used for topography, sketches, details of references for hubs, location and bearings of all property lines crossing or touching the right-of-way, names of property owners, and anything which will affect the cost of construction. All curves should be marked "open" or "blind," as the case may be.

An accuracy of at least 1 in 3,000 should be attained in the chaining. Double-reversal methods should be used at

TRANSIT NOTES

Point	Sta.	Deflections	Mag. Bear'g	Calc. Bear'g
		Note: The curve deflections to each full station point would also be given in this column.		
			N 44° 00′ E	N 42° 04′ E
S.T.	93 + 43.4			
C.S.	87 + 43.4			
P.I.	85 + 86.2	$\Delta = 49° 36' R$	Spiralized Circular Curve: $D_c = 5°$, $\theta_s = 15°$, $\Delta - 2\theta_s = 19° 36'$, $l_s = 600'$, $L_c = 392'$, $T_s = 834.8'$.	
S.C.	83 + 51.4			
T.S.	77 + 51.4			
			N 5° 30′ W	N 7° 32′ W
P.O.T.	75 + 00			
			N 5° 30′ W	N 7° 32′ W
P.T.	67 + 92.4			
P.I.	61 + 32.9	$\Delta = 13° 15' L$	Circular Curve Data: $D = 1°$, $R = 5730'$, $L = 1325'$, $T = 665.5'$, $E = 38.5'$.	
P.C.	54 + 67.4			

PLATE


Aug. 14, 1935.
A.D. Ray, Chief,
N.C. Smith, Inst.,
S.E. Boyd, Head chain,
Sam Brown, R. chain,
M.P Jones, Stakes.
Note: The details of references for hubs are given on this page, and unless a special note book for topography is used, this page would also include topography data
Tel. pole
51.3′
58.3′
Corner fence post
49.2′
Sta. 85+86.2
65.4′
Tel. pole
Hub with guard
Hub with guard
100.0′
100.0′
Hub with guard
Hub with guard
100.0′
Hub with guard
Sta. 75+00
100.0′
Hub with guard
Spike in 12″ oak
73.4′
61.3′
Fence post
Sta. 61+32.9
69.6′
Tel. pole

angle points and at intermediate transit points on long tangents. The calculated bearings should be based on sun or polestar observations, and "mean sea level" (as determined by U. S. Coast and Geodetic Survey) should be used as a datum for leveling if possible. Proceeding in this manner, the highway location survey notes, when properly tied in with prominent marks of the terrain, supply data for permanent maps of the country.

Transit points (hubs) should be referenced so that they may be relocated readily. In addition to the precise referencing of intersection points, the beginning and ending of all curves, points on long tangents, and the place of beginning and ending of a survey should be referenced generally as to location, by giving the approximate distances to near-by towns or other well-known local points.

If the center-line points come on an existing road, a parallel offset line should be established for guidance during construction.

On the front page of the notebook, opposite the flyleaf, should appear the name and number of the survey. Certain information of a very general nature will be given here, such as the description of the survey, the names of the members of the party, the notebooks used, dates, and the initials of the field computation checkers.

The indexing of field books should be kept up as each survey is completed, and all notes should be intelligently and clearly entered, so that the same may be readily worked up by others who are not familiar with the survey.

31. Equalities

Equalities in station numbers should be avoided as much as possible, but when they are absolutely necessary they should be carefully explained on the flyleaf, so that a total stranger can understand them. Thus: Sta. 374 + 12.5 of this line = Sta. 342 + 16.4 of the line ahead, or, if lines are designated by letters, Sta. 374 + 12.5 of line *A* = Sta. 342 + 16.4 of line *B*, ahead.

It is even more important still to avoid equalities in level datum. For this reason it is desirable, whenever

practicable, to base all levels on sea-level datum, as obtained from government bench marks. When this is impracticable, a datum may be assumed, but this datum must be adhered to throughout the survey, and no equalities in datum will be permitted which are due to errors in the levels, unless they are satisfactorily explained on the flyleaf, and even then they are very objectionable and could scarcely reflect credit upon the engineer responsible for same.

In "tying" one line into another, the new line should be run for at least two courses over the old line, and the equality of stationing noted at each hub in the field book, and otherwise explained, so that anyone can understand or plot the work. These equalities should also be marked on stakes securely driven near each hub.

Errors of closure in levels and alignment must be carefully and *conspicuously* noted on the flyleaf, and reference should be made to the book and page showing the original courses or bench marks at the place where the tie-in is made.

32. Soil and Material Notes

In the case of new construction, notes should be made of the type of materials at various points along the survey, as indicated by soundings or borings, for the purpose of estimating a preliminary classification of the relative quantities of earth, loose rock, hardpan, or rock to be encountered.

Available materials for construction, stone quarries, and gravel pits, with distances and character of road to be traveled over, should be noted. Often a pit or quarry unknown to county authorities or division engineers is located in this manner.

33. Right-of-way Notes

The following items are considered in preparing a right-of-way plan: location of boundary lines, full names of property owners, classification of lands through which the survey passes, area of land cut off, location of water affecting pasture rights, stations and pluses of property lines, area (in acres) of right-of-way to be purchased, legal description

of the same, ties to at least one monument, and net length of center line across the property.

Where trees, buildings, wells, and other features of similar nature, are within the right-of-way lines and the removal or changing of which affect property settlements, these items should be carefully shown on the plan to their true location and dimensions.

It is well to check the name of the property owner in the county records, and investigate title for liens. Also fill out the tax valuation section of the appraisal report from the county records.

Forms for easement titles to highway rights-of-way have been prescribed. This type of conveyance is usually a perpetual easement for highway purposes only. Property ownership, outside of municipal corporations, extends to the center of the road.

34. Cross Sections

Cross sections should be taken at each 100-ft. station, at all breaks in the center-line profile, and oftener if necessary to determine more accurately the volumes of excavation and embankment. Cross sections should extend for a distance on either side of the center line to allow the road templet to be superimposed upon them, whether the grade of the road at that point involves cut, fill, widening, superelevating, or shifting of center line.

Cross sections should also be taken at each culvert, and the distance out should be extended in case a relocation or cleaning out of the runoff channel is contemplated.

There is a tendency to take too few cross sections, particularly in rough country where rapid changes occur from cut to fill. The computation of quantities from cross sections is, at best, only approximate, and hence the field data should be as accurately representative as possible.

The use of a 10-ft. level board is recommended on very steep slopes. It is more accurate than the hand level and requires less time and fewer men in the party.

All projects which encounter a large amount of excavation or on which the director has a reason to believe the con-

tractor has not conformed to the specified cross sections, should be *re-cross-sectioned*, and the actual amount of yardage should be computed before the final estimate is paid. This cannot be done however with fairness to the contractor unless the original cross section was carefully taken, slope stakes accurately set, and the re-cross-section taken directly over the location of the original cross section.

A complete record should be kept of any changes of grade and alignment and of any authorized additions or deductions of yardage during the progress of the work.

Borrow pits, channel changes, etc., must be carefully cross-sectioned before the contractor is allowed to begin work on same. Upon completion of these, they should be re-cross-sectioned for final estimate of yardage.

35. The Grade Line

Trial grade lines should be drawn in pencil until the final line is established. As a rule this grade should be laid with a view of balancing cuts and fills. If the gradient is such that balancing can be accomplished without exceeding hauls of 500 ft., and the requirements of visibility and general appearance are satisfactory, then it is almost certain that the balancing will be economical.

Wherever possible, the balancing of cuts and fills should be made in such a manner as to avoid waste or borrow. Ordinarily, the quantities of cuts should exceed that of fills, from 10 to 20 per cent (see Art. 30). In light work, spongy soils, heavy sod, or vegetation, a special allowance should be made.

Information as to the unit cost of handling excavated material is necessary in order to determine whether balancing, wasting, or borrowing is most economical. Wasted materials can in nearly all cases be used to widen fills.

Level grades should be avoided if possible, as a slight fall insures better drainage. Vertical curves of considerable length should be introduced where the total change of grade exceeds one-half to one per cent. Short choppy grades should in all cases be avoided.

The grade line is often fixed by requirements of greater importance than that of balancing cuts and fills. Thus:

1. In rock work it is generally cheaper to have the embankment quantities in excess and borrow earth rather than cut down into the rock deep enough to obtain the balance.

2. It may be cheaper to borrow dirt rather than pay for the cost of overhaul.

3. In towns the grade may be adjusted for street intersections or lowered to receive drainage from the abutting property, while in flat, swampy country it may be raised so that the fills are in excess, in order to give better foundation and drainage.

4. Necessary maximum rate of grade to reach a summit.

5. Necessary approaches to bridges, underpasses, or intersecting roads.

36. Construction Stakes

When actual construction is about to begin, it is the engineer's duty to see that a sufficient number of substantial stakes are set correctly for the guidance of the contractor.

a. Center Line. The location of the center line as shown on the plans is the first operation of the construction survey. As the survey progresses, all angles, measurements, and bench marks should be checked; and the stakes that have been destroyed should be reset. All points of intersection should be carefully witnessed to permanent objects located outside of the working area of the project.

b. Slope Stakes. The slope stake (see page 82) is the contractor's guide in making his cuts and fills. These stakes are set for all heavy grading sections and on any section where the contour of the ground is such that the contractor would have difficulty in following grade stakes.

On sections having both cut and fill, the "grade point" should be staked.

The attention of the contractor should be called to the importance of observing the cuts and fills as shown on the

slope stakes where superelevated curves are encountered, to preclude the excavation being carried too deep.

c. Grade Stakes. In case the grading is light and no slope stakes are required, grade stakes are set before excavation begins, at 50-ft. intervals on each side of the road on tangents, and at 25-ft. intervals on superelevated curves. But if the grading is heavy, the grade stakes are set when the contractor is ready to begin his fine-grading operations.

In pavement construction, the top of the stake should represent the grade for the forms. When old macadam or hard materials are encountered, a steel drift pin should be used to make an adequate opening for the stake. When the stakes are driven far enough to insure their tightness, they may be sawed off so that the top is at grade.

37. Special Surveys

Surveys for minor improvements, such as partial reconstruction or resurfacing of the highway may come under this classification. Special surveys also include land surveys for (1) right-of-way descriptions (see Art. 33), (2) grade-crossing eliminations (see Art. 24), (3) site plans for drainage structures (see Art. 28), and (4) subsoil studies.

Plans and Profiles

38. Explanation

Much of the work in connection with highway planning is usually done on paper. This calls for maps of the area upon which the proposed route is tentatively projected before being finally established on the ground.

Then on the basis of field surveys, data are available for preparing a detailed set of plans (or maps) including title sheet, typical pavement cross sections, plans and profiles, cross sections of excavation and embankment, structural plans, summary sheets, and such departmental standards as are necessary for the project.

All plans in accordance with the U. S. Bureau of Public Roads Standards are on sheets measuring 22 by 36 in. with a binding margin two inches wide on the left-hand edge.

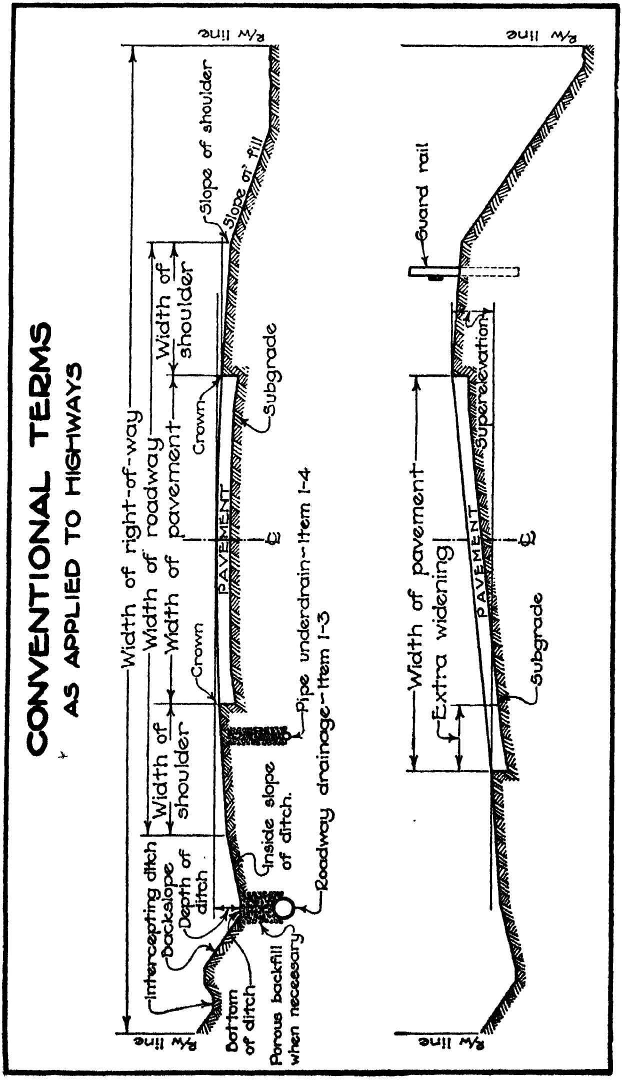

CONVENTIONAL TERMS
AS APPLIED TO HIGHWAYS
Width of right-of-way
Width of roadway
Width of pavement
Width of shoulder
Width of shoulder
Crown
Crown
PAVEMENT
Subgrade
Slope of shoulder
Slope of fill
R/W line
R/W line
Pipe underdrain—Item I-4
Roadway drainage—Item I-3
Inside slope of ditch.
Intercepting ditch
Backslope
Depth of ditch
Bottom of ditch
Porous backfill when necessary
Guard rail
Superelevation
Width of pavement
Extra widening
PAVEMENT
Subgrade
R/W line
R/W line

PLATE III.

39. Scales

The plan is usually drawn to a scale of 1″ = 100′; while the profile of the center line has a scale of 1″ = 100′ horizontally and 1″ = 10′ vertically. The yardage cross sections are drawn to a scale of 1″ = 5′ (or 1″ = 10′) both horizontally and vertically. Variations from these scales occur in special cases, depending on the topography, type of work, and the data required.

40. Typical Sections

A typical cross section of the pavement should be placed on the first sheet directly following the title page. This section, on a suitable scale, should show the various slopes for cut and fill, the crown of the finished pavement, the width of shoulders, the thickness of the various courses of pavement or resurfacing, the position of the profile grade line, and the center line of the right-of-way and of the pavement.

Widening and superelevation tables must be made for each curve and should be shown on special sheets immediately before the yardage cross-section sheets.

Plate III shows a typical cross section with conventional terms applied to highways.

41. Data on Plan

The plan of the road shall show the center line of construction; bearing of tangents; angles of intersection; station of beginning and ending of curves; full data for each curve including intersection angle; equations of stationing; streams and railroads on or within 500 ft. of the center line; property lines; names of property owners; intersecting roads; location, size and purpose of all pipe lines; location, size, and direction of flow of present and proposed culverts and bridges; structures affected by the proposed construction; the location of all unusual and special features, such as guard rail, retaining wall, ditch protection, manholes, catch basins, inlets, and connections.

The plan sheets shall show the balance points of yardage, together with a subsummary of the same between each

balance point. These plans shall also include excavation, embankment, waste, and borrow.

Such items as may be readily defined as to location, standard, or kind may be tabulated or noted, but the tabulation should be arranged in a uniform manner on each sheet concerned. For minor changes and for locations requiring new or additional right-of-way, the margins of the existing road shall be shown by broken lines. Where major relocations of roadway are made, the old road should be shown, for at least a distance of 500 ft. on either side of the center line of the proposed new road. Plans shall be platted with the stationing from left to right. A north point shall be placed upon each sheet.

Each set of plans should show the alignment and profile of the existing road for such distance beyond each end of the project as shall be necessary to avoid any future changes in alignment or grade in the project.

42. Data on Profile

All elevations shall be based on sea level data carried from the nearest U. S. Geological Survey bench mark. The profile shall show datum lines; the profile of the surface of the proposed center line; tops of rails of all grade crossings; elevations of all inlets and outlets of existing culverts; flow lines of all tile drains; profile of intersecting streams; low and high water marks, with data of same; elevations of existing bridge footers, seats, and floors; pipe lines and conduits; driveways; profile of intersecting roads for at least 500 ft. on each side of the center line; ground elevations at all buildings that may affect the grade of the improvements; clearance of all overhead structures and cables above grade; the proposed grade line together with the percentage of all grades and the location and elevation of all intersections thereof; the length of vertical curves; elevation of footers for new structures; elevation and complete description of all bench marks.

The profile of the existing road shall be delineated by broken straight lines between the points where the actual rod readings were taken. The grade line shall represent

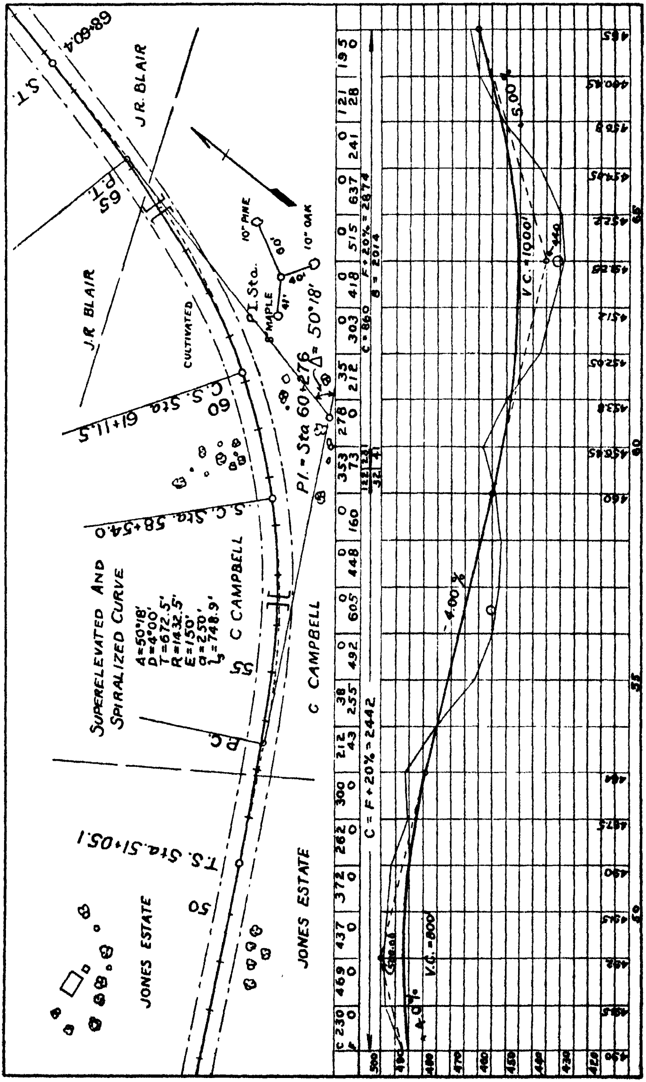


PLATE IV.—Plan and profile for a highway.

the crown profile of the finished pavement on tangents and be the basis of the elevation of the top edges of the pavement where widening or superelevation is applied.

Bridge Layout[1]

43. General

Bridges should be staked out as soon as the contractor is ready to start operations. Before beginning to stake out a bridge, go over the site, taking the plans with you, and compare carefully the location of all parts of the structure as shown on the plans. Try to get a mental picture of how the completed structure and all its parts will appear on the ground, and thereby avoid errors in staking.

44. Stakes

Set all stakes in such manner that they will be unmistakably understood by the contractor, and in such number as may be required by that part of the work.

For hubs, use 2 by 2 by 18 in., preferably of hard wood; for other work, use 1 by 2 by 18 in. dressed softwood. Use lath for temporary markers.

45. Alignment

Carefully establish the center line well back each way from the bridge and check all hubs until you establish the line correctly. Then proceed along the center line of the bridge to measure, line in, and set heavy wood hubs driven to practical refusal at face line of abutments at undercoping, and at center line of piers. The contractor's attention should be called to any offset batter distance for the neat lines at the bottom. Check and recheck every measurement. In your notebook, make a sketch of the plan of all masonry units and show the position of all hubs driven; and later, as you set them, show reference hubs.

46. Staking The Structure

From the center-line hubs, lay out the axis of each pier or bent and faces of abutments and wing walls. Watch your angles carefully and check them by measurement between your reference hubs as you set them. Set heavy reference hubs in both directions along the line of the face of abutment walls at undercoping and the axis of piers; or, if the ground is unsuitable, set

[1] A large part of the material in this section represents the practice of the Maryland State Roads Commission.

them on offset lines. Set one hub outside the end excavation lines, one at the right-of-way line, and one well back on private

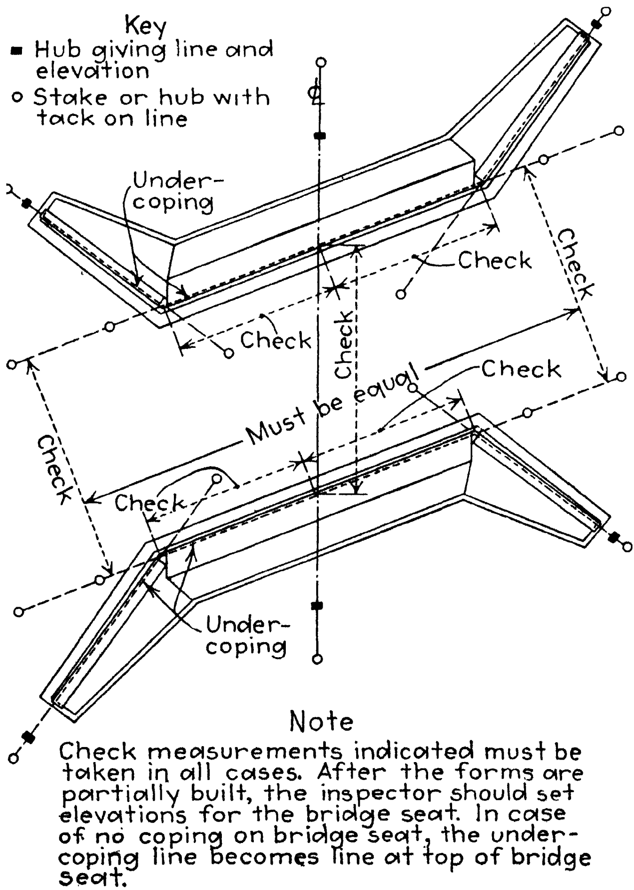


FIG. 6.—Method of staking bridge.

property, if you can do so without damage or with permission. Set these last hubs well back where they will not be disturbed. Drive all reference hubs well below the surface and set guard

stakes. Measure in all hubs with reference to the center line and record them in your notebook.

In the case of an arch bridge, the critical points to locate are the intersections of the springing lines with the center line of the bridge. This applies to single or multiple spans.

Fig. 6 shows the method of staking out a bridge.

a. Steel Bridge Substructure. The most important dimension in a steel bridge substructure is the distance back to back of curtain or back walls, as the superstructure has a definite and exact length which cannot be changed after the material is fabricated, to correct for errors in the substructure. Hence in the case of a single span, the abutments must be staked out using the intersections of the faces of the back walls and the center line as the critical points.

b. Multiple-span Bridge. In the case of a multiple-span bridge, the exact station for the center of each pier must be established, and the distances measured to the center of piers from the face of the back wall of the first abutment to be built. Likewise, the face of the back wall of the second abutment should be measured from the first. If, for any reason, a pier is built first, then the faces of the abutment back walls and the centers of the other piers should be located from this pier.

c. Wing Walls. In staking out wing walls, the inspector resident engineer should take care to check their lengths and their heights at the end, to make certain they properly fit the slope of the embankment fill and the edge of the channel. This is especially important on skewed abutments, because the wings then have different lengths and usually different end heights. Wings should have sufficient height to prevent the highway embankment, at a slope of $1\frac{1}{2}$:1, from encroaching on the channel, and the height of wall at the end should also be made to fit this slope.

The inspector should keep in mind that it is not always possible to see from one side of the stream to the other on center line during the course of construction, due to equipment or material piles. It is good practice, therefore, to set two points on center line on each side of the stream from which to take backsights.

47. Care in Chaining

Take pains that your chaining is accurate enough to locate the bridge exactly on the station where the plans show it. In staking small culverts, shifting a few feet may make no differ-

ence, and may even improve the drainage; but on larger structures across difficult streams, an error may land your bridge on foundations of a considerably different character from those for which it was designed. It is likewise a very serious matter if abutments are so built in error that they will not receive the steel superstructure, or arch footings so constructed that they cause a change in the span of the arch. Hence the necessity in staking out, for checking and rechecking by different methods all lines, measurements, and angles.

48. Setting Elevations

As soon as possible after getting organized, run a permanent line of bench marks and permanently reference all points on line. Establish bench marks at convenient places so that they may be reached for various parts of the structure by one set up. Accuracy in this work cannot be too greatly stressed, as the correct placement of your entire structure depends upon the proper starting point for either line or grade.

Carefully check your level work. Get your bridge to come out at the elevation it was designed to have. There is often very little leeway in which to make adjustments of approach grades, as the approach pavement may be already built to within a few feet of the new structure.

49. Taking Cross Sections

Cross-section the entire bridge site within the limits of the right-of-way. Take and record elevations at all angles, corners, and intermediate points along and within the lines of excavation for all foundations where breaks in the ground surface occur. Take careful and extensive cross sections at abutments. Make a sketch of the plan of each abutment and pier excavation in your notebook, and show all ground elevations within the area, and the location of the points where elevations were taken.

50. Skew Angles

Where skews are required, set your angles with extreme care, as it becomes almost impossible to erect steel on abutments improperly skewed.

51. Triangulation

Where there is a span across a stream too wide to measure accurately with tape line from center to center of the masonry, or in the case of piers of long bridges, the triangulation method must be used. This consists of laying out and measuring very carefully a base line on each side of the river at about right

angles to the center line and approximately equal in length to the distance across the river. In measuring the base line, extreme care must be taken, and corrections made for sag, pull, grade measurements, and temperature. The base-line measurement must be repeated several times. The triangles formed by the base lines and the center line are then completed by measuring carefully all possible angles from and to the ends of each of the base lines, the average of six to ten measurements of the same angle at each point being used. The triangles must then be carefully adjusted and computed. The length of the center line included in these triangles thus becomes known and fixed accurately. The center lines of the piers can then be located by transit line intersections from the ends of the base line or by direct measurement.

52. Excavation

Excavation should be carried to the depth shown on the plans unless unforeseen conditions are encountered, in which case the district engineer must be consulted. If, for any reason, excavation for an abutment or pier has been carried below the depth shown on the plans, the concrete should be started at this lower elevation. Unnecessary or excess width of the foundation should not be permitted; nor paid for, if done.

If explosives are used in removing rock or hardpan, the contractor should be warned not to overshoot the foundation; that is, disturb the underlying formation and spoil it for foundation purposes. Material below plan grade which has been loosened by shooting should be removed and replaced with concrete at the contractor's expense.

53. Cofferdams

Careless methods of constructing cofferdams, or the use of inferior or inadequate materials in their erection, should not be permitted. Check over the layout which the contractor has made on the ground for the cofferdam and see that he has read the stakes correctly, and that it is of sufficient size and in the correct location.

See that cofferdams are large enough to permit easy access to all parts of the foundation forms. Cofferdams should be as near watertight as possible. Means must be provided for the proper drainage of the area inside the neat lines so that the footing concrete above any seal coat may be poured in the dry. A distance of not less than 12 in. should be left from neat lines of footings to the inside of cofferdam for underwater work.

The framework in cofferdams must be heavy enough to take care of pressure with the least number of cross braces to interfere with pile driving or with other operations.

Where cofferdams are unwatered, a stake should be set for each pile to insure its being driven in the proper location.

54. Driving Sheeting

In driving sheeting, the best practice is ordinarily to keep the points a foot or two lower than the excavating operations, the sheeting to be set in place and driving begun before the excavation is started. When finally driven, the lower ends of the sheeting should be about two feet lower than the bottom of the excavation under ordinary conditions. Of course, where rock or other impenetrable strata are encountered, this cannot be done. Also, where fine sand or plastic soils exist at footing level it may be necessary to drive the sheeting much farther than this in order to cut off the flow of such material underneath the sheeting and up into the excavation. If the sounding record on the plans shows such soils, it is well to call the attention of the contractor to this fact before material for a cofferdam is ordered so that he may provide sheeting with extra length for this purpose.

55. Foundations

The foundation is the most vital part of the structure, therefore extreme care should be taken to secure a good foundation.

When possible, the excavated area is to be pumped dry so that a full inspection may be made before any concrete is placed. Where it is impossible to unwater the foundation, or to keep the water out during concreting of a footing, it may be necessary to place a seal course by means of a tremie or a bottom dump bucket. Such work should not be done without securing previous approval.

The use of a tremie must not be permitted until every precaution for handling it has been made. This work cannot be done with a makeshift outfit. Seal courses must be poured monolithic.

Special precautions are to be taken against foundation material which is apparently stable when dry but unstable when wet or saturated.

56. Excavation Record

This record, a sample of which is shown in Fig. 7, should be kept for each abutment and pier. It serves, not only for the purpose of record on the particular bridge for which it is taken

but when collected by the Bridge Department for different structures in the vicinity, these data give information of actual subsoil conditions. This is of great value in designing the substructure of future bridges.

The formula on page 97 shows how the volume of excavation is computed.

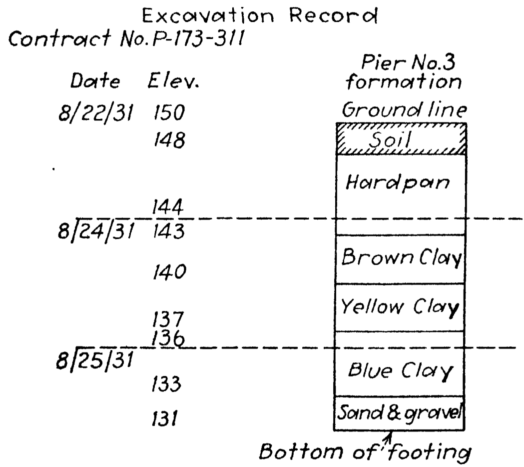


FIG. 7.—Soil excavation record.

57. Inspection of Foundation

Footings should be carefully inspected to insure proper bearing for the structure.

No concrete should be deposited in foundations until the district engineer has been advised of the depth, subsoil conditions, and other features having a bearing on the stability of the foundations. The inspector should then withhold concreting operations until duly authorized to proceed. *This is important* and must never be overlooked.

58. Driving Piles

A large percentage of the bridges built are on pile foundations. Inasmuch as the type of equipment used in pile driving is necessarily light, precaution must be exercised in the driving operations. Except where batter piles are specified, care must be taken to insure the pile being driven as nearly

vertical as possible, as then greater penetration can be secured. Frequently boulders or other obstructions are encountered which deflect the course of the pile or entirely prevent its driving. When it is apparent that the pile is brooming or failing, the driving should be stopped, as piling becomes ineffective by continued driving after the pile has come to refusal. When the penetration is ⅛ in. or less per blow, driving should be stopped, as such penetration can be considered refusal. Watch the piles carefully, and if they have a tendency to split, they should be provided with a metal collar. Watch for bounce at the pile head; and note particularly what the specifications say in regard to bounce when computing the bearing power of the pile. In the case of gravity hammers, be sure the hammer has a free fall.

All foundation excavation in which piles are to be driven, must be completed before pile driving is commenced. In foundations where piling are driven closely together, get an elevation on the top of each pile as driven, and as the driving proceeds, check these piles; they may at times *come up* several inches, due to heaving, as the ground compacts from displacements. If they do come up, finish driving all piles and then return and drive heaved piles back to place.

Piling shown on the plans must never be omitted without approval; but if you find that you are not securing a penetration of more than 6 or 8 ft. you should take up this matter at once with the district engineer. This small penetration may give enough bearing power to carry the load, but there is the possibility of scouring which must be considered.

The same thing is true when the piling goes down rapidly under the first few blows (say, 12 to 18 in. per stroke) and then begins to tighten up during the last few feet. In this case you are getting only a few feet of adequate bearing, which would not be enough in the case of heavy scouring.

Piling should be driven to the capacity specified, but the total penetration obtained must be satisfactory regardless of the fact that the required bearing capacity may have been obtained at a less penetration. If the pile is still going down well when the required capacity is obtained, you should consider the possibility of scour, and if this seems probable, continue to drive until refusal or until an ultimate penetration is noted.

It is well to remember that piling is used in foundation work on account of scouring, as well as for safe bearing.

Piles must not be spliced without written permission. If permitted, the splice should be examined carefully to see that a butt joint is made which has a uniform bearing over the entire area.

In general, the foregoing discussion on piling applies to precast concrete piling as well as to timber piling.

Extensions or splices in precast concrete piling should not be permitted.

Where pile foundations are provided, a plan sketch of each foundation should be made in the structure book, showing the actual number of piles placed and their locations. Each pile should be given a number, beginning with No. 1 for each foundation. The number should be placed in a small circle on the sketch to show clearly the pile to which it belongs. The pile driving record should then give, by corresponding number, the required information for each pile in the foundation.

CHAPTER II

EARTHWORK

General

59. Definitions

Aside from *roadway and drainage excavation* and *embankment;* the term *earthwork* includes *clearing*, *grubbing*, *borrow*, *overhaul*, *shoulders*, finishing and cleaning up *subgrade*, and any other items necessary to complete *grading* of the roadway.

Grading is measured by the cubic yard, and the unit price includes full compensation for excavation, formation of embankments, trimming slopes, disposing of surplus materials, and the preparation and completion of roadway, subgrade, and shoulders. Measurement of grading is made in excavation only, except where otherwise specifically directed. Some grading contracts provide for payment of *overhaul* (see page 101).

Grading operations are usually done with power shovels, scrapers, or special road-grading machinery.

The *embankment*, in order to support a pavement, must be of material which will remain in place and become firmly compacted. Sod, muck, and other unstable material from cuts or channel changes should be wasted and, if necessary, other material borrowed to replace it in the embankment. Specifications frequently require embankment material to be spread in horizontal layers and rolled before the next layer is placed, particularly if a pavement is to be constructed shortly afterward.

Borrow Pits. If earth from the near-by cuts is not sufficient to make the fills, the contractor is allowed to obtain it from *borrow pits*, which are cross-sectioned by the

engineer both before and after excavation, in order to measure the volume in cubic yards.

60. Classification of Excavated Material

Excavated material is usually classified as (*a*) *common excavation*, (*b*) *loose rock*, or (*c*) *solid rock*.

Common excavation is largely *earth*, or earth with detached boulders less than ½ cu. yd. *Loose rock* usually refers to rock which can be removed with pick and bar without blasting. *Solid rock* comprises rock in solid beds or masses of boulders measuring 1 cu. yd. or more.

61. Shrinkage and Swell Factors

Excavated earth when placed in a fill will usually become compacted so that its final volume is less than the space originally occupied in the embankment. This difference between the original volume in cut and the final volume in fill may be defined as the *shrinkage*.

Shrinkage may be allowed for by increasing the yardage of embankments. An average allowance is 10 to 15 per cent for fills over 4 ft. deep and 20 per cent for shallow fills.

The actual volume of freshly excavated material is at first increased due to its loose and porous condition. Then compaction and shrinkage begin to take place. The rapidity of shrinkage depends upon the character of the material, method of placing the fill, climatic conditions, and the intensity of loads on the fill before stability has been reached. Shrinkage generally includes wastage in transporting material from cut to fill and loss in material which washes beyond the toe of the slopes. A considerable amount of shrinkage usually takes place during construction.

Shrinkage should not be confused with *subsidence*, which is a settlement of the entire embankment due to unstable foundation conditions, such as might be expected if a heavy fill is placed on swampy soil.

In case of rock excavation, the material will occupy a *larger* volume; the increase called *swell*, amounting to 25 to 40 per cent. If, however, a comparatively small amount of rock is placed in an embankment so that the

voids are filled with earth, there is no "swell factor" to be considered.

62. Side Slopes

Earth slopes are usually 1 to 1 in cuts and $1\frac{1}{2}$ horizontally to 1 vertically in fills, except as modified by departmental standards or soil conditions. In sand cuts and fills and in cuts subject to slides, the slopes may be 2 horizontally to 1 vertically.

In cases where borrow becomes necessary to make the required fills, it may be advisable to widen the cut sections or flatten the slopes thereof to provide the necessary yardage in the fills. This in turn will increase the sight distances.

Slopes may be protected against *erosion* by sodding and planting grass seed, vines, or dwarf shrubbery. Under a law just enacted in New York State, all highway improvement contracts must include items of trees and shrubbery for this purpose.

63. Roadbed Sections

Figure 8 shows typical roadbed sections for a *railroad*, *highway*, and *street* (business and residential).

On *railroad* work, the subgrade is usually flat and not sloped on curves, superelevation being added in the ballast.

The *highway* subgrade is usually formed with shoulders and a trenched section to receive the pavement; the finished surface being crowned to facilitate drainage. On curves, the subgrade is banked; it is also widened for curves of 7° or sharper. The width of roadbed in cuts is wider than on fills to allow for the side ditches. The formation of the trenched subgrade and shoulders requires "fine grading" which is sometimes a separate item in the cost of "roadway grading."

On resurfacing projects, "reshaping and scarifying" to a depth of about 6 in. is often paid by the *square yard* instead of by the volume quantities.

The normal cross section of a *city street* has a crowned pavement with curb and gutter depressed sufficiently to receive drainage from the sidewalk and adjoining property.

In residential areas, a planting space for grass and trees should intervene between the sidewalks and curb.

64. Slope Stakes

Slope stakes are set at points where the side slopes of the graded road will intersect the ground surface; they mark the limits of excavation and embankment. The slope stakes are driven at points of zero cut or fill, but the numbers

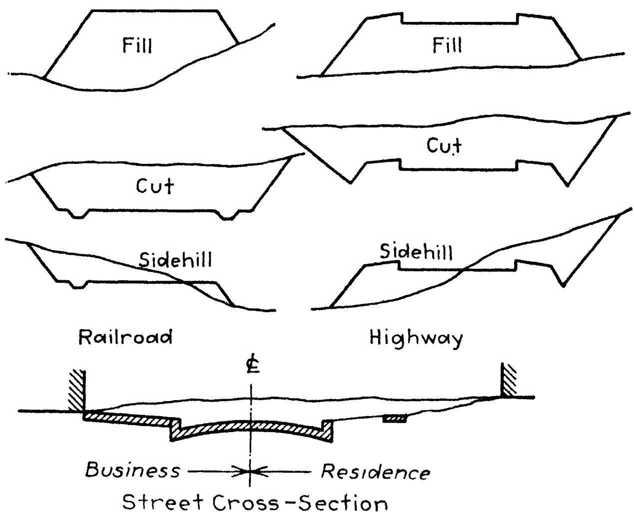


FIG. 8.—Typical roadbed sections for railroad, highway and street.

written on them give the vertical distance with reference to the finished plane of subgrade. On the inner side of the stakes is marked in feet and tenths the "cut" or "fill" as the case may be. Thus, "C 3.2" indicates that the center-line elevation of the roadbed is to be cut 3.2 ft. *below* the ground at the slope stake; and "F 4.6" indicates that the fill is to be 4.6 ft. *above* the slope stake.

The stakes are driven with the tops slanting outward, and with the sides upon which the cuts or fills are marked facing the roadway. The station number is marked on the other side of the stake.

EXAMPLE 1. *Part A.*

Referring to Fig. 9, which represents a fill on side-hill ground, let

$b = AB$ = width of roadbed;

s = "slope ratio" for the fill = ratio of horizontal to vertical;

$d = PC$ = depth of fill at the center;

x_1 = horizontal distance from P to D;

x_2 = horizontal distance from P to E;

$d + y_1$ = vertical distance from C to D;

and $d - y_2$ = vertical distance from C to E.

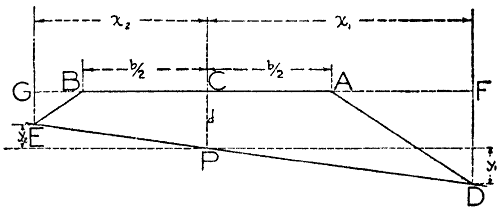


FIG. 9.

The slope stake at point D, on the right, is correctly established if

$$x_1 = \tfrac{1}{2}b + s(d + y_1).$$

Likewise, point E, on the left, is correctly established if

$$x_2 = \tfrac{1}{2}b + s(d - y_2).$$

In the foregoing equations, d and s are known, while x and y are measured and remeasured in the field until the equations are satisfied (trial and error method).

If the ground were *level*, $y_1 = y_2 = 0$; then $x_1 = x_2 = \tfrac{1}{2}b + sd$.

Part B. Referring to Fig. 9, let width of roadbed (b) = 30 ft., center fill (d) = 4.2 ft., and slope of fill is $1\tfrac{1}{2}$ horizontal to 1 vertical ($s = \tfrac{3}{2}$). It is required to locate the slope stakes at D and E.

PROCEDURE. Using a *level*, the rod reading, when rod is held on ground at center stake (P), is assumed to be 5.1 ft.

To locate slope stake D. Try a distance out of 28 ft.; the rod reads 9.5 ft., and hence $y_1 = 9.5 - 5.1 = 4.4$ ft.

Test: $x_1 = \tfrac{1}{2}(30) + \tfrac{3}{2}(4.2 + 4.4) = 27.9$ ft., which is almost exactly correct.

To locate slope stake E. Try a distance out of 17.1 ft.; the rod reads 2.3 ft., the level remaining in the same position, and hence $y_2 = 5.1 - 2.3 = 2.8$ ft.

Test: $x_2 = 15 + \frac{3}{2}(4.2 - 2.8) = 17.1$ ft., which is correct.

The slope-stake notes for this station are therefore:

Left	Center	Right
$\frac{F1.4}{17.1}$	$\frac{F4.2}{0.0}$	$\frac{F8.6}{28.0}$

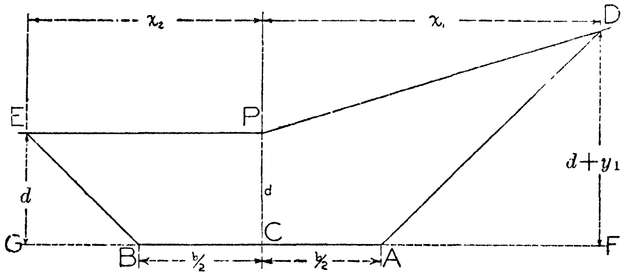


FIG. 10.

EXAMPLE 2. *Part A.*

Referring to Fig. 10, and letting s represent the "slope ratio" of the banks in cut, we have

x_1 = distance from center to slope stake $D = \frac{1}{2}b + s(d + y_1)$,

and

x_2 = distance from center to slope stake $E = \frac{1}{2}b + s(d + 0)$.

Part B. In Fig. 10, let width of roadbed $(b) = 30$ ft., center cut $(d) = 6.2$ ft., and assume 1 to 1 slopes $(s = 1)$.

PROCEDURE. Using a *level*, the rod reading on the ground at the center stake (P) is assumed to be 8.4 ft.

To locate slope stake D. Try a distance out of 29.0 ft.; the rod reads 0.4 ft., and hence $y_1 = 8.4 - 0.4 = 8.0$ ft.

Test: $x_1 = 15 + (6.2 + 8.0) = 29.2$ ft., which is about right.

Next, try $x_1 = 29.2$ ft., rod reads 0.35 ft., and hence

$$y_1 = 8.4 - 0.35 = 8.05 \text{ ft.}$$

Test: $x_1 = 15 + (6.2 + 8.05) = 29.25$ ft., which is correct.

To locate slope stake E. $x_2 = 15 + 6.2 = 21.2$ ft., if the ground is level from P to E, which it is in this case.

The slope-stake notes for this station are:

Left	Center	Right
$\dfrac{C6.2}{21.2}$	$\dfrac{C6.2}{0.0}$	$\dfrac{C14.2}{29.2}$

It should be understood that prior to setting slope stakes, the grade line must be established on the profile, from which is obtained the center-line cuts and fills (shown in the table below).

TABLE TO ACCOMPANY SLOPE-STAKE NOTES

Sta.	Elevation		Center	
	Ground	Grade	Cut	Fill
6 + 00	96.2	100.21	...	4.0
+ 40	101.8	101.81	...	0.0
7 + 00	106.5	104.21	2.3	
+ 50	110.4	106.21	4.2	
8 + 00	114.4	108.21	6.2	

65. Sections Passing from Cut to Fill

In passing from cut to fill, the junction rarely takes place in one cross section, but along an irregular line, as shown in Fig. 11. Three intermediate cross sections should be taken: (1) where the cut runs out on downhill side, (2) where the center line is at grade, and (3) where the fill runs out on uphill side.

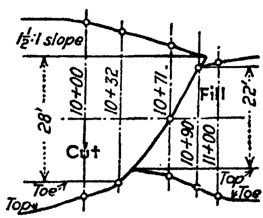


FIG. 11.—Sections passing from cut to fill.

AREA OF CROSS SECTIONS

66. Planimeter Method

When the cross sections of the ground and roadbed are plotted, the areas in cut (or fill) may be measured by means

of a planimeter. The scale of the cross sections is usually 1″ = 10′ or 1″ = 5′.

A transparent roadway templet is convenient for plotting subgrade and ditches. Some engineers prefer to use a "balanced grading section" with grade and width so adjusted that grading quantity will be equal to that for actual subgrade.

The planimeter should, prior to actual use, be carefully tested on a boundary of known area, and every area should be run over twice for a check.

Although the planimeter (or graphical) method is long and expensive, it has the advantage of representing a graphical picture of individual sections for reference or detailed study.

67. Computation Method

a. General Rule. The area of cross section may be computed for any case of irregular ground by means of the so-called *coördinate method*, which is a formula like that used to determine the area of any surveyed field, namely:

Proceeding in regular order around the boundary, *multiply each ordinate by the difference between the following and preceding abscissas, take one-half the algebraic sum of the products, giving the desired result.*

The foregoing rule will now be applied to the irregular cross section (Fig. 12). It is assumed that the cuts (y_1, y_2, y_3, . . .) and their corresponding distances (x_1, x_2, x_3, . . .) from the center line have been found in the field and recorded as fractions.

The following arrangement shows the ordinate and abscissa for each break in the surface.

$$\frac{0}{x_B} \quad \frac{y_5}{x_5} \quad \frac{y_4}{x_4} \quad \frac{y_3}{x_3} \quad \frac{y_2}{x_2} \quad \frac{y_1}{x_1} \quad \frac{0}{x_A}$$

In addition, on the right and left, are given the fractions, $0/x_A$ and $0/x_B$, which refer to the coordinates of A and B. The ordinates of points A and B (in the numerator) are each zero, since the origin is at C and the X-axis coincides with line AB.

The abscissas to the left of the vertical center line are *minus*, and those to the right are *plus*. Ordinates *above* the base grade in cuts are *plus*, while those *below* (as for side ditches) are *minus*.

Using the general rule, we have,

$$\text{Area} = \tfrac{1}{2}[y_5(x_4 - x_B) + y_4(x_3 - x_5) + y_3(x_2 - x_4) + y_2(x_1 - x_3) + y_1(x_A - x_2)]. \quad (2)$$

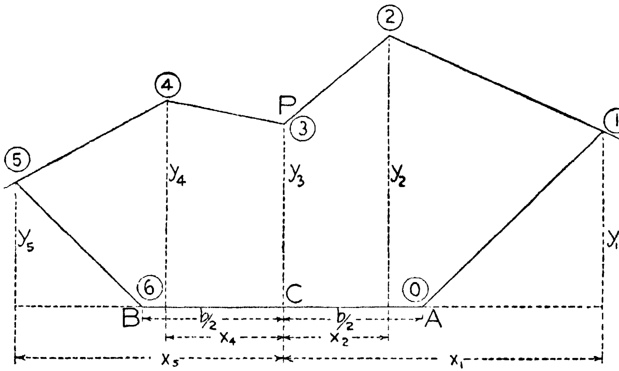


FIG. 12.

Substituting numbers for letters in Fig. 12, and noting that the width of roadbed is 30 ft., we have

$$\frac{0.0}{-15.0} \quad \frac{+7.5}{-30.2} \quad \frac{+7.8}{-14.1} \quad \frac{+5.5}{0.0} \quad \frac{+9.2}{+12.5} \quad \frac{+7.1}{29.0} \quad \frac{0.0}{+15.0},$$

from which

$$\text{Area} = \tfrac{1}{2}[7.5(-14.1 + 15) + 7.8(0 + 30.2) + 5.5(12.5 + 14.1) + 9.2(29.0 - 0) + 7.1(15 - 12.5)] = 337 \text{ sq. ft.}$$

To be consistent with the data involved, it is sufficiently close to record areas only to the nearest square foot.

Usually the ditches are of a uniform design, and the additional areas due to them can be added later when computing volumes.

The foregoing rule applies equally well to areas in *fill*.

b. Level Ground. For *level ground,* the area of cross section in cut (or fill) is merely that of a trapezoid.

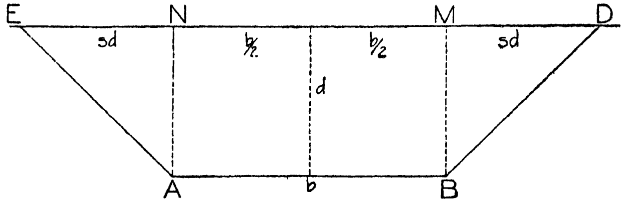


FIG. 13.

In Fig. 13, b = width of base AB,
d = center cut,
s = slope of banks = $MD/BM = NE/AN$,

hence

$$\text{Area} = \tfrac{1}{2}d(2b + 2sd) = d(b + sd) \qquad (3)$$

c. Three-level Ground. For regular ground, only three readings are usually taken in the field; one at the center stake and one at each slope stake.

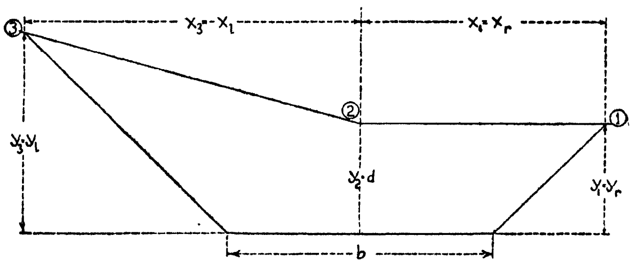


FIG. 14.

Letting the ordinates for points 1, 2 and 3(Fig. 14) be $y_1(= y_r)$, $y_2(= d)$ and $y_3(= y_l)$; and the corresponding abscissas $x_1(= x_r)$, $x_2(= 0)$ and $x_3(= -x_l)$; we have the following arrangement, in the form of fractions:

$$\frac{0}{-\frac{1}{2}b} \quad \frac{y_l}{-x} \quad \frac{d}{0} \quad \frac{y_r}{x_r} \quad \frac{0}{\frac{1}{2}b}.$$

Applying the *general rule* (given in *a*), we have

$$\textit{Area of three-level section} = \tfrac{1}{2}[y_l(0 + \tfrac{1}{2}b) + d(x_r + x_l) + y_r(\tfrac{1}{2}b + 0)] = \tfrac{1}{2}[d(x_r + x_l) + \tfrac{1}{2}b(y_r + y_l)]. \quad (4)$$

d. Side-hill Ground. When both *cut* and *fill* occur at any one cross section, as in side-hill construction, the quantities of excavation and embankment should be com-

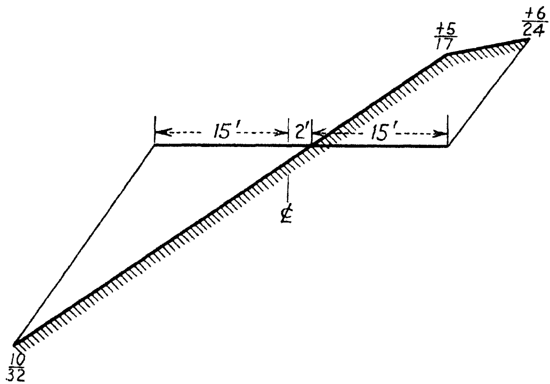


FIG. 15.—Side-hill ground.

puted separately. Knowing the center fill (or cut), the *grade point* (zero ordinate) can be determined by trial.

Referring to Fig. 15, the arrangement of fractions for applying the general rule is shown below.

$$\text{Fill: } \frac{0.0}{-15.0} \quad \frac{-10.0}{-32.0} \quad \frac{0.0}{+2.0}$$

$$\text{Cut: } \frac{0.0}{+2.0} \quad \frac{+5.0}{+17.0} \quad \frac{+6.0}{+24.0} \quad \frac{0.0}{+17.0}$$

The center line of roadway is used for the origin of abscissas in both cases.

The *areas* are as follows:

Area in fill $= \tfrac{1}{2}[10(2 + 15)] = 85$ sq. ft.

Area in cut $= \tfrac{1}{2}[5(24 - 2) + 6(17 - 17)] = 55$ sq. ft.

Volumes

68. Introduction

The volume of earthwork may be found by means of either the *average end area* or the *prismoidal* formulas. Although the former is less exact than the latter, it is generally accepted as the *standard earthwork formula*, on account of its simplicity.

69. The Average End Area Formula

The volume of a right prism equals the *average area* multiplied by the *length.* Assuming the average area to be same as the average *end* area, we have

$$\text{Volume} = V = \tfrac{1}{2}(A_1 + A_2)l(\text{cu. ft.}) = \tfrac{1}{54}(A_1 + A_2)l(\text{cu. yd.}). \quad (5)$$

in which A_1 and A_2 = area of end sections in square feet, and l = length of solid in feet.

This formula is applied to areas of any shape, but the results are slightly too large. The error is small if the sections do not change rapidly, as is the usual case in practice.

Table 18 gives volumes of 100-ft. earthwork sections computed by the average end area formula. For any other length, the values are in proportion.

70. The Prismoidal Formula

A prismoid is a solid whose ends are parallel and whose sides are plane or warped surfaces. Figure 16 represents a typical prismoid.

It may be shown by means of the calculus that the correct volume of a prismoid is

$$V = \frac{l}{6}(A_1 + 4A_m + A_2) \quad (6)$$

in which l is the distance between the two parallel bases A_1 and A_2, and A_m is a section midway between the two end bases and parallel to them. A_m is not an average of A_1 and A_2, but each of its linear dimensions is an average of the corresponding dimensions of A_1 and A_2.

71. Comparison of Average End Area and Prismoidal Formulas

Volumes computed by the average end area formula usually are slightly in excess of those by the prismoidal formula. The *difference* is called the *prismoidal correction.* If this correction is applied to volumes obtained by the more simple end area formula, then the results are as if the more exact prismoidal formula had been used.

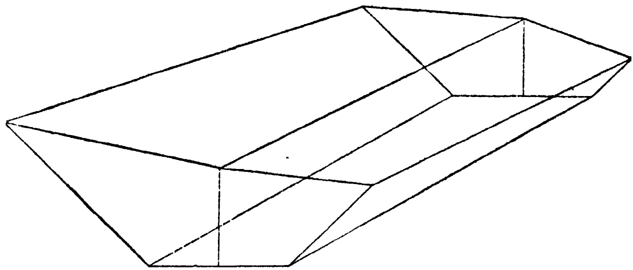


FIG. 16.—Typical prismoid.

Let V_a = volume given by end area formula;
V_p = volume given by prismoidal formula;
$C_v = V_a - V_p$ = prismoidal correction;
d_1 and d_2 = center cuts (or fills) at end sections A_1 and A_2;
X_1 and X_2 = total distance between slope stakes at A_1 and A_2.

For three-level sections, it can be shown that

$$C_v(\text{cu. yd.}) = \frac{l}{12 \times 27}(d_1 - d_2)(X_1 - X_2). \qquad (7)$$

For any form other than the typical three-level section, the foregoing expression is somewhat approximate.

Table 19 gives values of the prismoidal correction for lengths of 100 ft.

72. Borrow Pit Quantities

Excavations outside of the roadway are taken from so-called *borrow pits.* Data for the volume of material

moved from borrow pits are determined by taking cross sections at definite intervals before and after the work is done. Two methods of procedure are as follows.

Method A

1. Establish a base line to one side of the proposed pit. Take cross sections at right angles thereto at intervals of 10, 20, or 25 ft. extending well behind the probable limits of the pit.
2. After excavating, re-cross-section at the same points and intermediate ones if necessary.
3. Compute volumes excavated by the average end area formula.

Method B

1. Divide the original surface into squares, rectangles, or triangles, taking ground elevations at every corner.
2. Establish reference stakes that will not be disturbed during the construction.
3. After the borrow pit is abandoned, reproduce the same corners and take elevations on them again in the pit.
4. Compute volumes of the various vertical truncated prisms excavated, the altitudes being determined from the difference in levels at the corners, and bases from the dimensions of the squares or triangles (Art. 73).

Borrow pits may be avoided by widening the cut uniformly, but this will not be economical unless the material is easily obtainable and the haul is short.

73. Truncated Prisms

The volume of a rectangular prism truncated top and bottom is equal to $A\frac{h_1 + h_2 + h_3 + h_4}{4}$ cu. ft., where A is the area of right section and h_1, h_2, h_3, h_4 are the corner heights, determined from the difference in levels.

The volume of a triangular prism truncated top and bottom is equal to $A\frac{h_1 + h_2 + h_3}{3}$ cu. ft., where A is the

area of right section and h_1, h_2, h_3 are the corner heights, determined from the difference in levels.

Instead of computing the volume of each prism separately in the system of equal squares shown below, it will save time to substitute in a combined formula.

Let Σh_1 = sum of corner heights used only once.
Σh_2 = sum of corner heights used twice.
Σh_3 = sum of corner heights used three times.
Σh_4 = sum of corner heights used four times.

Then the total volume,

$$V = \frac{A}{27}\ \frac{\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4}{4} \text{ (cu. yd.).}$$

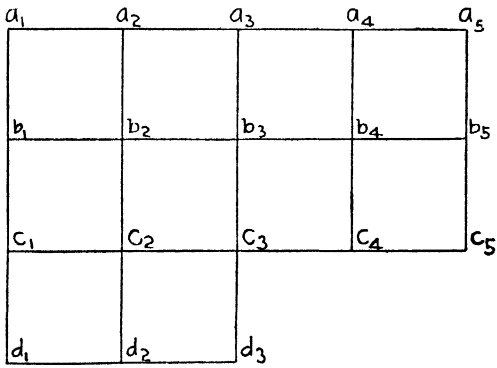


FIG. 17.

74. Correction for Road Curvature

The formulas for volumes of earthwork are based upon prismoids with parallel ends; but when applied to curved roadways with end sections that are radial (not parallel), there is apt to be some error. The difference between the true volume and that determined by the prismoidal formula is called the *curvature correction.*

By the theorem of Pappus and Guldinus (see texts on mechanics):

If a plane area is made to revolve about an axis in the same plane, the volume of the solid generated by the

revolution is equal to the product of the revolving area and the path described by the center of gravity of the plane area during the revolution.

It is only when the center of gravity remains in a vertical through the center of the road that the volumes as computed by the prismoidal formula are exact. If the gravity curve is *outside* the center-line curve, the true volume is *more* than that given by the prismoidal formula; if *inside*, it is *less*.

Let e = eccentricity of center of gravity of section
= horizontal distance from center line to a vertical line through center of gravity;
R = radius of center line.

Consider a curving section of earthwork of center-line length $= l$ ft. The volume as per straight sections equals

$$V = \tfrac{1}{2}[A_1 + A_2]l - C_v. \qquad (a)$$

If the end area (A_1) and its eccentricity (e_1) are constant throughout, then the corresponding true volume (V_1) is $V_1 = A_1 l_1$, where l_1 equals length circular arc of radius $(R + e_1)$.

From geometry, we have $\frac{l_1}{l} = \frac{R + e_1}{R}$, whence

$$l_1 = \left(\frac{R + e_1}{R}\right)l.$$

Therefore, $V_1 = A_1\left(\frac{R + e_1}{R}\right)l.$

Similarly, we have $V_2 = A_2\left(\frac{R + e_2}{R}\right)l$, as if the area (A_2) at the other end, and its eccentricity (e_2), were constant throughout.

The actual true volume is assumed to be

$$\frac{1}{2}(V_1 + V_2) - C_v = \frac{l}{2R}[A_1(R + e_1) + A_2(R + e_2)] - C_v \; (b)$$

Subtracting Eq. (*a*) from Eq. (*b*), we have

$$\textit{Curvature correction} = \frac{l}{2R}(A_1e_1 + A_2e_2) \text{ cu. ft.} = \frac{l}{54R}(A_1e_1 + A_2e_2) \text{ cu. yd.} \quad (8)$$

This formula has been derived in a somewhat similar manner by Webb in his "Railroad Construction." It also agrees very closely with an expression derived by calculus

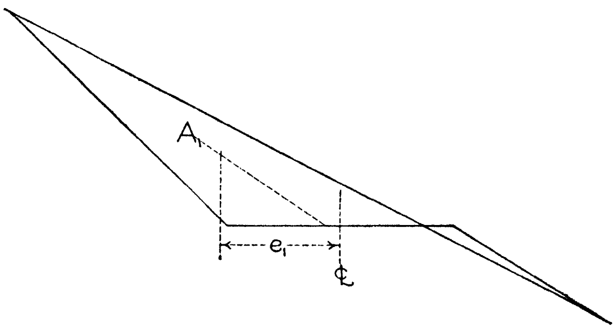


Fig. 18.

methods in Crandall's "Tables for the Computation of Railway and Other Earthwork."

The eccentricity (e) is *plus* when the greater area is on the outside of the center-line curve, and *minus* when the greater area is on the inside of the curve.

Formulas have been derived for the eccentricity of sections on three-level and side-hill ground. But for highway areas of sections, which are usually plotted on cross-section paper, it seems easier to find the center of gravity graphically (the area being usually considered as a triangle or trapezoid) and then measure to scale the eccentricity.

Correction for curvature is rarely ever considered except in special cases, such as side-hill construction or in rock excavation.

EXAMPLE. Given a curve with radius = 191 ft. (30° curve), width of road = 30 ft. and center-line length of earthwork solid = 25 ft. Fig. 18 represents the radial cross section at one end of the solid. The area A_1 of the excavation triangle is 380 sq. ft., and the eccentricity (center of gravity being determined from the intersection of the medians of the triangle) is scaled to be 16.0 ft.

Let A_2 and e_2 at the other end of the solid be 300 sq. ft., and 14.2 ft., respectively.

$$\textit{Correction for curvature} = \frac{25}{2 \times 191}(380 \times 16 + 300 \times 14.2) = 677 \text{ cu. ft.} = 25.1 \text{ cu. yd.}$$

The curve extends around a ravine and the center of gravity of section is outside the center line of road; hence the correction is added to the excavation quantities. If the curve extends around the end of a ridge and the center of gravity of section is inside the center line, the correction would be subtracted.

75. Culvert Quantities

a. General Considerations. For pipe culverts and small box culverts, standard plans are available for supplying all the necessary construction information, except special features, such as the length of barrel, the skew angle (if anything), etc.

The length of culvert is determined by drawing accurately to scale the highway embankment, taking into account (if necessary) the effect which the skew, extra widening and superelevation have on the side slopes, and adjusting the culvert to fit.

In case of appreciable skew, the layout plan for a box culvert should be supplemented by specially prepared detail drawings, showing mainly the difference in length of wings and other details.

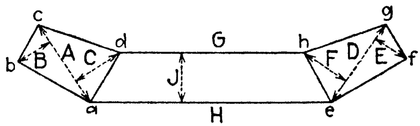
The flow line of culvert should conform to the gradient of the stream, and the correct elevations should be indicated at the ends of the barrel.

For arch culverts and large box culverts, complete detail drawings should be made for each situation. The box type

will be more frequently used, but arch culverts are economical and desirable where there is a very deep fill with a firm foundation.

Definitions for the *span* length of drainage structures should be *uniform*. Ohio practice is as follows:

The span of a *bridge* shall be considered as the clear span, measured between faces of piers and abutments in a direction parallel with the center line of the highway (that is, on skew bridges the span is not to be considered as measured perpendicular to faces of piers and abutments), except that for *steel bridges* the span will generally be expressed as center to center



Horizontal measurements $= A \cdot B \cdot C \cdot D \cdot E \cdot F \cdot G \cdot H \cdot J$

Vertical measurements $= a \cdot b \cdot c \cdot d \cdot e \cdot f \cdot g\, h$

Formula:

$$\left[\left(\frac{A\times B}{2}+\frac{A\times C}{2}\right)\times\left(\frac{b+c+a+d}{4}\right)\right]+\left[\left(J\times\frac{G+H}{2}\right)\times\left(\frac{a+e+d+h}{4}\right)\right]+\left[\left(\frac{D\times E}{2}+\frac{D\times F}{2}\right)\times\left(\frac{f+g+e+h}{4}\right)\right]$$

FIG. 19.—Formula for computing abutment excavation.

of pins or bearings. For *culverts*, however, the clear span will generally be understood to be the distance between faces of walls, measured at right angles to the faces. In stating the size of a culvert, the span should be given first and then the height. (Example: 4 by 3 ft. box culvert, means 4-ft. span and 3-ft. height.)

b. Abutment Excavation. Figure 19 gives a formula for computing abutment excavation, as prepared by the Maryland Roads Commission.

c. Headwalls for Pipe Culverts. Pipe culverts made of concrete, cast iron, vitrified clay, or corrugated metal are used for small openings. Except for vitrified pipe, the diameters may run up to about 60 in.

Two types of headwalls for pipe culverts, (1) straight and (2) flared, are given on pages 98 and 99.

Straight Headwalls. This simple type of headwall is used for small pipe. Figure 20 shows the design, and the accompanying table gives *volumes* and *dimensions.*

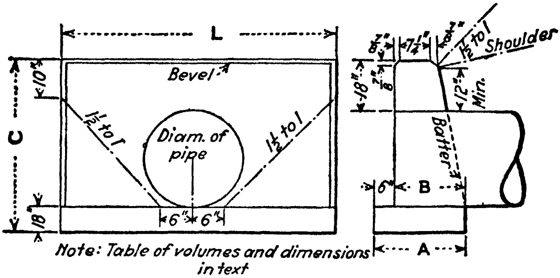


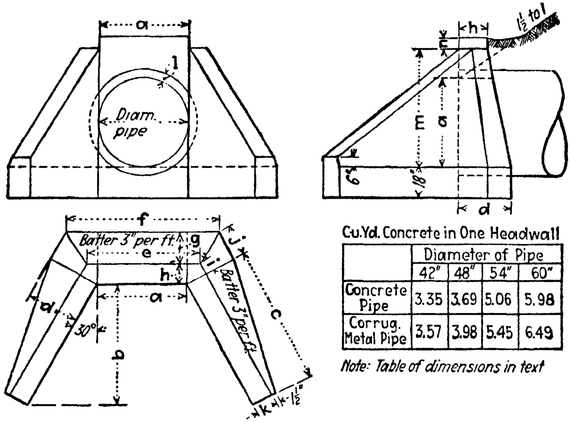
FIG. 20.—Headwall for pipe culvert (Kentucky practice).

VOLUMES AND DIMENSIONS OF HEADWALLS FOR PIPE CULVERTS
(Accompanying Fig. 20)

Diameter of pipe (in.)	Cu. yd. in one headwall	Dimension (in.)				Batter (in. per ft.)
		A	*B*	*C*	*L*	
12	1.06	20	14	48	72	2
15	1.25	20½	14½	51	81	2
18	1.51	21	15	54	90	2
24	2.00	22	16	60	108	2
36	3.25	24	18	72	144	2

Flared Headwall. This type of headwall is used for large pipe. Figure 21 shows the *design* for wings flared at 30° with axis of pipe, and also the *volumes.* The accompanying table gives the *dimensions.*

d. Box Culverts. Reinforced-concrete box culverts are generally advantageous for waterway areas exceeding 12 to 16 sq. ft. This type may be used up to 12 by 12 ft., although 8 by 8 ft. maximum is common; twin box or triple box culverts are sometimes built instead of a large single box.



Cu.Yd. Concrete in One Headwall

	Diameter of Pipe			
	42"	48"	54"	60"
Concrete Pipe	3.35	3.69	5.06	5.98
Corrug. Metal Pipe	3.57	3.98	5.45	6.49

FIG. 21.—Flared headwall (Virginia practice).

DIMENSIONS OF HEADWALLS FOR PIPE CULVERTS
(Accompanying Fig. 21)

Diam. of pipe ((in.)	*a* (in.)	*b* (ft.-in.)	*c* (ft.-in.)	*d* (in.)	*e* (ft.-in.)	*f* (ft.-in.)	*g* (in.)
42	42	5′ 0″	5′ 9¼″	23½	4′ 5½″	5′ 9″	13½
48	48	5′ 6″	6′ 4¼″	25	4′ 11½″	6′ 5″	15
54	54	6′ 0″	6′ 11⅛″	27½	5′ 6¾″	7′ 1¾″	16½
60	60	6′ 6″	7′ 6″	30	6′ 2″	7′ 10⅝″	18

Diam. of pipe (in.)	*h* (in.)	*i* (in.)	*j* (in.)	*k* (in.)	*l* (in.)	*m* (ft. -in.)	*n* (in.)
42	10	5¾	13½	10	4½	4′ 6″	3½
48	10	5¾	14½	10	5	5′ 0″	4
54	11	6⅜	15⅞	11	5½	5′ 6″	4½
60	12	7	17⅜	12	6	6′ 0″	5

Figure 22 shows a reinforced-concrete box culvert for sizes from 2 ft. by 1 ft. 6 in. to 12 by 12 ft., and the dimensions and quantities of concrete are given in the table below.

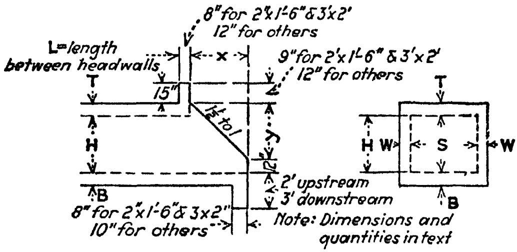


FIG. 22.—Box culvert (Missouri practice).

DIMENSIONS AND QUANTITIES OF BOX CULVERTS
(Accompanying Fig. 22)

S (ft.)	*H* (ft.)	*W* (in.)	*T* (in.)	*B* (in.)	*x* (ft.-in.)	*y* (ft.-in.)	Max. fill (ft.)	Concrete (cu. yd.)
2	1½	6	6	6	2′ 3″	1′ 6″	20	$1.25 + 0.17L$
3	2	6	6	6	3′ 0″	2′ 0″	14	$1.83 + 0.22L$
4	2½	6	6	6	3′ 4½″	2′ 3″	10	$2.79 + 0.28L$
5	3	7	8	7	4′ 4½″	2′ 11″	10	$4.22 + 0.42L$
6	4	8	9	7½	6′ 0″	4′ 0″	10	$6.49 + 0.57L$
4	6	10	7½	6	8′ 10″	5′ 10½″	10	$8.58 + 0.61L$
8	4	10	10	8½	6′ 1½″	4′ 1″	3	$9.0 + 0.80L$
8	6	10	10	8½	9′ 1½″	6′ 1″	3	$13.0 + 0.92L$
8	8	10	10	8½	12′ 1½″	8′ 1″	3	$17.8 + 1.05L$
10	6	12	12	10	9′ 4½″	6′ 3″	3	$17.5 + 1.26L$
10	8	12	12	10	12′ 4½″	8′ 3″	3	$23.7 + 1.41L$
10	10	12	12	10	15′ 4½″	10′ 3″	3	$30.8 + 1.55L$
12	6	12	14	11½	9′ 7½″	6′ 5″	3	$21.3 + 1.55L$
12	8	12	14	11½	12′ 7½″	8′ 5″	3	$28.3 + 1.70L$
12	10	12	14	11½	15′ 7½″	10′ 5″	3	$36.2 + 1.84L$
12	12	12	14	11½	18′ 7½″	12′ 5″	3	$45.0 + 1.99L$

HAUL AND OVERHAUL

76. Haul

In contracts for road grading it is usually stipulated that the contractor shall be paid a certain price per cubic

yard for excavating, hauling, and dumping the material, regardless of distance hauled, provided it does not exceed a specified limit called *free haul.* This free-haul distance may be 500 to 1,000 ft. as per agreement.

If there is an *overhaul* on some of the material, that is, the distance from excavation to embankment is beyond the free-haul limit, then an extra charge may be allowed. The unit overhaul price is based on hauling one cubic yard a distance of one station (100 ft.). This varies from about $1\frac{1}{2}$ to $2\frac{1}{2}$ cents.

As will be seen later, *overhaul* is expressed in terms of *cubic yards hauled* $\times$ *average excess distance* (in stations), which means so many *yard stations.* Thus if 30 cu. yd. are hauled an average distance of 1,500 ft. when the free-haul limit is 500 ft., the overhaul would be $30 \times 10 = 300$ yd.-sta.

A *mass diagram* (see Art. 78) is helpful in determining the amount of overhaul and the most economical distribution of the excavated material. It may also indicate whether in some cases waste and borrow is economical.

77. Limit of Economical Haul

Where there are long hauls, it may be more economical to waste and borrow material rather than pay for the cost of overhaul. Equating the cost of excavation plus overhaul to the cost of excavation from both the cut and the borrow pit, one can estimate the limit of economical haul for making an embankment. Thus, letting

c = cost of excavation per cubic yard;
h = cost of overhaul, on basis of 1 cu. yd. per station;
x = economical length of overhaul.

The cost to excavate and move 1 cu. yd. of material from cut to fill $= c + hx$ (*a*)

The cost to excavate from the cut, waste, borrow, and place 1 cu. yd. of material in the fill $= 2c$ (*b*)

Equating Eqs. (*a*) and (*b*), and solving for x, we have

$$x = \frac{c}{h} \text{ (stations)} \qquad (c)$$

EXAMPLE. The cost of excavation (c), is 25 cents per cubic yard, and the cost of overhaul (h) is 2 cents, what is the economical length of overhaul?

SOLUTION. Substituting in the foregoing Eq. (c), we have

$$x = \tfrac{25}{2} = 12.5 \text{ stations} = 1{,}250 \text{ ft.}$$

Hence, assuming a free-haul limit of 500 ft., the material may be hauled a distance of 1,750 feet for the same cost as to *waste* and *borrow* near by.

78. Mass Diagram

a. Definition. A mass diagram is a continuous curve showing the accumulated algebraic sum of the yardage [cuts (+) and fills (−)] from some initial station to any succeeding station. It is drawn to the same horizontal scale as the profile.

Preliminary to drawing the mass curve, it is convenient to tabulate the accumulated yardage of cuts and fills at each station, in turn, as indicated below.

Sta.	Cubic yards		Algebraic sum of cuts and fills
	Cut (+)	Fill plus shrinkage (−)	
12	...	...	0
	152		
+50	...	...	152
	132		
13	...	...	284
	210		
14	...	...	494
	...	89	
15	...	...	405
	...	133	
16	...	...	272

The points of the mass curve are plotted with reference to a horizontal scale of *distances* (same as the profile) and a vertical scale of *yards* (1 in. = 500 cu. yd., for example).

b. Characteristic Properties. A study of the mass diagram (or curve) shown in Fig. 23 will verify the following statements:

1. Within the limits of a single cut, the curve rises from left to right; within the limits of a single fill, it falls from left to right.

2. Sections where the yardage changes from cut to fill correspond to a maximum; sections where the yardage changes from fill to cut correspond to a minimum.

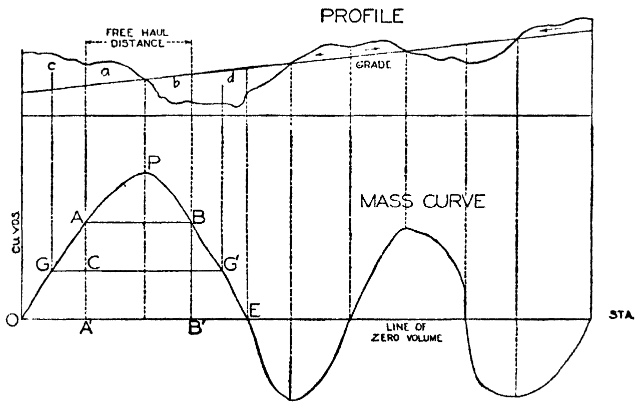


FIG. 23.

3. Any horizontal line, as AB, cutting off a loop of the mass curve intersects the curve at two points between which the cut is equal to the fill (adjusted for shrinkage), the volume of the cut being represented by the vertical distance from the line to the extreme point of the loop.

4. When the curve is above the initial line of zero yardage, it indicates that material must be hauled to the right; when below the line, material must be hauled to the left: as shown by the arrows.

5. The loops *convex* upward indicate that the haul from cut to fill is to be in one direction (to the *right* in this case), while loops *concave* upward indicate a reverse direction for haul.

c. Determination of Overhaul from the Mass Diagram. One of the important uses of the mass diagram, aside from balancing cuts and fills and indicating the most advantageous distribution of the same, is to establish definitely the overhaul distance and the portion of the total yardage which is to be regarded as hauled beyond the specified free-haul limit.

Referring to Fig. 23, proceed as follows:

1. Assuming the free-haul distance to be 500 ft., find by trial a horizontal line intersecting the curve at points A and B, such that $AB = 500$ ft.

2. Then the material above AB will be hauled with no extra cost. The amount of this material is given by the ordinate from line AB to point P, and it is the same as the volume in cut a, which makes the fill b.

3. On material OA between line AB and the base line, there may be paid *overhaul.* The amount of this material is given by the ordinate $A'A$, and it is the same as the volume in cut C, which makes the fill d. The average length of haul is evidently the distance between the center of gravity of cut c and fill d.

These gravity lines are easily found, thus: Bisect AA' at C and draw a horizontal line intersecting the curve at G and G'; these points are vertically below the desired centers of gravity; therefore the average haul is given by the length of line GG', and the *overhaul* is the distance GG' less the free-haul distance AB.

CHAPTER III

CIRCULAR CURVES

79. Classification

The surveyed center line of roads consists of a series of *straight lines* and *curves*. Technically, the straight lines are called *tangents*, and a curve uniting two intersecting tangents is known by its *radius* or by the angle subtended at the center of an *arc of* 100 *ft*.

Circular curves are classified as *simple*, *compound*, and *reversed*.

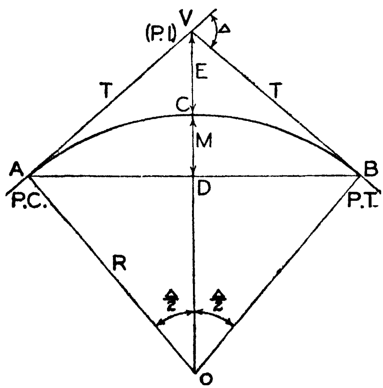


FIG. 24.

SIMPLE CURVES

80. Functions or Parts of Curves

A *simple curve* is a circular arc joining two tangents. Figure 24 shows such a curve with all its related parts, or functions, which will now be explained.

1. If the tangents be produced, they will meet at a *point of intersection*, called the P. I. (or vertex, V).

2. Proceeding from left to right around the curve, point A, the *beginning* of the curve is called the P. C. (*point of curvature*); while point B, the *end* of the curve, is called the P. T. (*point of tangency*).

3. The external angle of deflection between the tangents is called the *intersection angle*, or the *angle*. This angle (Δ) is equal to the central angle subtended by the arc AB.

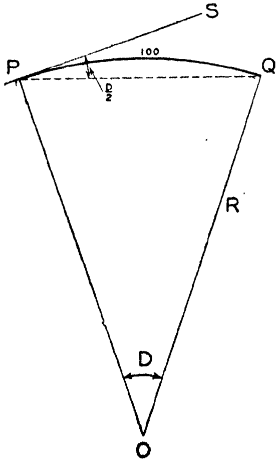


FIG. 25.

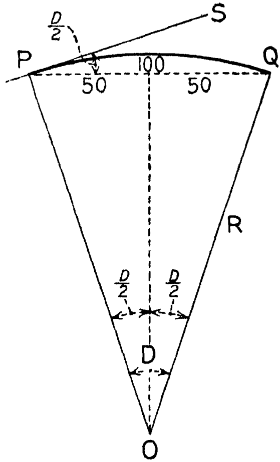


FIG. 26.

4. *a.* The angle at the center subtended by an *arc* of 100 ft. is called the *degree-of-curve*, D. That is, if the arc PQ (Fig. 25) is 100 ft. in length, the central angle POQ is D, the degree-of-curve.

b. In conformity with railroad practice, some highway engineers prefer the definition that the *degree-of-curve* is the central angle subtended by a *chord* of 100 ft. (see Fig. 26).

These two definitions will be discussed further in Art. 83.

5. The *radius* of the curve is called R. There will be a slight difference in R depending upon whether the (*a*) *arc* or (*b*) *chord* definition is used. See Table 3.

a. From Fig. 25, we have the familiar proportion:

$$100:2\pi R = D^\circ:360^\circ,$$

from which $$R = \frac{5{,}729.58}{D} \text{ (\textit{arc} definition)} \tag{9}$$

showing that R varies inversely as D.

Approximately, we have

$$R = \frac{5{,}730}{D}, \quad \text{or} \quad D = \frac{5{,}730}{R}, \tag{10}$$

giving results sufficiently close for most calculations.

Thus, if $D = 10^\circ$, $R = 573$ ft.; if $D = 2^\circ$, $R = 2{,}865$ ft., etc.

b. From Fig. 26, we have

$$\sin \frac{1}{2}D = \frac{50}{R},$$

from which $$R = \frac{50}{\sin \frac{1}{2}D} \text{ (\textit{chord} definition)} \tag{11}$$

6. The distance VA ($= VB$) from the P. I. to the P. C., (or to the P. T.) is called the *tangent distance*, T. Figure 24 shows that

$$T = R \tan \tfrac{1}{2}\Delta. \tag{12}$$

7. The straight-line distance, AB, from the beginning to the end of the curve is called the *long chord*, or L. C. From Fig. 24,

$$\sin \frac{1}{2}\Delta = \frac{AD}{OA} = \frac{\frac{1}{2}(\text{L. C.})}{R},$$

from which $$\text{L. C.} = 2R \sin \tfrac{1}{2}\Delta. \tag{13}$$

8. The *external distance*, E, is the distance from the P. I. to the middle of the curve. From Fig. 24,

$$\cos \frac{1}{2}\Delta = \frac{OA}{OV} = \frac{R}{R + E},$$

from which $$E = R(\sec \tfrac{1}{2}\Delta - 1) = R \text{ exsec } \tfrac{1}{2}\Delta. \tag{14}$$

9. The *middle ordinate*, M, is the length of the ordinate from the middle of the long chord to the middle of the curve. From Fig. 24,

$$\cos \frac{1}{2}\Delta = \frac{OD}{OA} = \frac{R - M}{R},$$

from which $$M = R(1 - \cos \tfrac{1}{2}\Delta) = R \text{ vers } \tfrac{1}{2}\Delta. \qquad (15)$$

10. The *length of curve*, L, is the arc length AB from P.C. to P.T.

Since any two central angles of the same circle are proportional to the corresponding intercepted arcs, we have

$$\frac{\Delta}{D} = \frac{L}{100},$$

from which $$L = 100\frac{\Delta}{D}. \qquad (16)$$

Equation (16) gives the exact length of arc, but if a series of chords totaling L feet, are used, and chosen small enough so that no chord differs appreciably from its arc, then the sum of the chords may be taken the same as L, the length of curve.

EXAMPLE. Given a 6° curve when the intersection angle is 54° 36′. What is the length of curve?

SOLUTION. $L = 100 \times \frac{54.6}{6} = 910$ ft.

Note: It is obvious that the foregoing formula for L holds true equally well for the *chord* definition of degree-of-curve. But L in this case really means the length along the curve of an inscribed polygon with sides of 100 ft. or less.

11. Referring to Fig. 25, it is seen that the angle SPQ between the tangent and chord is $\frac{1}{2}D$, or one-half the central angle. In general, an angle between the tangent to a curve and the chord to any point on the curve is called the *deflection angle.*

In Fig. 24, the *total deflection angle* (angle VAB) from P.C. to P.T. is $\frac{1}{2}\Delta$.

The *deflection angle* may be expressed in terms of the *arc* and *radius* by making use of the geometrical proportion:

$$\frac{\text{Central angle}}{\text{arc}} = \frac{360°}{2\pi R},$$

from which $$\text{Central angle} = \frac{360}{2\pi R} \times \text{arc};$$

hence Deflection angle (degrees) = $\frac{1}{2}$(central angle) =

$$\frac{28.64789}{R} \times \text{arc}.$$

Also, $$\text{Deflection angle (minutes)} = \frac{1718.873}{R} \times \text{arc}.$$

Obviously it is much simpler to express deflections in terms of the degree-of-curve rather than the radius, but this practice is not universal.

Table 4 (page 266) gives deflections and chords for even radius curves.

81. The Unit Curve

The *lengths* of any two curves with the *same central angle* (Δ) are in direct proportion to the corresponding radii. Hence

$$\frac{L}{L'} = \frac{R}{R'}.$$

Since $T = R \tan \frac{1}{2}\Delta$, and $T' = R' \tan \frac{1}{2}\Delta$, we have

$$\frac{T}{T'} = \frac{R}{R'} = \frac{L}{L'}.$$

Similarly, it may be shown that *all* parts of one curve are proportional to the corresponding parts of any other curve having the same central angle. But it should be noted that R varies *inversely* with D, hence

$$\frac{R}{R'} = \frac{D'}{D} = \frac{L}{L'}, \text{ etc.}$$

The functions of curves are therefore multiples of any one part which may be called the *unit curve.* Thus we may have (*a*) *the* 1° *curve* (which is widely used in American

practice), (*b*) *the curve of unit length,* and (*c*) *the curve of unit radius.*

a. The 1° *Curve.* If $D = 1°$, then $R = 5{,}729.58$ ft.

Hence $$T = 5{,}729.58 \tan \tfrac{1}{2}\Delta,$$
$$E = 5{,}729.58 \text{ exsec } \tfrac{1}{2}\Delta, \text{ etc.}$$

Table I (second part) gives the tangents and externals for a 1° curve when the intersection angle (Δ) is known. The values of T and E in this table are the same for either the *arc* or *chord* definition of degree-of-curve, since for a 1° *curve*, there is no appreciable difference between the arc and its subtended chord.

The proper values of T and E for any case are the tabular values *divided* by the actual degree-of-curve (D).

EXAMPLE. Given $\Delta = 47° 20'$, $D = 12°$. To find T and E. Referring to Table 1:

$$T = \frac{T \text{ (for } 1° \text{ curve)}}{12} = \frac{2{,}511.2}{12} = 209.27;$$

and $$E = \frac{E \text{ (for } 1° \text{ curve)}}{12} = \frac{526.1}{12} = 43.84.$$

The foregoing values of T and E are correct for the *arc* definition. But for the *chord* definition, a *correction* of 0.38 ft. is to be *added* to T (see Table 1*A*), giving $T = 209.65$ ft.

b. Unit Arc of 100 *Ft.* If the unit curve is defined as one whose length is 100 ft., then

$$L = 100 = 100\frac{\Delta}{D}; \qquad \text{from which } D = \Delta.$$

Then radius of unit curve (*arc* definition) $= 5{,}729.58/\Delta$.

Table 1 (first part) gives values of R, T, E, L. C., and M for the unit curve of length 100 ft. For curves longer than 100 ft., the tabular values must be *multiplied* and the degree-of-curve *divided* by the length of curve (in stations).

EXAMPLE. Given $\Delta = 40° 00'$ and $L = 500$ ft. To find R, T, E, L. C., M, and D.

Using Table 1, we have $R = 143.24 \times 5 = 716.2$; $T = 52.135 \times 5 = 260.7$; $E = 9.193 \times 5 = 46.0$; L. C. $= 97.982 \times 5 = 480.0$; $M = 8.638 \times 5 = 43.2$. Also, it follows that

$$D = \frac{40°}{5} = 8°.$$

This table is especially useful in laying out curves by the method of *offsets*.

c. Unit Radius ($R = 1$). Letting $\Delta°$ equal the central angle subtending any arc of radius (R), we have the familiar proportion:

$$\frac{Arc}{2\pi R} = \frac{\Delta°}{360°},$$

from which $$\text{Arc length} = \frac{\pi}{180} \times R\Delta = \frac{R\Delta}{57.2958}. \qquad (17)$$

If $R = 1$, Table 5 gives lengths of arcs for various values of the central angle (Δ).

Using Tables 1 and 5, the corresponding parts of a curve may be found for cases when Δ and R are given.

EXAMPLE. Given $\Delta = 60°\ 10'$ and $R = 500$ ft. To find L, T, and E. With $R = 1$ and $\Delta = 60°\ 10'$, Table 5 gives arc length $= 1.0501$.

Since the actual value of R is 500, actual arc length $= 525.05$.

Finally, entering Table 1 with $\Delta = 60°\ 10'$, we have $T = 55.165 \times 5.2505 = 289.6$; and $E = 14.824 \times 5.2505 = 77.8$.

The above values of T and E may be found another way, as follows:

$T = R \tan \frac{1}{2}\Delta = 500 \times 0.57929$ (from Table 21) $= 289.6$ ft.,
$E = R \operatorname{exsec} \frac{1}{2}\Delta = 500 \times 0.15567$ (from Table 22) $= 77.8$ ft.

82. Excess of Arc over Chord

The difference between any arc and its subtended chord is

$$R\theta - 2R \sin\frac{\theta}{2} = R\left(\theta - 2\sin\frac{\theta}{2}\right),$$

where θ is the central angle in radians.

Substituting $R = \text{arc/angle} = L/\theta$, and expressing the sine in terms of the angle,

$$\text{Arc excess} = \frac{L}{\theta}\left[\theta - 2\left(\frac{\theta}{2} - \frac{\theta^3}{48} + \cdots\right)\right] = \frac{L\theta^2}{24} = \frac{L^3}{24R^2}$$
(approx.) (18)

Letting $L = 100$ ft. and expressing R in terms of D,

$$\text{Arc excess} = 0.00127D^2 \text{ for arcs of 100 ft.}$$

For arcs of 50 ft., arc excess = $\frac{1}{8}$ (of that for 100 ft.); for arcs of 25 ft., arc excess = $\frac{1}{64}$ (of that for 100 ft.), etc.

Table 2 gives the length of chord subtended by various arcs.

An inspection of this table shows that the following chords may be assumed equal to the arcs without any appreciable error:

100-ft. chords up to	4° curves.
50-ft. chords up to	10° curves.
25-ft. chords up to	25° curves.
10-ft. chords up to	100° curves.

EXAMPLE. Suppose a 30° curve is to be laid out. What is the longest permissible chord length that will not differ appreciably from the subtended arc? Table 2 shows that 20-ft. chords are the longest advisable.

If, however, 50-ft. points along the curve are desired, then corresponding chord-lengths are 49.86 ft.

83. Discussion of the Definition, Degree-of-curve

Books on railroad location define the degree-of-curve as the central angle subtended by a *chord* of 100 ft. rather than an *arc* of 100 ft. In highway engineering practice, the tendency is to use the *arc* definition, since it seems simpler and leads to no confusion in dealing with short-radius curves. The real difference between the two definitions is expressed by the formulas for radii:

$$R_1 = \frac{5{,}729.58}{D}, \text{ for "arc" definition,}$$

$$R_2 = \frac{50}{\sin \frac{1}{2}D}, \text{ for "chord" definition.}$$

Table 3 gives comparative values of D and R on the basis of both definitions. The difference is negligible for flat curves but quite appreciable for sharp curves.

If $D = 1°$, then R_1 (*arc* definition) = 5,729.58, and R_2 (*chord* definition) = 5,729.65. These values of R_1 and R_2 are so nearly the same that Table 1 (second part), giving functions of a 1° curve, applies to either definition with sufficient accuracy.

The formula, $L = 100\frac{\Delta}{D}$, for actual length of curve is *exact* according to the *arc* definition, but only approximate for the *chord* definition; and it departs too far from the truth for sharp curves.

To illustrate the confusion which is apt to occur, suppose $\Delta = 90°$ and a 40° curve is to be run in the field:

$R_1 = 143.24$ and $R_2 = 146.19$, a difference of practically 3 ft.

$$\text{Tangent distance} = \frac{40)5{,}729.6}{143.2},$$

$$\text{External distance} = \frac{40)2{,}373.3}{59.33}.$$

The above values of T and E are the same for either definition, but they exactly fit an arc of length

$$L = \frac{100\Delta}{D} = \frac{100 \times 90}{40} = 225.0 \text{ feet},$$

which is the true length of curve of radius = 143.24 ft.

If short chords coinciding practically with the arc (or long chords reduced as per Table 2) are used, then the field work should check, and there is no correction of any kind necessary.

But according to the "chord definition," the nominal length of curve (the perimeter of an inscribed polygon with sides of 100 ft.) is

$$L = \frac{100 \times 90}{40} = 225 \text{ ft.}$$

Now the actual length of arc corresponding to the two chords of 100 ft. and the subchord of 25 ft. is 229.6 ft., and its radius is 146.19 ft. In order to make the above tangents fit this arc, *corrections* must be added. In this case (for $\Delta = 90°$), the correction given in Table 1*A* is 2.94 ft. Hence, corrected tangent = 143.22 + 2.94 = 146.2 ft.

The use of the "correction" implies 100-ft. chords, but chords of such great length would not properly define a 40° curve. Therefore, if short chords are used in laying out a curve with its P.C. and P.T. established by adding "corrections" to the tangents, then, of course, the field work will not check. There would probably be confusion, too, as to how the P.T. stake should be labeled, resulting in a possible error of a few feet in distance.

The foregoing illustrative example is an extreme case. Fortunately, the majority of highway curves are much flatter, so that the adoption of either definition leads to practically the same results.

Tables 1 to 16 given herein are adaptable to either definition.

The author prefers the *arc* definition for the following reasons: (1) Radius and degree-of-curve are in exact inverse proportion; (2) formula $L = 100\Delta/D$ gives true length of curve; (3) the field work in laying out sharp curves offers no more difficulty than for flat curves.

84. Problem

a. To Find R When D Is Known. Suppose $D = 4°\ 30'$, then from Table 3, R (*arc* definition) = 1,273.2 ft., and R (*chord* definition) = 1,273.6 ft.

b. To Find D When R Is Known. Suppose R = 550 ft., then D (*arc* definition) = $\frac{5730}{550}$ = 10.42° = 10° 25′; and $\sin \frac{1}{2}D$ (*chord* definition) = $\frac{50}{550}$ = 0.09091, whence D = 10° 26′.

c. To Find T and E When Δ and R Are Known. Suppose Δ = 40° 12′ and R = 1,000 ft.,

then $\quad T = R \tan \frac{1}{2}\Delta = 1{,}000 \times 0.36595 = 365.95$ ft.,

and $\quad E = R \operatorname{exsec} \frac{1}{2}\Delta = 1{,}000 \times 0.06486 = 64.86$ ft.

85. Problem

To Find D (or R) When Δ and E Are Known. **Referring to Fig. 24, let the two tangents *AV* and *VB* intersect at an angle**

$\Delta = 42° 30'$. Suppose, after careful examination of the ground, point C opposite vertex is selected as a point through which center of road should run.

Measure $VC = 42$ ft.

From Table 1, E (for 1° curve) $= 418.0$.

Hence $D = 418.0/42 = 9.95$. Let $D = 10°$; and hence revised $E = \frac{418}{10} = 41.8$ ft.

86. Problem

To Find D (or R) When Δ and T Are Known. Given $\Delta = 42°$ $30'$ and suppose the tangent lengths (T) are limited to 203 ft.

$$T \text{ (for 1° curve)} = 2{,}228.1.$$

Hence $$D = \frac{2{,}228.1}{203} = 10.98°.$$

For convenience, let $D = 11°$.

Then $$T = \frac{2{,}228.1}{11} = 202.55 \text{ ft.}$$

87. Problem

To Find All the Functions When Δ and E Are Known. Given $\Delta = 25° 06'$ and $E = 45$ feet (more or less).

We shall first decide upon the value of D from which the other functions can be determined easily.

From Table 1, $$D = \frac{E \text{ (for 1° curve)}}{45} = \frac{140.25}{45} = 3.12°.$$

For convenience, let

$$D = 3°, \qquad \text{then} \qquad E = \frac{140.25}{3} = 46.75,$$

which is permissible.

Hence, $$T = \frac{T \text{ (for 1° curve)}}{3} = \frac{1{,}275.5}{3} = 425.2.$$

Also, we have $$L = 100\frac{\Delta}{D} = 100 \times \frac{25.1}{3} = 836.67.$$

Then, multiplying the values of L.C. and M (from Table 1) by the length of curve, in stations, we have

$$\text{L.C.} = 99.202 \times 8.3667 = 830.0,$$

and $$M = 5.454 \times 8.3667 = 45.6.$$

88. Field Procedure in Locating P.C. or P. T.

Existing conditions may vary the field procedure, but ordinarily the tangents are first run to an intersection, and the angle Δ is carefully read.

The degree-of-curve (D) is then usually determined indirectly from controlling values of E or T, bearing in mind that it is desirable to make the curve as flat as considerations of topography and economy will permit.

The P.C. and P.T. should be established independently by measuring the tangent distances VA and VB (Fig. 24) while the transit is set at the P.I. It is important that these points should be exactly in line with distant points (P.O.T.) on the tangents VA and VB produced.

For special cases, such as inaccessible P.I., the foregoing procedure must be modified, as will be shown later.

89. Locating Curves by the Deflection Method

a. When Transit Is Set at the P.C. Preliminary to laying out curve, the following steps are assumed to be taken: (1) tangents run to an intersection at the P.I.; (2) deflection angle (Δ) measured; (3) length and degree-of-curve decided upon either by means of a fixed E or T, or an assumed D or R; (4) the P.T. and P.C. established by measuring the tangent distance from the P.I.; (5) the proper station numbers of P.C. and P.T. recorded. (The example which follows will show how this is done.)

Procedure. Referring to Fig. 25 for a moment, it is seen that the angle SPQ between the tangent and chord is $\frac{1}{2}D$. That is, deflection angle from the tangent to any point is one-half the central angle.

Assume transit set at P.C. (Fig. 27) with vernier reading 0° on initial tangent AV.

Turn telescope until vernier reads angle $VA1$ (one-half central angle for arc a), then measure the proper chord length a and swing tape about A as center until extremity of chord falls in line of sight. This locates point 1 on the curve.

Next, turn telescope until vernier reads angle $VA2$ (this angle will equal the preceding deflection plus $\frac{1}{2}D$ if arc c is 100 ft.). Measure chord c and swing tape about point 1 as center until

extremity of chord falls in line of sight. This locates point 2 on the curve.

The next points in order are located in a similar manner. It is important to mark the curve points accurately in order to avoid cumulative errors. The final deflection angle to point B(P.T.) should equal $\frac{1}{2}\Delta$, and the last chord length should reach the P.T.

b. When Transit Is Set at the P.T. If the entire curve is visible from the P.T., one setup of the instrument may be

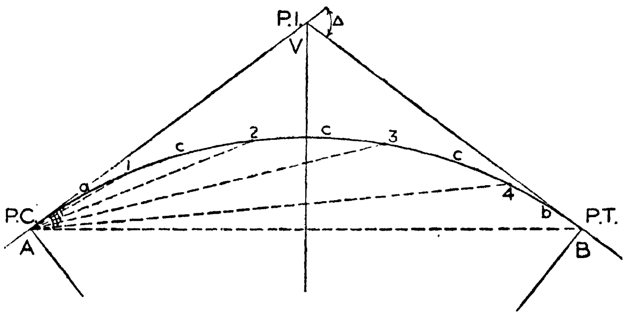


FIG. 27.

avoided if the curve is run in entirely by deflections from the P.T. instead of the P.C., since the instrument would be in position for proceeding forward on the next tangent.

When the transit set at the P.T., with the telescope in its normal position, the instrument is properly oriented either by sighting on the P.I., vernier reading $\frac{1}{2}\Delta$, or by sighting on the P.C., vernier reading 0° 00′. Then it follows that *any point on the curve has the same deflection from the* P.T. *as it would have if located from the* P.C.

Assuming that the instrument is oriented as stated above, the curve notes are the same whether deflections are from the P.C. or P.T. (See example which follows.)

For long curves, it is better to run in the second half by starting the measurements and deflections from the P.T.; for in this way, any small errors of surveying can be adjusted

at the middle of the curve where a slight deviation in the alignment is of less consequence than at points of tangency to the curve.

EXAMPLE. Given $\Delta = 52° 12'$ and P.I. at Sta. 47 + 05.4. To find complete curve data.

From Table 1, T (for 1° curve) = 2,807.0.
E (for 1° curve) = 650.6.

To make curve fit the ground in the most feasible way, actual measurement shows that E should equal 54 ft., approximately.

$$\text{Hence} \qquad D = \frac{650.6}{54} = 12.05°.$$

$$\text{Let } D = 12°, \text{ then } E = \frac{12)650.6}{54.22}$$

$$T = \frac{12)2{,}807.0}{233.92.}$$

$$L = 100 \times \frac{52.2}{12} = 435.0, \qquad R = \frac{5{,}729.58}{12} = 477.5.$$

$$\begin{aligned} \text{P.I.} &= \text{Sta. } 47 + 05.4 \\ T &= \quad 2 + 33.9 \\ \hline \text{P.C.} &= \text{Sta. } 44 + 71.5 \\ L &= \quad 4 + 35.0 \\ \hline \text{P.T.} &= \text{Sta. } 49 + 06.5 \end{aligned}$$

Deflection per 100 ft. $= \frac{1}{2}D = 6° 00' = 360'$
Deflection per 1 ft. $= 3.6'$
Transit at P.C., (*or* P.T.), *vernier reading* 0° *on tangent.*

Sta.	Deflection
44 + 71.5 (P.C.)	0° 00′
45 + 00	1° 42.6′
+ 50	4° 42.6′
46 + 00	7° 42.6′
+ 50	10° 42.6′
47 + 00	13° 42.6′
+ 50	16° 42.6′
48 + 00	19° 42.6′
+ 50	22° 42.6′
49 + 00	25° 42.6′
+ 06.5 (P.T.)	26° 06.0′ (check) = $\frac{1}{2}\Delta$

c. When Transit Is Set at Any Intermediate Point on the Curve. Assume the part AC of curve (Fig. 28) to be located by deflections from point A (P.C.). If station point D is not visible from the P.C., then the instrument may be moved to point C and the location of curve continued by the method of deflections. If, however, the obstruction is a small object, as a tree, then points on either side of D and the remainder of curve may be located by deflections from the P.C., or the instrument may be moved to the P.T. and the curve including point D run in by means of deflections from the P.T.

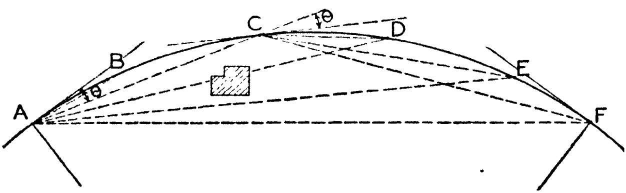


Fig. 28.

Assume transit set at point C.

There are two methods of procedure, depending upon how the vernier is set for a backsight on preceding curve point A. Both methods involve a proposition in geometry, namely, The angle between a chord and a tangent is measured by one-half of the intercepted arc. Therefore angle θ between chord AC extended and the tangent at C is the same as the total deflection angle from tangent at A to point C.

Method a. Backsight on A with the telescope inverted and the vernier reading the total deflection from A to C (angle θ), the angle being reset on the opposite side of 0°, reverse the telescope, unclamp the alidade and turn until the angle reads 0°; then the line of sight is on the forward tangent to curve at C. The forward points D, E, etc. are located by setting off angles equal to one-half the central angles for arcs CD, CE, etc.

Method b. Backsight on A with telescope inverted and vernier reading 0°, reverse telescope, unclamp alidade, and set vernier to read the deflection to the next point as if it were located from A; this locates point D. In a similar manner the other forward points are located.

This method has the advantage of using the deflections which were calculated for laying out the entire curve from the P.C.

If a second move (*i.e.*, a third setup) is necessary, the P.C. may be visible from that point and should be used if possible. However, any previously located point may be used for a backsight with vernier set at the deflection of the point sighted on as calculated for laying out from the P.C. In other words, *if the transit is at any point on a curve and is sighted to any other point on the curve with vernier set at the deflection from* P.C. *to the point sighted, then the deflection to any desired point on the curve will be that originally calculated from the* P.C. *to the point desired.* Hence transit at any point should read 0° on P.C., first deflection on first point, . . . fifth deflection on fifth point and so on.

d. When Transit Is Set at the P.I. Referring to Fig. 29, the transit is set at V, the point of intersection of the tangents, and the telescope is directed to any point P on the circular arc AB.

Let α = angle PVB (= PVA when point P is on the other side of VM).

$$\theta = \text{angle } POB, \text{ etc.}$$

$$\tan \alpha = \frac{PN}{VN} = \frac{DB}{VB - NB} = \frac{R \text{ vers } \theta}{T - R \sin \theta}. \qquad (19)$$

Substituting $T = R \tan \frac{1}{2}\Delta$ and dividing numerator and denominator by R,

$$\tan \alpha = \frac{\text{vers } \theta}{\tan \frac{1}{2}\Delta - \sin \theta}. \qquad (20)$$

Solution of the above formula gives deflections from the tangent VB (or VA) to any point on the curve. It is independent of the radius or length of the curve.

Imagine the entire curve divided into 10 equal parts (see Fig. 30), starting at either the P.T. or P.C.

Let α_1 and θ_1 represent the corresponding angles to point 1; α_2 and θ_2 the angles to point 2; etc.

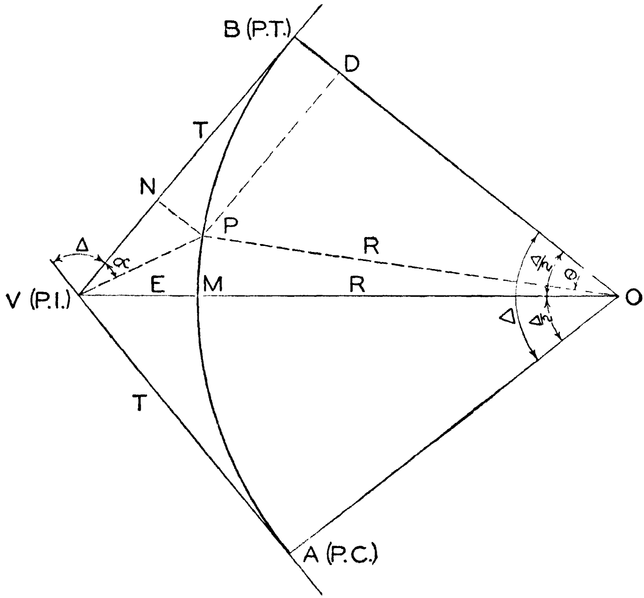


FIG. 29.

It is obvious from Fig. 30 that $\theta_1 = \frac{1}{10}\Delta$, $\theta_2 = \frac{2}{10}\Delta$, etc.

By means of Eq. (20), the deflections α_1, α_2, etc., to each of the 10 points of equal division along the circular arc for various values of Δ have been computed and compiled in Table 28. Interpolations over ranges of 1° in Δ, give results as closely as 1 minute in the deflection angle.

The middle point of the curve (point 5) is located independently by the deflection angle α_5 or its equivalent $(90 - \frac{1}{2}\Delta)$, and the external distance (E). All other points on the curve are located by deflections taken directly from

Table 28, and chord measurements *along the curve* (not from the P.I.).

Assuming the P.I. to be accessible, this method has the advantage that the curve may be staked out in one operation while the transit is set and oriented at the P.I. for the purpose of finding the angle Δ, thus saving much time.

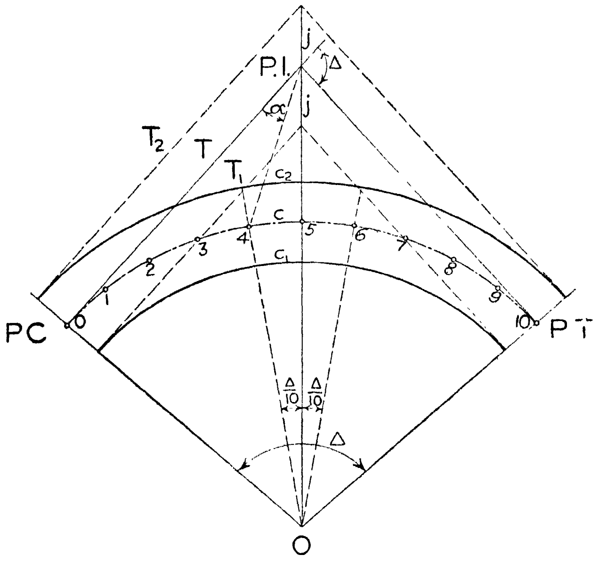


FIG. 30.

The points on the curve are at equal distances apart, but they are not *full stations.* For this reason, it is *not recommended as the best general method of locating highway curves.*

EXAMPLE. Given $\Delta = 40° \; 00'$, P.I. at Sta. $62 + 11.8$. It is required to write down curve data on the basis of location from the P.I.

From Table 1, for $L = 100$ ft.; $E = 9.193$, $T = 52.135$.

Suppose $E = 46.5$ fits the ground.

Hence Ratio $= \frac{46.5}{9.2} = 5.05$.

Let $L = 500$ ft., then $E = 9.193 \times 5 = 45.96$.

Also, $T = 52.135 \times 5 = 260.7$, and $D = \frac{40}{5} = 8°$.

$$\begin{aligned} \text{P.I.} &= 62 + 11.8 \\ T &= \underline{\ \ 2 + 60.7} \\ \text{P.C.} &= 59 + 51.1 \\ L &= \underline{\ \ 5\qquad\quad} \\ \text{P.T.} &= 64 + 51.1 \end{aligned}$$

Referring to Table 28 with $\Delta = 40°\ 00'$ as argument, the deflections are as given below, and the equal chords are each 50 ft.

Transit set at the P.I., vernier reading 0° 00′ *on initial tangent.*

Point	Sta.	Deflection
P.C.	59 + 51.1	0° 00′
1	60 + 01.1	0° 29′
2	60 + 51.1	2° 29′
3	61 + 01.1	7° 58′
4	61 + 51.1	23° 41′
5	62 + 01.1 ($E = 46.0$)	70° 00′
6	62 + 51.1	116° 19′
7	63 + 01.1	132° 02′
8	63 + 51.1	137° 31′
9	64 + 01.1	139° 31′
10 (P.T.)	64 + 51.1	140° 00′

As a check in taking deflections from the table, it should be noted that the first + ninth = second + eighth = third + seventh = fourth + sixth = $180 - \Delta = 140°$.

90. Locating Curves by the Offset Method

a. Offsets from the Tangent. In Fig. 31, it is required to locate any point P along curve ACB by means of the distance AN ($= x$) along the tangent and the perpendicular offset NP ($= y$).

If θ is the central angle for any arc AP, then $x = AN = DP = \frac{1}{2}$ (long chord) for arc subtending angle 2θ.

Also, $y = NP$ = middle ordinate for arc subtending angle 2θ.

Proceed as follows:

(1) Knowing the central angle (2θ) of twice the arc AP, Table 1 gives values of L.C. and M on the basis of 100-ft. arc lengths; (2) Multiply the tabular value of L.C.

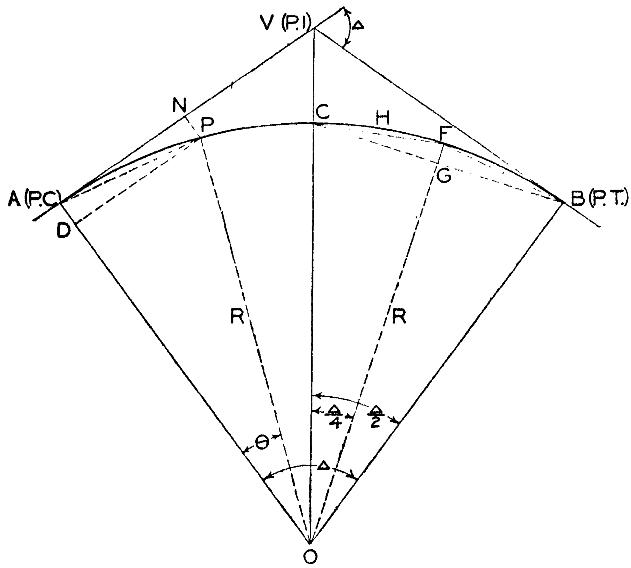


Fig. 31.

by the arc distance AP (in stations), giving x. (3) Multiply the tabular value of M by *twice* the arc distance AP (in stations), giving y.

Thus, if $\Delta = 30° \ 30'$, $D = 5°$, $L = 610$ ft., and P.C. is at Sta. 47 + 60; the values of x and y locating Sta. 48 (40 ft. ahead) are obtained as follows:

Entering Table 1 with central angle of 4° 00′ (subtended by 40-ft. arc *doubled*), we have

$$x = 99.980 \times 0.4 = 39.99 \text{ ft.,}$$

and $$y = 0.873 \times 0.8 = 0.70 \text{ ft.}$$

Proceeding in this way, the entire curve may be located, as will be seen in the example to follow.

EXAMPLE. Given $\Delta = 30° 30'$, $D = 5°$, $L = 610'$, P.C. at Sta. 47 + 60.

It is required to find the coordinates of station points along the curve; the first half of the curve is referred to tangent AV and the second half to tangent BV. It is possible to refer all points to one tangent, but offsets to the second half of the curve would be comparatively long, which is objectionable.

The necessary data for computing x and y should be tabulated as shown below.

x = (L.C. from Table 1) × (arc length) in stations.
y = (M from Table 1) × (twice arc length) in stations.

DATA FOR LOCATING CURVE OF OFFSETS FROM TANGENTS

Sta.	(1) Distance (stations)	(2) Twice distance (stations)	Twice central angle	From Table 1 (3) L.C.	From Table 1 (4) M	x (3) × (1)	y (4) × (2)
47 + 60 P.C.	0	0	0	0	0	0	0
48	0.4	0.8	4°	99.980	0.873	39.99	0.70
49	1.4	2.8	14°	99.751	3.050	139.65	8.54
50	2.4	4.8	24°	99.271	5.217	238.25	25.04
51	2.7	5.4	27°	99.077	5.863	267.51	31.66
52	1.7	3.4	17°	99.634	3.702	169.38	12.59
53	0.7	1.4	7°	99.938	1.526	69.96	2.14
53 + 70 P.T.	0	0	0	0	0	0	0

Note: Stations 51, 52, and 53 are to be located by offsets from the *second* tangent (at P.T.).

An approximate value of the tangent offset (y) may be obtained as follows:

In Fig. 31, $y = NP = AD = R \text{ vers } \theta$.

Expressing the angle (θ) in radians, and expanding vers θ, we have

$$y = \frac{l\theta}{2} - \frac{l\theta^3}{24} + \cdots = \frac{l\theta}{2} \text{ (approx.)} = \frac{l^2}{2R} \text{ (approx.)} \qquad (21)$$

in which l is any arc length (AP) of radius (R).

Hence the tangent offsets vary approximately as the *square* of the distance along the curve.

b. Offsets from Chords. The middle ordinates for arcs with the *same* central angle are exactly *proportional.*

Hence, in Fig. 31, the middle ordinate (GF) for any arc (BC) equals

(M in Table 1) $\times$ length of arc BC (in stations).

Middle ordinates for the half arcs CF and FB are $\frac{1}{4}(GF)$ (closely).

If arc lengths are not excessive, the middle ordinates vary as the square of the arcs.

For any chord (c) subtending a central angle (Δ), the middle ordinate is

$$M = \tfrac{1}{2}c \tan \tfrac{1}{4}\Delta.$$

Thus, for a 12° curve, M for a 100-ft. chord (or arc) is $50 \times \tan 3°\ 00' = 2.62$ ft., and M for a 50-ft. chord (or arc) is $25 \times \tan 1°\ 30' = 0.65$ ft., etc.

91. Inner and Outer Curves

Frequently, it is necessary to lay out inner or outer parallel circular arcs that are not very far apart radially, such as property lines or offset curves for reference during construction. The *lengths* of these outer and inner arcs may also be desired.

In Fig. 32, let Δ = central angle (in radians) of all or *any part* of the curve;
w_1 = radial width from center to inside curve;
R_1 = radius of inside curve;
l_1 = length of inside curve.

w_2, R_2, l_2 and D_2 are corresponding parts of outside curve. l and R are length and radius of center-line curve. Then

$$l = R \times \text{angle } \Delta;$$
$$R_1 = R - w_1 \quad \text{and} \quad R_2 = R + w_2;$$
also $l_1 = R_1 \times \text{angle } \Delta = l - w_1 \times \text{angle } \Delta = l - a_1$; (22)
and $l_2 = R_2 \times \text{angle } \Delta = l + w_2 \times \text{angle } \Delta = l + a_2$ (23)

If w_1 or w_2 = 10 ft., then a_1 or a_2 may be found in Table 6 for various values of the central angle.

For any value of w_1 or w_2 other than 10 ft., the corresponding values of a_1 or a_2 are proportional.

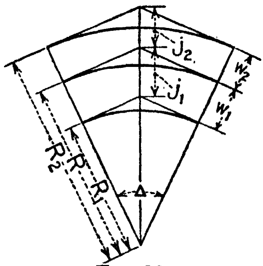


FIG. 32.

EXAMPLE 1. Given $\Delta = 44° 20'$, $w_1 = 12$ ft., and $w_2 = 10$ ft. It is required to find R_1, R_2, l_1, and l_2.

SOLUTION. $R_1 = R - 12$, $R_2 = R + 10$,

$$l_1 = l - a_1 = l - (\text{value of } a \text{ in Table 6}) \times 1.2 =$$
$$l - 7.74 \times 1.2 = l - 9.29;$$
and
$$l_2 = l + a_2 = l + 7.74.$$

EXAMPLE 2. Given the same data as in Example 1, it is required to find the skew distances j_1 and j_2 (see Fig. 32).

SOLUTION. j_1 = (value of j in Table 6) $\times$ 1.2 = 10.80 $\times$ 1.2 = 12.96; and j_2 = 10.80.

92. To Locate Curve When P.I. Is Inaccessible

In Fig. 33, it is required to determine Δ and other data necessary for locating a curve connecting the given tangents AV and VB, point V being inaccessible.

Select two points N and D on tangents AV and VB. Measure angle VND, distance ND and angle VDN.

Then deflection angle $\Delta = VND + VDN$.

Solve triangle VND for NV and VD by the law of sines.

Hence $AN = T - NV$ and $BD = T - DV$, where T depends upon the degree-of-curve adopted; or if it is desirable that the curve should begin at (or near) some point A, then $T = AN + NV$, from which D is determined.

EXAMPLE. In Fig. 33. suppose the following field measurements are taken: $\alpha = 18° 00'$, $ND = 312.2$ ft., and $\beta = 32° 12'$. It is required to find Δ, T, and D.

SOLUTION. $\Delta = \alpha + \beta = 50° \ 12'$. Solving triangle VND, we have

$$NV = \frac{ND \sin \beta}{\sin \Delta} = \frac{312.2 \times 0.53288}{0.76828} = 216.54;$$

also, $$DV = \frac{ND \sin \alpha}{\sin \Delta} = \frac{312.2 \times 0.30902}{0.76828} = 125.57.$$

If the curve starts at (or near) A, then if $AN = 300$ ft., $T = AN + NV = 300 + 216.54 = 516.54$ ft.

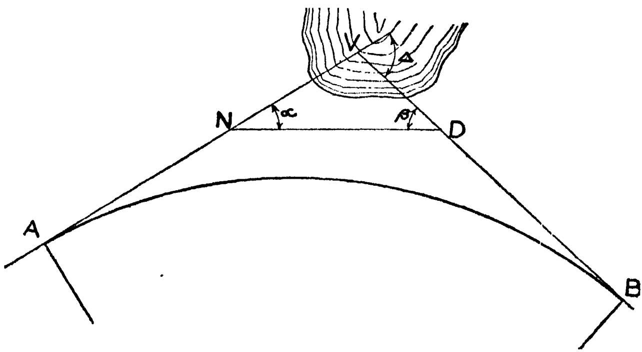


FIG. 33.

Then entering Table 1 with $\Delta = 50° \ 12'$, as argument, we have

$$D = \frac{T \text{ (for } 1° \text{ curve)}}{516.54} = \frac{2683.9}{516.54} = 5.2°.$$

Let $D = 5°$, for convenience, then $T = 2683.9/5 = 536.8$ ft., which is permissible.

93. To Find One Curve That Will Replace Two Curves United by a Tangent

Two curves with a common tangent, as $ABCD$ in Fig. 34, are known as broken-back curves. They are unsightly and should be replaced with one spiralized curve.

Let $R_1 = O_1B$ = radius of the sharper curve;
$R_2 = O_2C$ = radius of the flatter curve;
d = tangent BC;

and $R = OA = OD$ = given radius of new curve which is to be flatter than either of the two given curves.

It is required to find points A and D where the selected curve AD will become compounded with the given curves.

The problem, therefore, is: Given R_1, R_2, d, and R, to find Δ_1 and Δ_2, the angles subtended by arcs AB and CD.

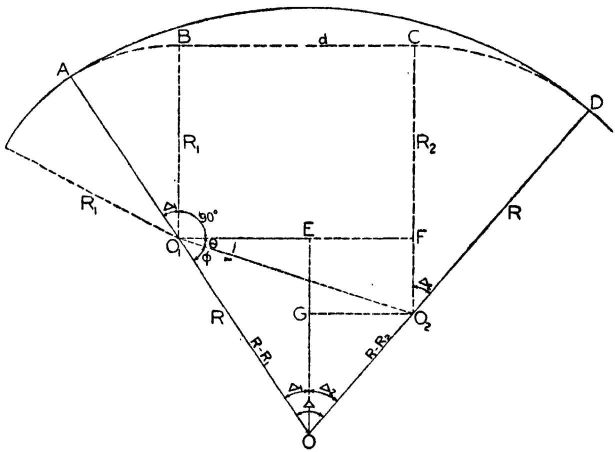


FIG. 34.

PROCEDURE. A preliminary graphical construction (or sketch) should be made to establish point O, the center of curve of radius R, as follows: Strike an arc with center O_1 and radius $R - R_1$ so as to intersect an arc with center O_2 and radius $R - R_2$; this gives point O. Prolong OO_1 and OO_2 to intersections with the given curves, thus fixing points A and D.

The angles Δ_1 and Δ_2 will now be determined analytically.

Join O_1 and O_2, and draw line O_1F parallel to BC.

In the right triangle O_1O_2F, the side O_1F equals d and the side O_2F equals $R_2 - R_1$; hence the angle $O_2O_1F(= \theta)$ and the side O_1O_2 can be computed.

The three sides of triangle O_1O_2O are now known. Compute the angles at O_1 and O, giving $OO_1O_2(= \varphi)$ and $O_1OO_2(= \Delta_1 + \Delta_2)$.

In the triangle OO_1E (obtained by drawing OE perpendicular to O_1F), it is seen that $\Delta_1 = 90 - \theta - \varphi$. Knowing the value of Δ_1, then $\Delta_2 = O_1OO_2 - \Delta_1$.

Finally, the curve lengths BA and CD can be computed, thus giving the stationing of points A and D.

EXAMPLE. A broken-back curve consists of 10° curve separated from 6° curve by a tangent distance of 200 ft. It is required to substitute a 4° curve which will be compounded with the given curves.

SOLUTION. Using the foregoing notation, and applying the formula, $R = 5{,}730/D$, we have $R_1 = 573.0$, $R_2 = 955.0$ and $R = 1432.5$.

Then $R_2 - R_1 = 382.0$; and since $d = 200$, we have $\tan \theta = 382.0/200 = 1.910$, giving $\theta = 62° \, 22'$.

$$\text{Also,} \qquad O_1O_2 = \sqrt{(200)^2 + (382)^2} = 431.2.$$

Since $R - R_1 = 859.5$ and $R - R_2 = 477.5$, the solution of triangle O_1O_2O gives $\phi = 19° \, 58'$ and $\Delta = 17° \, 58'$.

$$\text{Then} \qquad \Delta_1 = 7° \, 40' \qquad \text{and} \qquad \Delta_2 = 10° \, 18'.$$

Finally, arc $BA = 100\frac{\Delta_1}{10} = 76.7$, and arc $CD = 100\frac{\Delta_2}{6} =$ 171.7 ft.

94. Metric Curves

In measuring distances under the metric system it is usually customary to use a 20-meter tape. If a station is assumed to be 10 meters, then a full tape length is two stations, and ordinarily only every other station need be staked.

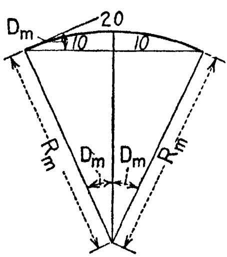


FIG. 35.

In some foreign countries where the metric system is legally adopted, a curve is designated by its *radius*.

But many have adopted the definition that *degree-of-curve* is the *deflection angle* for a chord (or arc) of 20 *meters*. *Degree-of-curve will here be defined as the deflection angle for an arc of* 20 *meters, or the central angle subtended by an arc of* 10 *meters.*

Accordingly, in Fig. 35, if D_m = degree-of-curve and R_m = radius (meters), we have the proportion:

$$2D_m{}^\circ:360^\circ = 20:2\pi R_m,$$

giving
$$D_m = \frac{572.958}{R_m}, \tag{24}$$

or
$$R_m = \frac{572.958}{D_m} \text{ (meters)} \tag{25}$$

If $D_m = 1^\circ$, $R_m = 572.958$ meters.

Therefore the functions (in meters) of the unit 1° *metric curve* are $\frac{1}{10}$ of the corresponding functions (in feet) of the unit 1° *foot curve.*

Hence Table 1 may be used to find the tangents and externals of the 1° metric curve, or the functions of the unit curve of length 100 meters.

Letting L_m = length of curve (in meters), we have the proportion,

$$L_m:10 = \Delta: D_m,$$

giving
$$L_m = 10\frac{\Delta}{D_m}. \tag{26}$$

EXAMPLE. Given $\Delta = 25^\circ\ 00'$ and external distance (E) from P.I. to point where middle of curve should run is 7.0 meters. It is required to find complete curve data.

SOLUTION. Entering Table 1 with $\Delta = 25^\circ\ 00'$ as argument, we have E (in meters) for 1° curve $= \frac{1}{10}(139.11) = 13.91$.

Hence $D_m = \dfrac{13.91}{7.0} = 1.99^\circ$. Let $D_m = 2^\circ$, then $E = \dfrac{13.911}{2}$ $= 6.95$ meters.

Also, $$T = \frac{\frac{1}{10}(T \text{ for } 1^\circ \text{ curve})}{2} = \frac{127.02}{2} = 63.51 \text{ meters;}$$

and $$L_m = 10\frac{\Delta}{D_m} = \frac{10 \times 25.0}{2} = 125.0 \text{ meters.}$$

The values of L.C. and M may be found by multiplying the tabular values by $L_m/100$; thus,

$$\text{L.C.} = 99.209 \times 1.25 = 124.01 \text{ meters,}$$

and
$$M = 5.432 \times 1.25 = 6.79 \text{ meters.}$$

The curve will be located by the usual method of deflections, using 20-meter chords, and setting stakes at every other station.

Deflection for 20 meters = 2° 00′ = 120′.

Deflection for 1 meter = 6′.

Thus, if the P.C. is at Sta. 41 + 6.24 (410 + 6.24 meters), and the transit is set at the P.C., vernier reading 0° 00′ on tangent, the deflections are as follows:

Sta. (1 Sta. = 10 meters)	Deflections
41 + 6.24 (P.C.)	0° 00′
42	0° 22.6′
44	2° 22.6′
46	4° 22.6′
48	6° 22.6′
50	8° 22.6′
52	10° 22.6′
54	12° 22.6′
54 + 1.24 (P.T.)	12° 30.0′ check.

$$\begin{aligned} \text{P.C.} &= \text{Sta. } 41 + 6.24 \\ L_m &= \underline{\quad\ \ 12 + 5.0\ } \\ \text{P.T.} &= \text{Sta. } 54 + 1.24 \end{aligned}$$

A comparison between *metric* curves and *foot* curves may be made as follows: For the same radius, *the ratio of metric curves to foot curves* is $\dfrac{10 \text{ meters}}{100 \text{ ft.}} = \dfrac{32.808 \text{ ft.}}{100 \text{ ft.}} = 0.32808$, or about as 1 to 3. That is, D_m is about $\frac{1}{3}D$.

In order that the chord may not differ appreciably from the corresponding arc, the following chords may be used:

20-meter chords up to 2° curves of radius = 286.5 meters (= 940 ft.).

15-meter chords up to 3° curves of radius = 191.0 meters (= 627 ft.).

12-meter chords up to 4° curves of radius = 143.2 meters (= 470 ft.).

10-meter chords up to 5° curves of radius = 114.6 meters (= 376 ft.).

8-meter chords up to 7.5° curves of radius = 76.4 meters (= 251 ft.).

6-meter chords up to 10.5° curves of radius = 54.6 meters (= 179 ft.).

5-meter chords up to 12° curves of radius = 47.7 meters (= 156 ft.).

4-meter chords up to 15° curves of radius = 38.2 meters (= 125 ft.).

2-meter chords for curves sharper than 15°.

It should be noted that all the circular and spiral curve tables in the United States, or foot system, are applicable to the metric-system-curves, where the length of unit curve is 100 meters instead of 100 feet.

Compound and Reversed Curves

95. Compound Curves

If two circular curves AC and CB (Fig. 36), having different radii, are tangent to each other at point C, both curves being on the same side of the tangent, the combination is defined as a *compound curve.* The point of compound curve is called P.C.C.

Compound curves are advantageous in placing the road to fit the ground. They should be avoided, however, except where topographic conditions make them necessary.

The two curves AC and CB are laid out in the field as separate circular curves, the P.T. of first curve being identical with P.C. of the second curve.

For the sharper curve AC, let

$$R_1 = \text{radius and } \Delta_1 = \text{intersection angle;}$$

and for the flatter curve CB, let

$$R_2 = \text{radius and } \Delta_2 = \text{intersection angle.}$$

Also, let

$$T_1 = AV_1 + V_1V. \quad \text{and} \quad T_2 = BV_2 + V_2V.$$

Some of these data may be obtained from preliminary paper locations.

96. Problems

Case 1

Given Δ_1, R_1, Δ_2 and R_2. To find Δ, T_1 and T_2.

SOLUTION. The total intersection angle Δ at V equals $\Delta_1 + \Delta_2$.

$$AV_1 = V_1C = R_1 \tan \tfrac{1}{2}\Delta_1;$$

also

$$CV_2 = V_2B = R_2 \tan \tfrac{1}{2}\Delta_2.$$

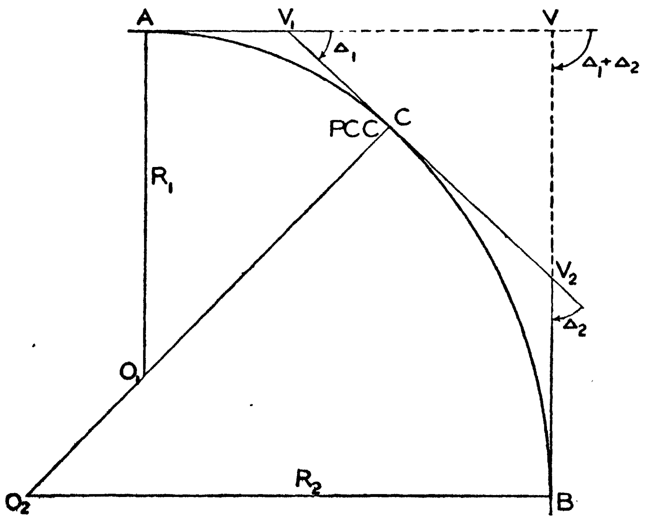


FIG. 36.

Since D_1 and D_2 are known ($D = 5{,}730/R$), the foregoing tangent distances may also be found from Table 1 with Δ_1 and Δ_2 as arguments.

In the triangle V_1VV_2, the angles and one side are known. Solve for V_1V and V_2V.

Then

$$T_1 = AV = AV_1 + V_1V,$$

and

$$T_2 = BV = BV_2 + V_2V.$$

Case 2

Given T_1, R_1, Δ_1 and Δ. To find Δ_2, R_2 and T_2.

SOLUTION. $\Delta_2 = \Delta - \Delta_1$. Knowing Δ_2, the value of R_2 and T_2 may be determined from (*a*) field measurements or (*b*) computations.

a. Field Measurements. Set up the transit at the P.C.C., vernier reading 0° 00′ on forward tangent CV_2, turn off angle $\frac{1}{2}\Delta_2$ and locate point B in the line VB. Measure the long chord CB, from which radius R_2 and tangents $CV_2(= V_2B)$ may be found (using Table 1). Then move transit to point B, read angle VBC as a check (it should be $\frac{1}{2}\Delta_2$) and measure distance $BV = T_2$.

b. Computations. In triangle V_1VV_2 the three angles and side V_1V are known. Solve for V_1V_2 and VV_2.

Then $CV_2 = V_1V_2 - V_1C$, and $T_2 = VB = VV_2 + V_2B = VV_2 + CV_2$.

Also $R_2 = CV_2 \cot \frac{1}{2}\Delta_2$.

Case 3

Given all the parts of a compound curve and a tangent parallel to the terminal tangent, as shown in Fig. 37.

It is required to change the position of the P.C.C. and P.T. so that a curve of the same radius as the terminal curve will end in the parallel tangent $B'D'$.

SOLUTION. Let p = the known perpendicular offset from original to parallel tangent.

With the usual notation for the parts of the original curves, the corresponding parts of the new curves will be designated by primed letters. Thus Δ_1' and Δ_2' are the central angles of the new curves; B is the original P.T., while B' is the new P.T. on a parallel tangent.

Assuming the original compound curve to be drawn, lay off $O_2N = p$.

From point O_1 as center, strike an arc of radius $R_2 - R_1$ intersecting a line drawn through N parallel to $B'D'$, thus locating point O_2', the new center of the flatter curve.

Draw a line from O_2' to O_1 prolonged to an intersection with the sharper curve at C'. This locates the new P.C.C.

As will be shown on the next page,

$$\cos \Delta_2' = \cos \Delta_2 - \frac{p}{R_2 - R_1},$$ from which Δ_2' can be found.

Then $\Delta_1' = \Delta - \Delta_2'$.

Also $q = (R_2 - R_1)(\sin \Delta_2' - \sin \Delta_2)$.

This completes the problem.

Since $O_1O_2 = O_1O_2' = R_2 - R_1$, the proof of the foregoing depends on the following steps:

Draw O_1KH parallel to $B'D'$, then $O_1K = O_1O_2 \sin \Delta_2 = (R_2 - R_1) \sin \Delta_2$; $O_2K = (R_2 - R_1) \cos \Delta_2$; $O_2'H =$

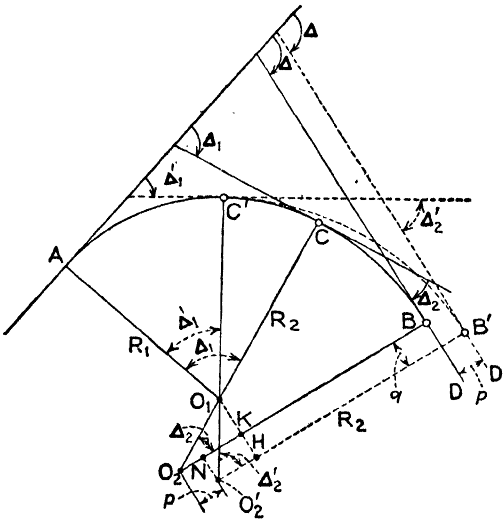


FIG. 37.—Compound curve with terminal tangent shifted. C' = new P.C.C., B' = new P.T.

$O_2K - O_2N = (R_2 - R_1) \cos \Delta_2 - p$; $\cos \Delta_2' = \dfrac{O_2'H}{R_2 - R_1} = \dfrac{(R_2 - R_1) \cos \Delta_2 - p}{R_2 - R_1} = \cos \Delta_2 - \dfrac{p}{R_2 - R_1}$; $O_1H = (R_2 - R_1) \sin \Delta_2'$; and $q = O_1H - O_1K = (R_2 - R_1) \sin \Delta_2' - (R_2 - R_1) \sin \Delta_2 = (R_2 - R_1)(\sin \Delta_2' - \sin \Delta_2)$.

EXAMPLE. Given P.C. at Sta. $40 + 20$, $\Delta_1 = 40°$, $\Delta_2 = 20°$, $D_1 = 6°$, $D_2 = 4°$ and $p = 50$ ft. To find Δ_2', Δ_1' and q; also the proper station number of the P.T.

SOLUTION. $\Delta = \Delta_1 + \Delta_2 = 60°$.

$R_1 = \dfrac{5{,}730}{D_1} = 955.0$, $R_2 = \dfrac{5{,}730}{D_2} = 1{,}432.5$; then $R_2 - R_1 = 477.5$.

$$\text{Cos } \Delta_2' = \cos 20° - \frac{50}{477.5} = 0.83498, \text{giving } \Delta_2' = 33° \ 23.2'.$$

Hence $\Delta_1' = 60° - 33° \ 23.2' = 26° \ 36.8'$. Also,

$$q = 477.5\ (0.55029 - 0.34202) = 99.45 \text{ ft.}$$

$L_1 = 100\frac{\Delta_1}{D_1} = 666.7$, $L_2 = 100\frac{\Delta_2}{D_2} = 500.0$; then original P.T. is at Sta. 40 + 20 + 1,166.7 ft. = Sta. 51 + 86.7.

$L_1' = 100\frac{\Delta_1'}{D_1} = 443.55$, $L_2' = 100\frac{\Delta_2'}{D_2} = 834.75$; hence new P.T. is at Sta. 40 + 20 + 1,278.3 ft. = Sta. 52 + 98.3.

When the terminal tangent is shifted *inward* to a parallel position, the solution is very similar to the foregoing.

97. Reversed Curves

Two circular curves AC and CB (Fig. 38) on opposite sides of a common tangent constitute a *reversed curve*. The

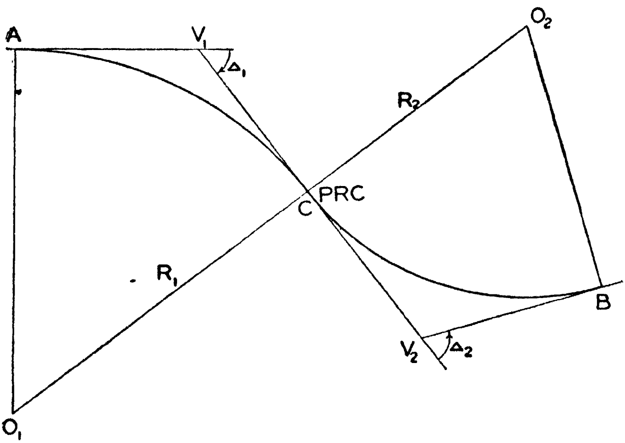


FIG. 38.

common point of reversed (or opposite) curvature is called P.R.C.

Reversed curves are undesirable and should be avoided on major routes. Nevertheless they are frequently encountered in difficult country.

98. Problems

Case 1

Given V_1V_2, the length of the common tangent, and the angles Δ_1 and Δ_2. It is required to find R_1 and R_2.

Solution. Divide V_1V_2 into parts that are equal or unequal, giving tangents V_1C and CV_2. Then by using Table 1, R_1 and R_2 may be found.

Case 2

Given parallel tangents (that is, $\Delta_2 = \Delta_1 = \Delta$), R_1 and R_2. It is required to find the relative positions of points A and B with reference to the tangents.

Solution. No special drawing was made for this case, but it should be obvious from Fig. 38 that the projection of line AB on the parallel tangents is $R_1 \sin \Delta + R_2 \sin \Delta = (R_1 + R_2) \sin \Delta$; and the perpendicular distance between parallel tangents is $R_1 \text{ vers } \Delta + R_2 \text{ vers } \Delta = (R_1 + R_2) \text{ vers } \Delta$.

Example. If two parallel tangents are 50 ft. apart and the radius of each curve is 200 ft., what is the distance (along the tangents) from P.C. of first curve to P.T. of second reversed curve?

Solution. $50 = (R + R) \text{ vers } \Delta = 400 \text{ vers } \Delta$, giving $\Delta = 28° 57'$. Hence required distance $= 400 \sin \Delta = 193.6$ ft.

Problem 1. Given $\Delta = 43° 05'$ and $E = 172$ ft. (approx.). What is the degree-of-curve? *Ans.:* $D = 2° 30'$.

Problem 2. If $\Delta = 30° 12'$ and $D = 2° 00'$, what is the length of curve? *Ans.:* $L = 1510.0$ ft.

Problem 3. Using the data of Problem 2, and assuming the P.I. to be at Sta. 422 + 16.6, what is the correct station number for the P.T.? *Ans.:* Sta. 429 + 54.4.

Problem 4. If the radius of a curve is 4,000 ft., what is the deflection angle for a 40-ft. arc? *Ans.:* 0° 17.2′ (Table 4).

Problem 5. What is the middle ordinate (M) for an arc (a chord) of 100 ft., which subtends a central angle of 8° 00′?

Ans.: $M = \frac{1}{2}C \tan \frac{1}{4}\Delta = 1.75$ ft.

Problem 6. Verify the answer to Problem 5 by referring to Table 1.

Problem 7. Given $D = 6° 00'$ and the P.C. at Sta. 21 + 00. What are the coordinates x and y locating Sta. 23 + 00?

Ans.: Referring to Art. 90, $x = 198.54$ ft., $y = 20.87$ ft.

CHAPTER IV

PARABOLIC CURVES

99. Adaptability

Parabolic curves, owing to their easement qualities and adaptability to the method of offsets, are used exclusively for vertical curves connecting highway and railroad grade profile tangents. Also, they are used widely in designing crowns, or cross profiles, for pavements.

For horizontal curves, however, the parabola fails to possess the advantages of the circle or spiral; since it cannot be laid out readily by deflections from a tangent, nor is its radius known at all points along the curve. In laying out comparatively short curves, such as paths, without the aid of an instrument, the parabola is easily adaptable.

100. Useful Properties

The equation of the ordinary parabola is $y = kx^2$. In general, the coordinate axes are oblique; the X-axis coincides with a tangent to the curve (as AV, in Fig. 39) and the Y-axis is parallel to a so-called diameter, which is a line from the vertex (V) to (M) the middle of a chord AB.

For any ordinate LR (Fig. 39) parallel to the diameter VM, we have

$$\frac{LR}{VC} = \left(\frac{AL}{AV}\right)^2 \quad \text{or} \quad LR = \left(\frac{AL}{AV}\right)^2 \times (VC).$$

That is, if $AL = \frac{3}{4}AV$, then $LR = (\frac{3}{4})^2 \times (VC) = \frac{9}{16}e$.

The foregoing illustrates a useful property of the parabola, namely; *the offsets from a tangent to the curve vary as the square of the distance along the tangent from the point of tangency.*

Observing that the tangents AV and VB may be unequal, and that a diameter extends from the vertex to the middle

of any chord, another useful property of the parabola is the following:

A point of the curve opposite the vertex is at the middle of a diameter.

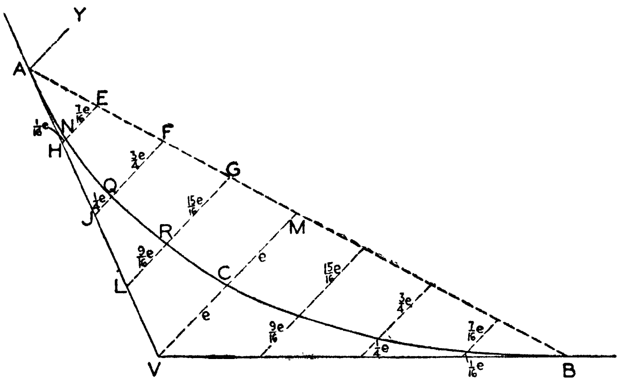


FIG. 39.

101. Horizontal Curves

Let it be required to locate a parabolic horizontal curve joining the tangents AV and VB at A and B (Fig. 39), the tangent lengths being not necessarily equal.

PROCEDURE. Referring to Fig. 39, bisect chord AB at M, then bisect VM at C, giving a point on the parabola. As many intermediate points as are desired may be obtained as follows:

Divide MA and VA each into the same number of equal parts (four parts, for example). Let distance $VC = CM = e$.

$$\text{Ordinate } HN \text{ from tangent to curve} = \left(\frac{AH}{AV}\right)^2 \times VC = \frac{1}{16}e.$$

$$\text{Ordinate } JQ \text{ from tangent to curve} = \left(\frac{AJ}{AV}\right)^2 \times VC = \frac{1}{4}e, \text{ etc.}$$

Similarly, points defining portion CB of the parabola may be established.

VERTICAL CURVES

102. The use of vertical curves for connecting the grade lines of the profile contributes to the safety, comfort, and

appearance of the highway in just as vital a manner as do horizontal curves in the alignment.

Only when the difference of the intersecting grades is less than ½ per cent, are vertical curves considered unnecessary.

103. Factors Governing the Length of Vertical Curves

In selecting the lengths of vertical curves over "crests" or into "sags," the algebraic difference of gradients enters into the calculations; but the controlling factor for crests is often an ample *sight distance.*

From the standpoint of easy riding grades and pleasing appearance, vertical curves should have sufficient length

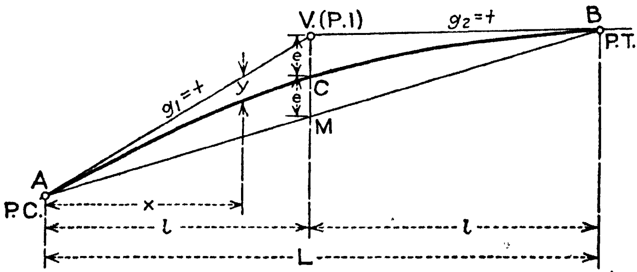


Fig. 40.

so that the "rate of change of grade" (*which is uniform on a parabola*) is held to a minimum. No limiting values for rate of change have been generally agreed upon for highways; but the American Railway Engineering Association recommends that on Class A railroads, "rates of change of 0.10 per station on summits and 0.05 per station in sags" should not be exceeded; while double these values are allowed on less important roads. Thus if a $+1.00$ per cent grade meets a -0.20 per cent grade, the total change of grade is $1.00 - (-0.20) = 1.20$; hence to satisfy the A. R. E. A. requirement, $1.20/L = 0.10$, or $L = 12$ stations (1,200 ft.).

104. Offset from Vertex to Middle of Curve

In Fig. 40, AV and VB represent two intersecting grade lines. For the vertical curves connecting these grade

tangents, a parabola (ACB) will be assumed, as is the usual custom.

It is required to derive an expression for the offset from the vertex to the middle of the curve, when the tangents are equal, as is the usual case.

Let g_1 and g_2 = per cent grades of tangents AV and VB respectively, using *plus* values for grades ascending forward and *minus* for those descending.

Also, $g = g_1 - g_2$ = algebraic difference in the grades (per cent);

$l = AC = CB$ = half length of curve (in stations);

$L = 2l$ = total length of curve (in stations);

$e = VC$ = offset from vertex to middle of curve (assumed vertical).

The middle point C of the parabola is halfway between V and M; and point M bisects chord AB.

For convenience of demonstration, let elevation of point A be *zero*. Then,

$$\text{Elev. of } V = 0 + g_1 l;$$

and

$$\text{Elev. of } B = g_1 l + g_2 l.$$

Therefore, since elev. of $M = \frac{1}{2}(\text{elev. } A + \text{elev. } B) = \frac{1}{2}(g_1 l + g_2 l)$; and also since $VC = \frac{1}{2}(\text{elev. } V - \text{elev. } M)$, we have

$$e = VC = \tfrac{1}{2}[g_1 l - \tfrac{1}{2}(g_1 l + g_2 l)] = \tfrac{1}{4}l(g_1 - g_2),$$

or

$$e = \tfrac{1}{8}Lg. \tag{27}$$

Equation (27) shows that the distance from vertex to middle of parabola is $\frac{1}{8}$(*length of curve in stations*) $\times$ (*total change in grade*).

Since the offsets from a tangent to the parabola vary as the *square* of the distance from the P. C. (or P. T.), all intermediate offsets can be readily computed in terms of the central offset (e). Thus in Fig. 40, we have the proportion: $y:e = x^2:l^2$; from which any offset y can be found. The intersection angle is small, so that it is sufficiently exact to measure all offsets vertically.

Strict attention must be given to the *signs* of the per cent grades while using Eq. (27). Thus if $g_1 = -3$, $g_2 = +5$ and $L = 600$ ft., then $g = -3 - 5 = -8$, and hence $e = -\frac{1}{8} \times 6.00 \times 8 = -6.00$ ft.; if $g_1 = -1$, $g_2 = -5$ and $L = 500$ ft., then $g = -1.00 - (-5.00) = +4.00$, and hence $e = \frac{1}{8} \times 5.00 \times 4.00 = +2.50$ ft.

Figure 41 shows four possible arrangements of vertical curves. Cases 1 and 2 are "summits," while Cases 3 and 4 are "sags."

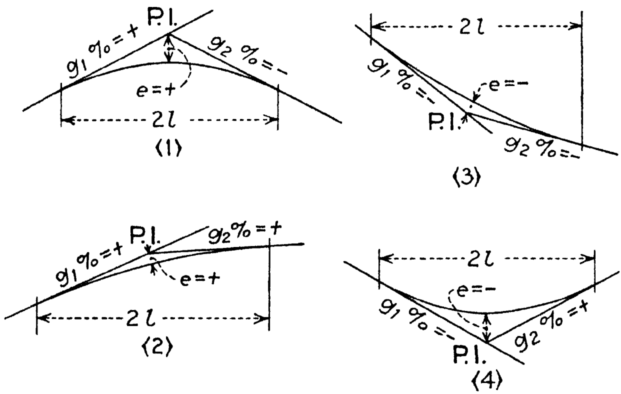


FIG. 41.—Four cases of vertical curves.

If e is *plus*, the P.I. is above the curve; and hence offsets are to be *subtracted* from the elevation of the stations along the grade tangents. If e is *minus*, the P.I. is below the curve; and hence offsets are to be *added* to the elevations of the stations along the grade tangents.

In Fig. 41, e is (+) for curves (1) and (2); and (−) for curves (3) and (4).

105. Computation of Offsets

Assuming a vertical curve with equal tangents, two methods will be presented here for computing tangent offsets, Method A and Method B. These methods are quite similar.

Method A. The offsets are computed from the formula

$$y = \left(\frac{x}{l}\right)^2 e,$$

in which

$$e = \frac{Lg}{8}.$$

Method B. Combining the foregoing formulas, we have

$$y = \left(\frac{x}{l}\right)^2 \frac{Lg}{8} = Cg,$$

in which C is a *coefficient* given in the table on page 146.

The offsets for various cases are therefore obtained by merely multiplying the tabular coefficients by (g), the algebraic difference in rates of grades.

Example 1. (Method A.) A grade of +3 per cent meets a −1.0 per cent grade at Sta. 27 + 00 whose elevation is 600.20. A length of vertical curve equal to 500 ft. will be used. For each station, it is required to compute: (1) elevation of tangent, (2) offset from tangent to curve, and (3) elevation of curve. The length of each tangent is 250 ft.

	Sta.		Elevation
P.I.	= 27 + 00 =		600.20.
l	= 2 + 50		
P.C.	= 24 + 50 =	600.20 − 7.50 =	592.70.
L	= 5 + 00		
P.T.	= 29 + 50 =	600.20 − 2.50 =	597.70.

$$g = g_1 - g_2 = 3.0 - (-1.0) = +4.0.$$

Offset from vertex to middle of curve $= e = \frac{1}{8} \times 5 \times 4.0 = 2.50$.

Offsets from tangents to any point on curve $= y = (x/l)^2 \times 2.50$.

The required curve data are given in the table on page 145.

DATA FOR VERTICAL CURVE

$L = 500, \quad l = 250, \quad g_1 = +3.0, \quad g_2 = -1.0, \quad e = 2.50$

Sta.	Elev. on tangent	x/l	Offsets (−)	Elev. on curve
24 + 50 (P.C.)	592.70	0	0.00	592.70
25	594.20	$\frac{1}{5}$	0.10	594.10
26	597.20	$\frac{3}{5}$	0.90	596.30
27 (P.I.)	600.20	1	2.50	597.70
		x'/l		
28	599.20	$\frac{3}{5}$	0.90	598.30
29	598.20	$\frac{1}{5}$	0.10	598.10
29 + 50 (P.T.)	597.70	0	0.00	597.70

x' is the distance from the P.T.

Note: As a check in the computations of the elevations for parabola, the *second differences* for full stations should be *constant*. Thus for Sta. 25 to 29, we have:

Differences	
1st	2nd
−2.2	
	−0.8
−1.4	
	−0.8
−0.6	
	−0.8
+0.2	

Example 2. (Method B.) Given the same data as those of Example 1; that is, $g = 4.00$, $L = 500$ ft., and P.I. at Sta. 27 + 00, whose elevation is 600.20.

Referring to the table on page 146, we have the following coefficients which are each multiplied by the value of $g(= 4)$:

Sta.	Distance from P.C. (ft.)	Distance from P.T. (ft.)	Offsets (ft.)
24 + 50	0	. . .	0
25	50	. . .	$0.0250 \times 4 = 0.10$
26	150	. . .	$0.2250 \times 4 = 0.90$
27 (P.I.)	250	. . .	$0.6250 \times 4 = 2.50$
28	. . .	150	$0.2250 \times 4 = 0.90$
29	. . .	50	$0.0250 \times 4 = 0.10$
29 + 50	. . .	0	0

COEFFICIENTS FOR OFFSETS FROM TANGENT TO VERTICAL CURVES

Dist. from P.C. (ft.)	Length of vertical curve (ft.)								Dist. from P.T. (ft.)
	100	150	200	250	300	350	400	450	
25	.03125	.0208	.0156	.0125	.0104	.0089	.0078	.0069	25
50	.12500	.0833	.0625	.0500	.0417	.0357	.0313	.0278	50
75	......	.1875	.1406	.1125	.0937	.0804	.0703	.0625	75
100	......	.....	.2500	.2000	.1667	.1429	.1250	.1111	100
125	......	.....	.....	.3125	.2604	.2232	.1953	.1736	125
150	.....	.....	.....	.....	.3750	.3214	.2812	.2500	150
175	......	.....	.....	.....	.....	.4375	.3828	.3403	175
200	......	.....	.....	.....	.....	.....	.5000	.4444	200
225	......	.....	.....	.....	.....	.....	.....	.5625	225

Dist. from P.C. (ft.)	Length of vertical curve (ft.)								Dist. from P.T. (ft.)
	500	550	600	700	800	900	1,000	1,100	
25	.0063	.0057	.0052	.0045	.0039	.0035	.0031	.0028	25
50	.0250	.0227	.0208	.0179	.0156	.0139	.0125	.0114	50
75	.0563	.0511	.0469	.0402	.0352	.0312	.0281	.0256	75
100	.1000	.0909	.0833	.0714	.0625	.0556	.0500	.0455	100
125	.1563	.1420	.1302	.1116	.0977	.0868	.0781	.0710	125
150	.2250	.2045	.1875	.1607	.1406	.1250	.1125	.1023	150
175	.3063	.2784	.2552	.2188	.1914	.1701	.1531	.1392	175
200	.4000	.3636	.3333	.2857	.2500	.2222	.2000	.1818	200
225	.5063	.4602	.4219	.3616	.3164	.2812	.2531	.2301	225
250	.6250	.5682	.5208	.4464	.3906	.3472	.3125	.2841	250
275	.....	.6875	.6302	.5402	.4727	.4201	.3781	.3438	275
300	.....	.....	.7500	.6429	.5625	.5000	.4500	.4091	300
325	.....	.....	.....	.7545	.6602	.5868	.5281	.4801	325
350	.....	.....	.....	.8750	.7656	.6806	.6125	.5568	350
375	.....	.....	.....	.....	.8789	.7812	.7031	.6392	375
400	.....	.....	.....	.....	1.0000	.8889	.8000	.7273	400
425	.....	.....	.....	.....	.....	1.0035	.9031	.8210	425
450	.....	.....	.....	.....	.....	1.1250	1.0125	.9205	450
475	.....	.....	.....	.....	.....	.....	1.1281	1.0256	475
500	.....	.....	.....	.....	.....	.....	1.2500	1.1364	500
525	.....	.....	.....	.....	.....	.....	.....	1.2528	525
550	.....	.....	.....	.....	.....	.....	.....	1.3750	550

Multiply figures in table by algebraic differences in per cent rates of grades to get offsets at intervals of 25 feet. (See page 144.)

106. Offset from Vertex to Unsymmetrical Curve

Figure 42 shows an unsymmetrical parabolic vertical curve connecting the tangents AV and VB, which are unequal. Frequently an unsymmetrical curve will fit more closely certain imposed requirements than the usual symmetrical curve.

Let a vertical from the vertex (V) intersect the curve at (C) and the chord (AB) at M. Obviously points C and M

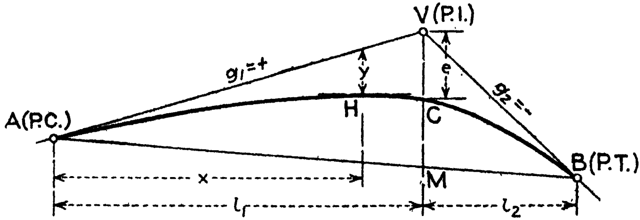


FIG. 42.

are not mid-points of curve and chord, nor is C the highest point of the curve, but C bisects the vertical VM.

It is required to derive an expression for the offset from vertex to curve.

As before, $g = g_1 - g_2$ = algebraic difference in the grades of tangents AV and VB, and $e = VC$ = offset from vertex to curve.

Also, let l_1 and l_2 = lengths (in stations) of curves AC and CB respectively;

$l_1 + l_2$ = total length of curve (in stations);

and for convenience of demonstration, let elevation of point A be *zero*. Then

$$\text{Elev. of } V = 0 + g_1 l_1;$$

and
$$\text{Elev. of } B = g_1 l_1 + g_2 l_2.$$

$$\text{Also, elev. of } M = \text{elev. A} + \frac{l_1}{l_1 + l_2}(\text{elev. } B - \text{elev. } A)$$

$$= \frac{l_1}{l_1 + l_2}(g_1 l_1 + g_2 l_2).$$

Finally, $e = VC = \frac{1}{2}(\text{elev. } V - \text{elev. } M)$

$$= \frac{1}{2}\left[g_1 l_1 - \frac{l_1}{l_1 + l_2}(g_1 l_1 + g_2 l_2) \right],$$

from which
$$e = \frac{l_1 l_2}{2(l_1 + l_2)}(g_1 - g_2) \qquad (28)$$

If $l_1 = l_2 = l$, Eq. (28) becomes

$$e = \frac{l^2}{4l}(g_1 - g_2) = \frac{1}{8}Lg,$$

which is the same as Eq. (27) for a symmetrical curve.

The intermediate offsets are calculated for each tangent independently. Thus for offset, y, locating point H in Fig. 42, we may use the proportion: $y:e = x^2:l_1^2$.

EXAMPLE. Figure 43 shows a +5.0 per cent grade meeting a −7.0 per cent grade at Sta. 32 + 00, whose elevation is 115.00. The tangents are unequal; $l_1 = 300$ ft. and $l_2 = 400$ ft. It is required to calculate offsets for each station along the unsymmetrical curve.

DATA FOR UNSYMMETRICAL CURVE

Sta.	Elev. on tangent	x/l_1	Offsets (−)	Elev. on curve
29 + 00 (P.C.)	100.00	0	0.00	100.00
30	105.00	$\frac{1}{3}$	1.14	103.86
31	110.00	$\frac{2}{3}$	4.57	105.43
32 (P.I.)	115.00	1	10.29	104.71
Sta.	**Elev. on tangent**	**x'/l_2**	**Offsets (−)**	**Elev. on curve**
33	108.00	$\frac{3}{4}$	5.79	102.21
34	101.00	$\frac{1}{2}$	2.57	98.43
35	94.00	$\frac{1}{4}$	0.64	93.36
36 + 00 (P.T.)	87.00	0	0.00	87.00

x' is measured from the P.T.

SOLUTION. $g_1 - g_2 = 5.0 - (-7.0) = 12.0$. Applying Eq. (28), we have $e = \frac{3 \times 4}{2(3+4)} \times 12 = 10.29$.

The offsets as recorded on page 148 are computed for each tangent independently, using the formulas, $y = (x/l_1)^2 \times 10.29$ and $y = (x/l_2)^2 \times 10.29$.

107. Turning Point on Vertical Curve

The turning point is the *highest* (or lowest) point on a vertical curve. This point is usually not vertically below (or above) the vertex, but on the right or left branch of the curve.

It remains to determine not only the *position* of the turning point but its *actual elevation*, as well.

Treating first the *unsymmetrical* curve, expressions will be derived, which in a simplified form, apply also to the curve with equal tangents.

a. Position of Turning Point. In Fig. 42, the highest point of the curve is point H on the left branch.

The tangent offset to any point on the left branch of the curve is

$$y = \left(\frac{x}{l_1}\right)^2 e. \tag{29}$$

Letting K = elevation of this point relative to that of the P.C., we have

$$K = g_1 x - \left(\frac{x}{l_1}\right)^2 e. \tag{30}$$

For maximum, $\frac{dK}{dx} = 0$, giving $g_1 - \frac{2xe}{l_1^2} = 0$. Hence, solving for x, we obtain the distance (x_1) from P.C. to the turning point, H thus:

$$x_1 = \frac{l_1^2 g_1}{2e}. \tag{31}$$

Likewise for cases when the turning point occurs on the right branch, we have

$$x_1' = \frac{l_2^2 g_2}{2e}, \tag{32}$$

in which x_1' is measured from the P.T.

For vertical "sag" curves, Eqs. (31) and (32) will give the position of the *lowest* point.

When the *tangents are equal*, $l_1 = l_2 = l = \frac{1}{2}L$, and $e = \frac{L(g_1 - g_2)}{8}$. Hence substituting in Eq. (31), we have the distance from P.C. to turning point, thus:

$$x_1 = \frac{l^2 g_1}{2} \frac{8}{L(g_1 - g_2)} = \frac{L g_1}{g_1 - g_2}. \tag{33}$$

Similarly, the distance from P.T. to the turning point is

$$x_1' = \frac{L g_2}{g_1 - g_2}. \tag{34}$$

b. Elevation of Turning Point. Substituting in Eq. (30) the value of x from Eq. (31), and reducing, we have

$$K \text{ (relative to P.C.)} = \frac{l_1^2 g_1^2}{4e}. \tag{35}$$

Similarly, for the right branch of curve, we have

$$K' = \frac{l_2^2 g_2^2}{4e}, \tag{36}$$

where K' is elevation relative to that of the P.T.

When the *tangents are equal*, Eqs. (35) and (36) become

$$K \text{ (relative to P.C.)} = \frac{L g_1^2}{2(g_1 - g_2)}, \tag{37}$$

and

$$K' \text{ (relative to P.T.)} = \frac{L g_2^2}{2(g_1 - g_2)}. \tag{38}$$

EXAMPLE. Referring to Fig. 43, it is required to find the position of the highest point and its elevation.

Assuming the highest point to be on the left branch of the curve, Eq. (31) gives $x_1 = \frac{l_1^2 g_1}{2e} = \frac{9 \times 5}{2 \times 10.29} = 2.187$ stations = 218.7 ft. from the P.C. to the highest point.

The elevation of the highest point is $K = \frac{l_1^2 g_1^2}{4e} = \frac{9 \times 25}{4 \times 10.29}$ $= 5.47$ ft. *above the P.C.*, or at elevation 105.47.

Applying Eq. (32), we find $x_1' = 5.442$ stations, which, being beyond the limits of the second branch, cannot locate the highest point of the curve.

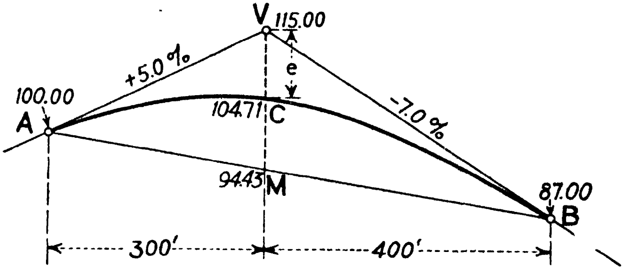


FIG. 43.

108. Sight Distances

The sight distance on vertical curves over summits should be far enough ahead to enable motorists to take appropriate action in case of emergency, when the cars are moving in opposite directions at rapid speeds. On two-lane trunk highways, the minimum sight distance should be 800 ft.

a. Formulas. Formulas for sight distances will now be developed for two cases: (1) when sight distance is entirely on the curve, and (2) when sight distance overlaps the curve and extends on to the tangents.

Let S = sight distance in stations, and
h = height of line of vision above roadway at either side of summit, usually taken to be 5 ft.

L, g, and e refer to the vertical curve (see Art. 104).

Case 1. $S < L$.

The equations of any two parabolas connecting the same tangents are $y = kx^2$ and $y_1 = kx_1^2$. Then, dividing, we have

$$\frac{y}{y_1} = \frac{x^2}{x_1^2}.$$

Hence, in Fig. 44, since the lengths S and L with their corresponding middle offsets h and e represent portions

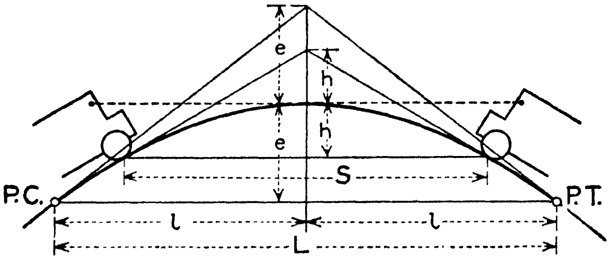


FIG. 44.

of the same parabola, we have, by analogy:

$$\frac{h}{e} = \frac{S^2}{L^2},$$

from which

$$S^2 = \frac{hL^2}{e}. \tag{39}$$

Substituting the value of e from Eq. (27), and reducing, we have

$$S^2 = \frac{8hL}{g}. \tag{40}$$

Letting $h = 5$ ft., Eq. (40) becomes

$$S^2 = \frac{40L}{g}. \tag{41}$$

Case 2. $S > L$.

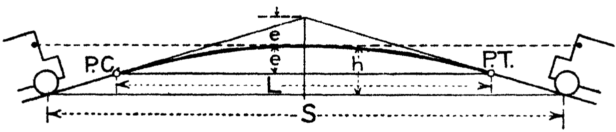


FIG. 45.

In Fig. 45, from similar triangles, we have

$$\frac{S}{L} = \frac{e + h}{2e},$$

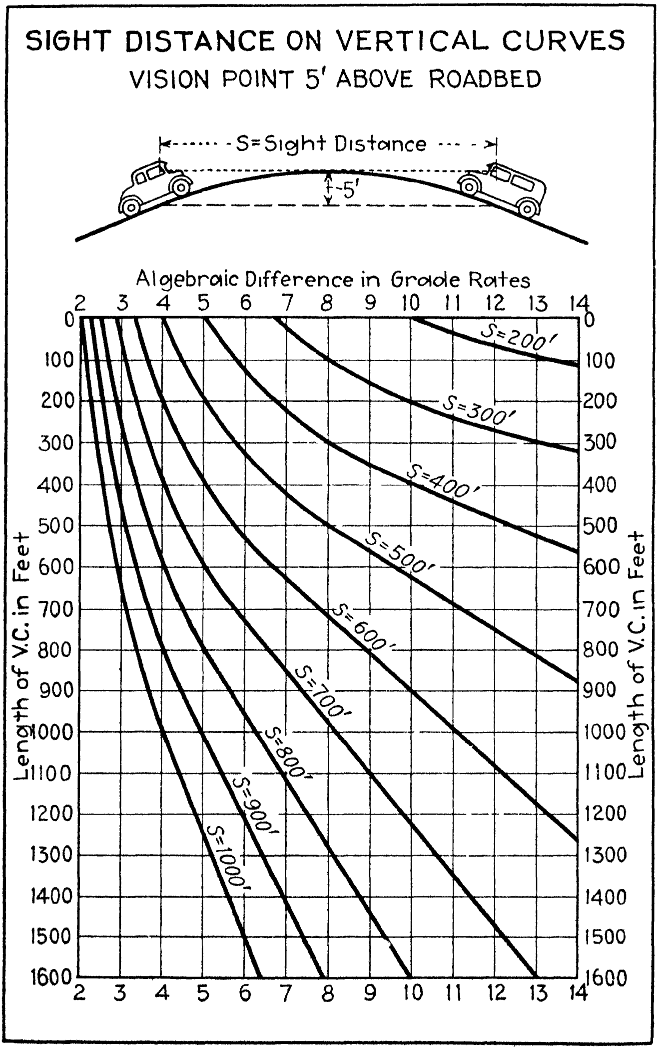

SIGHT DISTANCE ON VERTICAL CURVES
VISION POINT 5' ABOVE ROADBED
S=Sight Distance
5'
Algebraic Difference in Grade Rates
Length of V.C. in Feet
S=200'
S=300'
S=400'
S=500'
S=600'
S=700'
S=800'
S=900'
S=1000'
2 3 4 5 6 7 8 9 10 11 12 13 14
0 100 200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600

FIG. 46.

from which $$S = \frac{L}{2}\left(1 + \frac{h}{e}\right).$$

Substituting the value of e from Eq. (27), and reducing, we have

$$S = \frac{1}{2}L + \frac{4h}{g}. \tag{42}$$

Letting $h = 5$ ft., Eq. (42) becomes

$$S = \frac{1}{2}L + \frac{20}{g}. \tag{43}$$

b. Diagrams. Using Eqs. (41) and (43), tables or diagrams giving sight distances for various values of g and L, may be compiled. The diagram given in Fig. 46 was prepared by the California Highway Commission.

Thus if the algebraic difference in the rates of grade is 8.00 and L is 600 ft., the sight distance is 550 ft.

REVIEW PROBLEMS

Problem 1. The intersecting profile grades consist of a +5 per cent grade meeting a −4 per cent grade at Sta. 92 + 00, and the length of the vertical curve is 800 ft.

(*a*) What is the amount of the offset (e) from vertex to middle of curve?

(*b*) What is the tangent offset locating Sta. 90 + 00 on the curve? *Ans.:* (*a*) 9.00 ft.; (*b*) 2.25 ft.

Problem 2. Two intersecting profile grades consist of a −3 per cent grade meeting a +5 per cent grade at Sta. 112 + 00, whose elevation is 674.5. The length of vertical curve is 600 ft.

(*a*) What is the amount of the offset (e) from vertex to middle of curve?

(*b*) What is the elevation of Sta. 111 + 00 on the curve?

Ans.: (*a*) 6.00 ft.; (*b*) 680.17 ft.

Problem 3. Two intersecting profile grades consist of a +6 per cent grade meeting a +2 per cent grade at Sta. 49 + 00, whose elevation is 842.2; and the length of vertical curve is 400 ft.

(*a*) What is the amount of the offset (*e*) from vertex to middle of curve?

(*b*) What is the elevation of Sta. 50 + 00 on the curve?

Ans.: (*a*) 2.00 ft.; (*b*) 843.70 ft.

Problem 4. (Unsymmetrical Curve.) Two intersecting profile grades consist of a +5 per cent grade meeting a −5 per cent grade at Sta. 20 + 00, elevation 100.00 ft. An unsymmetrical vertical curve of length 1,000 ft. (10 stations) is used, where $l_1 = 6$ and $l_2 = 4$ stations.

(*a*) What is the amount of the offset (*e*) from vertex to the vertical curve?

(*b*) At what station does the highest point occur?

(*c*) What is the elevation of the highest point on the curve?

Ans.: (*a*) 12.0 ft.; (*b*) Sta. 20 + 66.7; (*c*) 88.33 ft.

Problem 5. Two intersecting profile grades consist of a −6 per cent grade meeting a +4 per cent grade at Sta. 160 + 00, elevation 400.20 ft. A symmetrical vertical curve of length 800 ft. (8 stations) is used.

(*a*) What is the elevation of the middle point of the curve?

(*b*) At what station does the lowest point occur?

(*c*) What is the elevation of the lowest point of the curve?

Ans.: (*a*) 410.20 ft.; (*b*) Sta. 160 + 80; (*c*) 409.80 ft.

Problem 6. Given $g_1 = +5$, $g_2 = -3$, and S (desired sight distance) = 8 stations or 800 ft. Applying Eq. (41) and using the diagram in Fig. 46 as a check, determine the required length of vertical curve. *Ans.:* L = 12.8 sta. = 1280 ft.

CHAPTER V

TRANSITION SPIRALS

109. To minimize the tendency of skidding and to ease somewhat the lurch experienced by motorists in passing abruptly from a *straight* to a *circular* alignment, *transition spirals* are desirable; since they provide a gradual change in curvature from a straight line (zero curvature) to a circle (full curvature).

Such easement curves have long been found necessary on trunk railroads both from the standpoint of more comfortable riding qualities and of gradually bringing about the full superelevation of the outer rail on curves.

From the standpoint of pleasing appearance and ease of transition from a straight to a circular path, spirals should be as long as possible. The question of spiral length, however, will be discussed in Chap. VI on *Superelevation.*

When a car reaches a circular path, the steering wheel must be set at a new angle depending on the radius of curve. Obviously, the steering wheel cannot be moved instantaneously, but in a definitely measurable time, such as two or three seconds; thus creating a demand for a transition curve.

110. Notation and Formulas

T.S. = point of change from tangent to spiral.
= beginning of spiral.
S.T. = point of change from spiral to tangent.
S.C. = point of change from spiral to circle.
C.S. = point of change from circle to spiral.
l = length in feet from the T.S. along the curve to any point on the spiral.
l_s = total length of spiral from T.S. to S.C.
x and y = coordinates of any point on spiral with reference to T.S. and initial tangent.

x_c and y_c = coordinates of the S.C.

θ = central angle of spiral from the T.S. to any point.

θ_s = "spiral angle" = central angle of spiral from T.S. to S.C.

ϕ = spiral deflection angle at the T.S. from initial tangent to any point on spiral.

D_s = degree-of-curve of the spiral at any point.

D_c = degree-of-curve of the osculating circle to which the spiral becomes tangent at the S.C. See Fig. 48.

D = degree-of-curve of the circular curve whose middle portion remains unchanged in position, while the end portions are sharpened to make room for the spiral ($D_c = \frac{4}{3}D$, approx.) See Fig. 49.

Δ = total central angle of the circular curve, the spiraled curve.

p = ordinate from the initial tangent to the P.C. of the osculating circle.

k = abscissa of the offset P.C. referred to T.S. .

a = distance from T.S. to P.C. of the original unoffset circular curve with which the spiral is compounded at the S.C.

T_s = total tangent distance = distance from P.I. to T.S.

E_s = total external distance.

T and E = tangent and external for the same Δ and D.

$$\theta^\circ = \left(\frac{l}{l_s}\right)^2 \theta_s^\circ.$$

$$l_s = 200\frac{\theta_s}{D_c} \text{ (see Table 12).}$$

$$D_c = \frac{200\theta_s}{l_s}.$$

$$\phi^\circ = \frac{\theta^\circ}{3} = \frac{\theta_s^\circ}{3}\left(\frac{l}{l_s}\right)^2 \text{ (see Tables 9 and 10).}$$

$\phi' = \delta + \phi =$ deflection *forward* from auxiliary tangent at intermediate point (see page 170).

$\phi'' = \delta - \phi =$ deflection *backward* from auxiliary tangent at intermediate point (see page 170).

$p = y_c - R_c$ vers θ_s (Table 8).

$k = x_c - R_c \sin \theta_s$ (Table 8).

$a =$ (value in Table 12 for 1° curve) divided by D.

$T_s = p \tan \frac{1}{2}\Delta + T + k$ or

$T_s = T + a$ (Table 12).

$E_s = E + p \sec \frac{1}{2}\Delta$.

x and $y =$ (values in Table 7 for 100-ft. spiral) $\times$ l_s (in stations).

111. Definitions

The *transition spiral* may be defined as a curve whose radius (or degree-of-curve) changes uniformly with the distance from some point of reference.

Figure 47 shows a spiral curve APC connecting a straight line AX with some point C (S.C.) on a circle (called *osculating circle,* since it is shifted inward to point of tangency with the spiral). At point A on the tangent where the spiral *begins,* the (T.S.), its radius is infinity; at the point of tangency with the circular arc, its radius is R_c and its degree-of-curve is D_c, the same as that of the circle; at any intermediate point of the spiral arc (point P), its radius is R and its degree-of-curve is D_s, where $D_s : D_c =$ arc AP : arc AC.

112. Development of Spiral Theory

In Fig. 47, let

$R =$ variable radius at any point P of spiral ($R = R_c$ at the S.C.);

$l =$ length of spiral arc AP ($l = l_s$ at the S.C.);

$\theta =$ central angle up to any point P ($\theta = \theta_s$ at the S.C.);

x and $y =$ coordinates of point P with reference to the T.S. and axis AX (x and y become x_c and y_c at the S.C.).

Since the degree-of-curve of the spiral varies directly as the distance (l) from the T.S., its radius must vary

inversely as the distance; hence $R = K/l$, in which K is a constant.

At point C (S.C.), the end of spiral, the above equation becomes $R_c = K/l_s$.

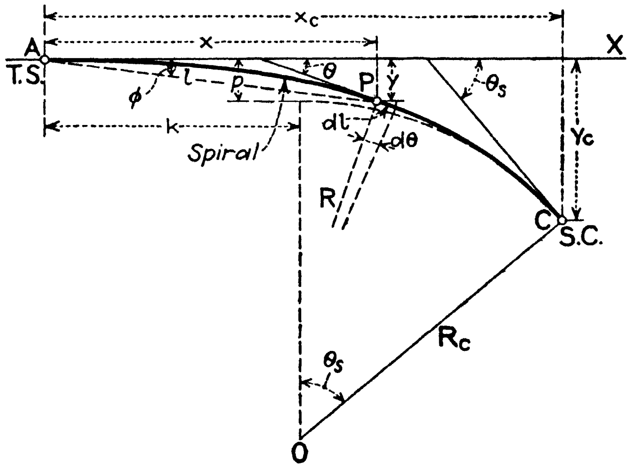


FIG. 47.

Eliminating K by division, we have

$$R = \frac{R_c l_s}{l}. \tag{44}$$

From the differential sector at point P, we have $d\theta = dl/R$.

Substituting the value of R from Eq. (44) gives

$$d\theta = \frac{l}{R_c l_s} dl.$$

Integrating,
$$\theta = \frac{l^2}{2R_c l_s}. \tag{45}$$

Since R_c and l_s are constants, it is seen that θ varies as the *square of the distance* from the T.S.

At point C, the S.C., $\theta = \theta_s$, which is called the *spiral angle.*
Letting $\theta = \theta_s$, then $l = l_s$; hence Eq. (45) becomes

$$\theta_s = \frac{l_s^2}{2R_c l_s} = \frac{l_s}{2R_c}, \qquad \text{or} \qquad l_s = 2R_c\theta_s, \tag{46}$$

in which θ_s is measured in radians.

Substituting $5{,}730/D_c$ for R_c, and expressing θ_s in degrees, Eq. (46) becomes

$$l_s = 200\frac{\theta_s}{D_c}. \tag{47}$$

Hence the total length of spiral is twice the circular arc of central angle (θ_s) and radius R_c (and degree-of-curve, D_c).

Comparing Eqs. (45) and (46), we have

$$\theta:\theta_s = l^2:l_s^2,$$

or

$$\theta = \left(\frac{l}{l_s}\right)^2\theta_s. \tag{48}$$

Equation (48) is the most convenient formula for finding θ at any point along the spiral. Since l/l_s is an abstract ratio, θ and θ_s may be expressed in either radians or degrees, the latter being preferable, of course, in practice.

From the differential triangle at point P (Fig. 47), we have $dy = dl \sin\theta$ and $dx = dl \cos\theta$.

Expressing the sine and cosine as infinite series, integrating, and reducing, we get

$$y = l\left[\frac{\theta}{3} - \frac{\theta^3}{42} + \frac{\theta^5}{1{,}320} - \frac{\theta^7}{75{,}600} + \cdots\right], \tag{49}$$

$$x = l - l\left[\frac{\theta^2}{10} - \frac{\theta^4}{216} + \frac{\theta^6}{9{,}360} - \cdots\right]. \tag{50}$$

The foregoing equations, although unsuitable for everyday use in the office or field, are fundamental as a basis for compiling spiral tables such as are given herewith.

113. Functions of the Spiral and Circle

In Fig. 48, the two spiral arcs extending from the T.S. to the S.C., and from the C.S. to the S.T., are joined by an intermediate circular arc subtending a central angle of $(\Delta - 2\theta_s)$ degrees.

If the circular arc is extended in each direction to points F and F', HF and $H'F'$ $(= p)$ represent the distance that

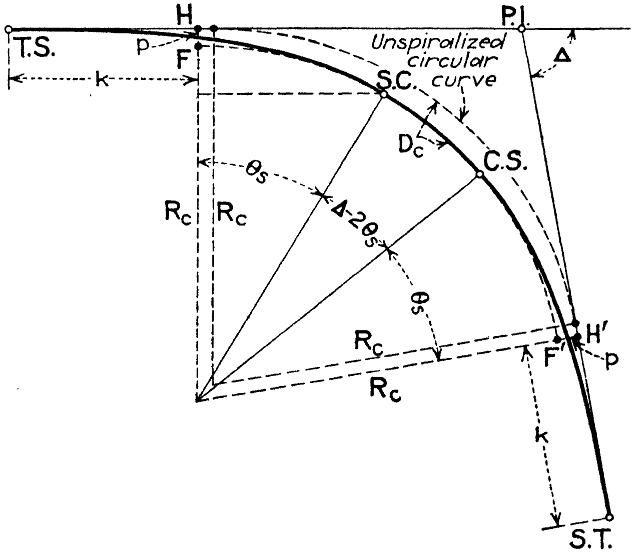


FIG. 48.

the circular curve has been shifted from the original tangents in order to make room for the spiral.

Referring to Fig. 48, certain parts (or functions) of the spiral will now be explained.

1. The *spiral angle* $= \theta_s =$ central angle of each spiral arc.

2. The *length of spiral* from T.S. to S.C. $= l_s = 200\theta_s/D_c$.

3. $y_c =$ perpendicular offset from initial tangent to the S.C.; $x_c =$ abscissa of the S.C. (origin at the T.S.).

4. $p = HF = y_c - R_c \text{ vers } \theta_s$. (51)

It is a useful fact that the offset from H to the spiral is very closely $\frac{1}{2}HF = \frac{1}{2}p$. Also, line HF approximately bisects the spiral.

5. E_s = *external distance* $= E + p \sec \frac{1}{2}\Delta$. (52)
6. $k = x_c - R_c \sin \theta_s$ = distance from T.S. to pt. H. (53)
7. $T_s = p \tan \frac{1}{2}\Delta + T + k$ = distance from P.I. to T.S. (54)

R, E, and T in the foregoing formulas are the *radius*, *external*, and *tangent* of the osculating circle FF' (Fig. 48).

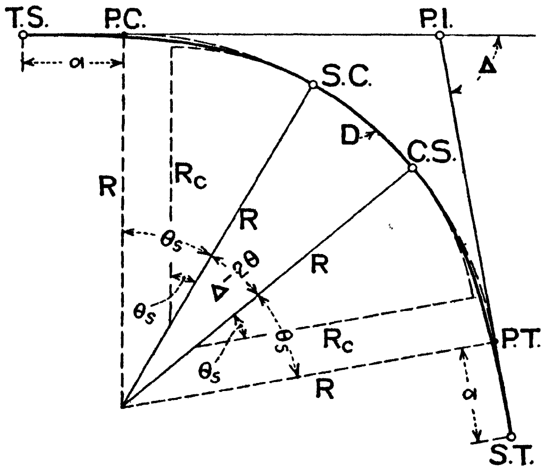


FIG. 49.

As a special procedure, it will be shown in Art. 118 that $T_s = T + a$, where a (in Table 12) is the distance along the initial tangent from P.C. to T.S. of the spiral which is compounded with the circle at the S.C., so that the middle portion of the circular curve remains unchanged (that is, not shifted inward), as shown in Fig. 49.

114. General Procedure for Laying Out Spiral and Circle By means of tables given herewith, the field work of laying out a spiralized circular curve is greatly simplified.

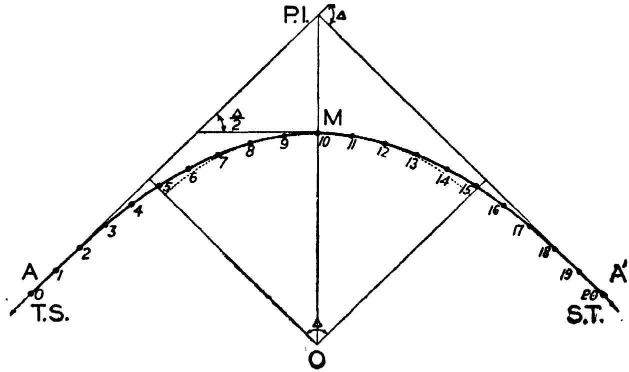


FIG. 50.—Double-spiral curve.

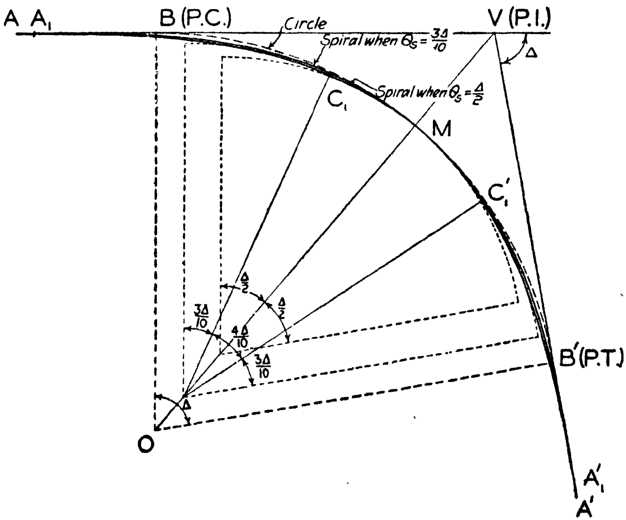


FIG. 51.—Position of unspiralized circular curve relative to spirals illustrated in Figs. 49 and 50.

The most general method of procedure, will now be presented, although *much simpler methods* to be explained later are *recommended.*

1. Given a circular curve of degree (D_c), select a spiral to join it at some point, then use the formula for length of spiral,

$$l_s = \frac{200\theta_s}{D_c}$$

2. Assuming the circle to be moved inward the required amount, determine p and k, from either formulas or Table 8.

3. Compute the total tangent distance (T_s) so as to locate the T.S. and the S.T.

4. With transit set at the T.S. lay out the first spiral, using deflection angles from Table 9, if the spiral is divided in 10 equal parts (as for a railroad); or take deflections from Table 10, if full stations are desired, as is ordinarily the case in highway location; or use the formula, $\phi = \frac{1}{3}\theta$; otherwise use method of offsets.

5. Set up transit at the S.C., vernier reading $(\theta_s - \phi_c)°$ on *other* side of 0°; take backsight on the T.S. with telescope in a reversed position, plunge telescope, unclamp alidade, and turn until vernier reads 0° 00′, thus putting the line of sight on auxiliary tangent at the S.C.

6. Run in circular curve from S.C. to C.S. by the deflection method, the central angle of the curve being $\Delta - 2\theta_s$.

7. Set transit at the S.T., vernier reading 0° 00′ on tangent, and then lay out the second spiral, checking on the C.S.

115. Deflection Angles

Let ϕ = angle PAX, in Fig. 47, = *deflection angle* at the T.S. between initial tangent and any point on the spiral. This angle is determined by its tangent, thus:

$$\tan \phi = \frac{y}{x}.$$

Substituting Eqs. (49) and (50), and dividing,

$$\tan \phi = \frac{\theta}{3} \text{ (approx.)}$$

Since for small angles the tangent is very closely equal to the angle (in radians), we have, approximately:

$$\phi = \frac{\theta}{3}, \qquad \text{or} \qquad \phi^\circ = \frac{\theta^\circ}{3} - C, \tag{55}$$

in which C is a small correction given below.

The correction in Eq. (55) is negligible for values of θ under 20°, as is shown in the table below.

Substituting the value of θ° from Eq. (48), we have

$$\phi^\circ = \frac{\theta_s^\circ}{3}\left(\frac{l}{l_s}\right)^2 - C. \tag{55a}$$

Equation (55*a*) shows that the deflections vary as *square of the distance* from the T.S.

VALUES OF THE CORRECTION (C) IN THE FORMULA: $\phi^\circ = \dfrac{\theta^\circ}{3} - C$

θ (degrees)	C (minutes)	θ (degrees)	C (minutes)
20	0.4	49	6.2
25	0.8	50	6.6
30	1.3	51	7.0
35	2.1	52	7.5
40	3.2	53	7.9
41	3.4	54	8.4
42	3.7	55	8.8
43	3.9	56	9.3
44	4.2	57	9.8
45	4.5	58	10.4
46	4.9	59	10.9
47	5.3	60	11.4
48	5.8	61	12.0

Note: Inasmuch as Tables 9 and 10 automatically take into account this small correction; and, too, since most values of θ are under 25°, this factor C will be considered negligible and therefore removed from the formula; thus we have $\phi = \theta/3$.

Table 10 gives values of ϕ, or the deflections from the T.S. to *any* point along a spiral; while Table 9 gives these deflections to each of the 10 points of equal division along a spiral. Table 9 is more adaptable to railroad spirals.

With Tables 9 and 10 available, Eq. (55*a*) would rarely be needed.

Letting ϕ_c = total deflection angle from a tangent at T.S. to the S.C., then the angle at the S.C. between a tangent and a chord to the T.S. (angle ACS in Fig. 52) is $\theta_s - \phi_c = \frac{2}{3}\theta_s + C$, in which C is negligible if θ_s is not over 20°.

EXAMPLE 1. Given $D_c = 10°$, $\theta_s = 20°$, $l_s = 400$ ft., and the T.S. at Sta. 37 + 10.2. It is required to locate a spiral by deflections from a tangent at the T.S.

Three or more ways are open for finding the deflection angles:

1. Take deflections for full station points directly from Table 10, as recorded in the table below.

$$\theta_s = \text{spiral angle} = 20°$$

Transit at Sta. 37 + 10.2, vernier reading 0° 00′ on tangent.

Sta.	Distance ratio l/l_s	Deflections
37 + 10.2	0.000	0° 00′
+ 50	0.100	0° 04.0′
38 + 00	0.224	0° 20.0′
+ 50	0.350	0° 49.0′
39 + 00	0.474	1° 30.0′
+ 50	0.600	2° 24.0′
40 + 00	0.724	3° 29.6′
+ 50	0.850	4° 48.8′
41 + 00	0.975	6° 20.0′
+ 10.2	1.000	6° 39.6′

2. Using the formula, $\phi = \frac{1}{3}\theta = \frac{1}{3}(l/l_s)^2\theta_s$, it is best to express $\frac{1}{3}\theta_s$ in *minutes*, thus $\frac{1}{3}(20°) = 400$ minutes. Then

$$\phi = \left(\frac{l}{l_s}\right)^2 \times 400 \text{ (minutes).}$$

A table of squares (Table 25) is convenient.
The results are given in the table below.

Sta.	$\frac{l}{l_s}$	$\left(\frac{l}{l_s}\right)^2$	Deflections
37 + 10.2 (T.S.)	0.000	0.000	0° 00′
+ 50	0.100	0.010	0° 04′
38 + 00	0.224	0.050	0° 20′
+ 50	0.350	0.122	0° 49′
39 + 00	0.474	0.225	1° 30′
+ 50	0.600	0.360	2° 24′
40 + 00	0.724	0.524	3° 30′
+ 50	0.850	0.722	4° 49′
41 + 00	0.975	0.951	6° 20′
+ 10.2 (S.C.)	1.000	1.000	6° 40′

3. If the spiral is divided in 10 equal parts of 40 ft. each, the deflections to the points of equal division are given directly in Table 9 for $\theta_s = 20°$. This is the simplest method of procedure, but *full stations* are not located, as would usually be desired in highway location.

Thus, with transit at the T.S., vernier reading 0° 00′ on initial tangent, the deflections are as follows:

Sta.	Deflections
37 + 10.2 (T.S.)	0° 00′
+ 50.2	0° 04′
+ 90.2	0° 16′
38 + 30.2	0° 36′
+ 70.2	1° 04′
39 + 10.2	1° 40′
+ 50.2	2° 24′
+ 90.2	3° 16′
40 + 30.2	4° 16′
+ 70.2	5° 24′
41 + 10.2 (S.C.)	6° 40′

EXAMPLE 2. Given $\Delta = 32° 10'$, and the desired external distance (distance from the P.I. to the middle of curve) is about 48 ft. The P.I. is at Sta. 87 + 12.2. It is required to determine the curve data.

SOLUTION. A *double-spiral* curve will be assumed. Accordingly, Table 11 gives E_s (for 100-ft. spiral) = 9.683. Ratio: $\frac{48}{9.683} = 4.99 = 5.00$, say. Hence, length of spiral = 5 stations = 500 feet. Then, more exactly, $E_s = 9.683 \times 5 = 48.4$ ft., also $T_s = 101.897 \times 5 = 509.5$ ft.

The stations are established as follows.

$$
\begin{array}{rl}
\text{P.I.} = & \text{Sta. } 87 + 12.2 \\
T_s = & 5 + 09.5 \\
\hline
\text{T.S.} = & \text{Sta. } 82 + 02.7 \\
l_s = & 5 + 00.0 \\
\hline
\text{S.C.} = & \text{Sta. } 87 + 02.7 \\
l_s = & 5 + 00.0 \\
\hline
\text{S.T.} = & \text{Sta. } 92 + 02.7
\end{array}
$$

The curve may be run in by deflections taken from Table 10. It is better to locate the first half from the T.S., and then the second half from the S.T., checking at the center.

116. Angular and Linear Divergence Property

Since a straight line and a circle each have *constant* curvature and the transition spiral is a curve of *uniformly changing* curvature, it is true that the *spiral diverges in angle and offset from the circular curve, for a given distance, at the same rate as from the initial tangent.* This is a useful property of the spiral.

Figure 52 shows a spiral diverging from the straight line *AH* with the same offsets as from the circle *CF*. If the spiral is divided in 10 equal parts, the offsets from points 1, 2, 3, etc., on the circle to corresponding points 9, 8, 7, etc. on the spiral are the same practically as the perpendicular offsets from spiral points 1, 2, 3, etc., respectively, to the initial tangent.

The error in applying this principle of equal offsets is not appreciable in practice. Even if the circular arc is extended

to point J, so that the arc CJ is equal to the spiral arc CA; the offset JA from circle to spiral is very closely equal to the perpendicular offset y_c from tangent to spiral at point C. This extreme offset (which would never be used as such in practice) is within 1 per cent of accuracy even for spiral angles as large as 45°.

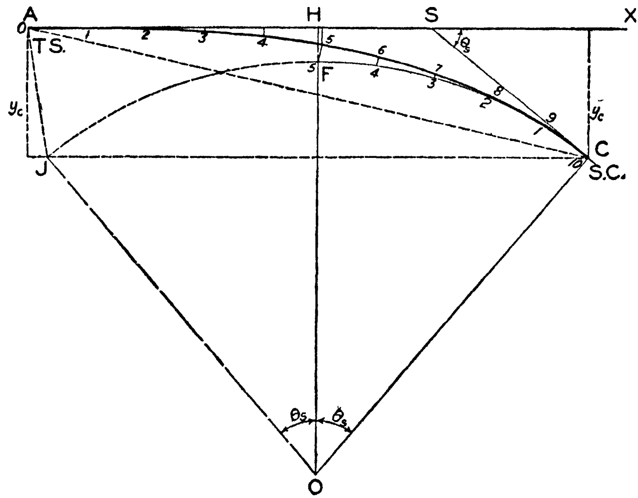


Fig. 52.

117. Deflections from Intermediate Point on Spiral

The fact that "the spiral diverges from a circle at the same rate as from the initial tangent" provides a method for locating forward or backward stations from any point.

Thus, if the transit is set at point P (Fig. 53), vernier reading 0° 00′ on auxiliary tangent, PS, the deflection to the *forward* point P' is $\delta + \phi$ = deflection for circular arc PC' ($= PP'$) of radius R *plus* deflection for spiral arc PP', as if point P were the T.S.

Also, the deflection from P to any *backward* point P'' is $\delta - \phi$.

EXAMPLE. Given spiral angle $\theta_s = 16°$, $D_c = 4°$, and the T.S. at Sta. 31 + 12.4. Assuming that stations 32–36 have been established by deflections from the T.S., it is required to locate the balance of the spiral by deflections from Sta. 36 + 00.

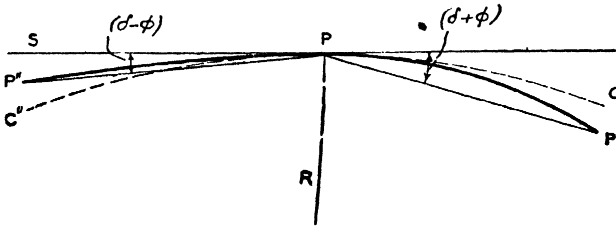


FIG. 53.

SOLUTION. $l_s = 200 \times \frac{16}{4} = 800$ ft.

$$\begin{array}{rl} \text{T.S.} = & \text{Sta. } 31 + 12.4 \\ l_s = & \quad\ \ 8 + 00.0 \\ \hline \text{S.C.} = & \text{Sta. } 39 + 12.4 \end{array}$$

Letting D_s = degree-of-curve of spiral at any point (as Sta. 36), we have $D_s : D_c = l : l_s$.

Hence $D_s = D_c \times l/l_s = 4 \times 487.6/800 = 2.438° = 2° \ 26.28'$

Then the circular curve deflection for Sta. 37, 100 ft. ahead, is $\frac{1}{2}D_s = 1° \ 13.14' = 73.14'$.

Using the formula, $\phi = \left(\frac{l}{l_s}\right)^2 \frac{\theta_s}{3}$, the spiral deflection for a distance of 100 ft. is

$$\phi = \left(\tfrac{100}{800}\right)^2 \times \tfrac{16}{3} \text{ (degrees)} = 5 \text{ (minutes)}$$

Therefore, with transit at Sta. 36, vernier reading 0° 00′ on auxiliary tangent, the deflections are as follows:

Sta.	Distance (ft.)	Deflections $\delta + \varphi$
37	100	1° 13.14′ + 0° 05.0′ = 1° 18.1′
38	200	2° 26.28′ + 0° 20.0′ = 2° 46.3′
39	300	3° 39.42′ + 0° 45.0′ = 4° 24.4′
+ 12.4 (S.C.)	312.4	3° 48.50′ + 0° 48.8′ = 4° 37.3′

Note: It will be observed that the circular curve deflections (δ) vary *directly* as the distance from the point of tangency, while the spiral deflections (ϕ) vary as the *square* of this distance.

In order to orient the instrument at Sta. 36, it is necessary to know the total spiral deflection from the T.S. to Sta. 36, a distance ratio of 0.610. This deflection (from Table 10) is 1° 59′.

Figure 54 shows that the backsight angle from any point P to the T.S. is $\theta - \phi = 2\phi$ (since $\phi = \frac{1}{3}\theta$). Hence the backsight angle is 3° 58′; and when the vernier is turned to 0° 00′, the telescope is on the auxiliary tangent.

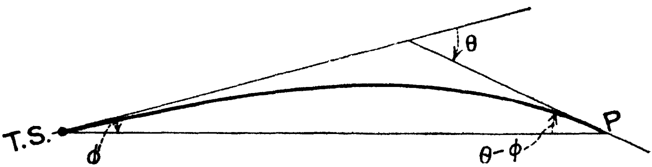


FIG. 54.

118. Spiral Compounded with Circular Curve Whose Central Portion Remains Unchanged

In Fig. 55, it will be assumed that the circular curve of radius (R) and degree-of-curve (D) is laid out on the ground (or is projected on paper) so that the middle portion, opposite vertex, cannot be shifted further inward.

Assuming the middle portion of the circular curve to be unchangeable, some point on it (the S.C.) will be selected, preferably where the spiral angle is an integral number of degrees; then it is required to find the spiral which will be compounded with this circle at the S.C.

Since, in Fig. 55, $y_c = R \text{ vers } \theta_s$, it is possible to find the length and other functions of the spiral which satisfy the foregoing requirements.

Table 12 gives functions of the spiral which is compounded with a 1° circular curve.

It will be seen that the *length* of this spiral is about $150\frac{\theta_s}{D}$, or *three-fourths* of the nominal length $\left(200\frac{\theta_s}{D}\right)$; and

the distance a, from P.C. to T.S., is about $50\frac{\theta_s}{D}$, or *one-third* length of spiral. Also, it follows that the radius (R_c) of the sharpened osculating circle is about $\frac{3}{4}R$.

For convenience in computing the "distance ratio" when taking deflection angles from Table 10, the values of $100/l_s$ are given.

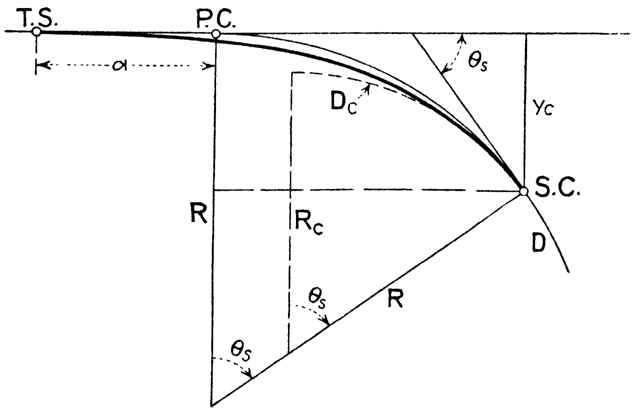


FIG. 55.

In case the transit must be moved to an intermediate point on the curve, the values of D_s and ϕ, given in Table 12, would be useful.

The foregoing method is very easy to apply, and it is particularly adaptable to highway location.

EXAMPLE 1. Given $D = 4°$, $\Delta = 42° \; 12'$, and the P.C. at Sta. 81 + 11.4. It is required to lay out a spiral which will be compounded with the circular curve whose central portion remains unchanged.

SOLUTION. Letting the central angle of each spiral = 16°, there remains $\Delta - 2 \times 16° = 10° \; 12'$ for the intermediate circular arc.

Referring to Table 12 with spiral angle = 16°, as argument, and remembering that $D = 4°$, we have:

a = distance from P.C. to T.S. $= \frac{799.9}{4} = 200.0$ ft. (same as $50 \times \frac{\theta_s}{D}$); and

l_s = length of spiral $= \frac{2397.7}{4} = 599.4$ $\left(\text{or about } 150 \times \frac{\theta_s}{D}\right)$.

$$\begin{array}{rrr} \text{P.C.} = & \text{Sta.} & 81 + 11.4 \\ a = & & 2 + 00.0 \\ \hline \text{T.S.} = & \text{Sta.} & 79 + 11.4 \\ l_s = & & 5 + 99.4 \\ \hline \text{S.C.} = & \text{Sta.} & 85 + 10.8 \end{array}$$

The "distance ratio" for 100 ft. is 0.04171 (from Table 12) $\times$ 4 = 0.16684. This quantity will be added for each station in succession.

With transit set at the T.S., vernier reading 0° 00′ on tangent, the deflections may be taken directly from Table 10 as follows:

SPIRAL ANGLE = 16°

Sta.	Distance ratio	Deflections
79 + 11.4 (T.S.)	0.000	0° 00.0′
80	0.148	0° 07.0′
81	0.315	0° 31.8′
82	0.482	1° 14.3′
83	0.649	2° 14.8′
84	0.815	3° 32.5′
85	0.982	5° 08.4′
85 + 10.8 (S.C.)	1.000 (check)	5° 19.8′

From T.S. to S.T. by way of the spiral and intermediate circular arc, the distance is $599.4 + \frac{100 \times 10.2}{4} + 599.4 =$ 1453.8 ft.

From T.S. to S.T. by way of the original circular curve, the distance is $200 + \frac{100 \times 42.2}{4} + 200 = 1455.0$ ft.

Hence the spiral path is 1.2 ft. shorter.

The tangent offset from P.C. to spiral is $\frac{8.33 \text{ (from Table 12)}}{4}$ = 2.08 ft.

As will be seen from Fig. 55, the deviation from the original circle would be more than 2.08 ft. further on along the curve.

EXAMPLE 2. Same as Example 1 (page 172) except that it is assumed that the transit is set up at Sta. 83, and the balance of the curve is to be run out from that position.

From Table 12, the degree-of-curve (D_s) 100 ft. from the T.S. = 0.05566 × (4)² = 0.89056°. Then at Sta. 83, which is 3.886 stations ahead of the T.S., D_s = 0.89056 × 3.886 = 3.4607° = 3° 27.6′. Hence the circular curve deflection at Sta. 83 per 100 ft. is $\frac{1}{2}D_s$ = 1° 43.8′.

Also, from Table 12, spiral deflection (ϕ) for first 100 ft. = 0.55662 × (4)² = 8.906′.

Hence with vernier reading 0° 00′ on auxiliary tangent at Sta. 83, we have the following deflections:

Sta.	Distance (ft.)	Deflections $\delta + \varphi$
84	100.0	1° 43.8′ + 0° 08.9′ = 1° 52.7′
85	200.0	3° 27.6′ + 0° 35.6′ = 4° 03.2′
+ 10.8 (S.C.)	210.8	3° 38.8′ + 0° 39.6′ = 4° 18.4′

EXAMPLE 3. Given R = 2,000 ft., Δ = 30° 20′ and the P.I. at Sta. 52 + 00. It is required to locate a spiral which will be compounded with the circular curve whose central portion remains unchanged.

SOLUTION. $T = R \tan \frac{1}{2}\Delta$ = 2,000 × 0.27107 = 542.14 ft. Hence the P.C. is at Sta. 52 + 00 − (5 + 42.14) = Sta. 46 + 57.9.

Letting the central angle of each spiral = 14°, there remains Δ − 2 × 14° = 2° 20′ for the intermediate circular arc.

Referring to Table 13 with spiral angle = 14°, as argument, and using the ratio, 10,000/2,000 = 5, we have a = 1,221.6/5 = 244.3 ft., and l_s = 3,662.6/5 = 732.5 ft.

Then P.C.	= Sta.	46 + 57.9
a	=	2 + 44.3
T.S.	= Sta.	44 + 13.6
l_s	=	7 + 32.5
S.C.	= Sta.	51 + 46.1

With the transit set at the T.S., vernier reading 0° 00′ on tangent, the deflections locating full stations may be taken from Table 10, as was done in Example 1.

119. Location of Spiral by Offsets

a. Offsets from Tangent. If the central angle (θ) at any point of a spiral is known, Table 7 gives the coordinates x and y referred to the T.S. and the initial tangent. It is therefore readily possible to locate the spiral by means of offsets (y) from the tangent.

The values of θ vary as the *square* of the distance ratio; that is,

$$\theta = \left(\frac{l}{l_s}\right)^2 \theta_s.$$

The example which follows fully explains the method of procedure.

EXAMPLE. Same data as Example 1, page 172. It is required to find x and y for each full station.

SOLUTION. Having found the central angle (θ) at each station, the corresponding unit values of x and y are taken from Table 7. These are to be multiplied by the spiral arc length (in stations) from the T.S. to each point in turn.

The results in the following table (see page 176) indicate the general method of procedure.

b. Offsets from Tangent and Circle. Conforming somewhat to the practice of the Indiana Highway Commission, Table 14 gives offsets from the original tangent and circle to the new center line of a modified spiral (see Fig. 56) which starts on the tangent, 100 ft. back of the P.C., and extends to a point on the offset circle, 100 ft. ahead, thus giving a spiral length of approximately 200 ft.

Although simple to lay out, this method has the theoretical disadvantage of adopting the same length spiral for all radii.

Making use of Table 14, the outer and inner edges of spiraled pavement may be located with reference to the original tangent and circle as follows; distance to *outer*

SPIRAL ANGLE = θ_s = 16°

Sta.	Distance (stations)	$\left(\frac{l}{l_s}\right)^2$	θ	x (ft.)	y (ft.)
79 + 11.4 (T.S.)	0	0	0	0	0
80	0.886	$(0.148)^2$ = .0219	0.350° = 0° 21.0′	100.00 × 0.886 = 88.6	0.204 × 0.886 = 0.18
81	1.886	$(0.315)^2$ = .0992	1.587° = 1° 35.2′	99.992 × 1.886 = 188.6	0.922 × 1.886 = 1.74
82	2.886	$(0.482)^2$ = .2323	3.717° = 3° 43.0′	99.957 × 2.886 = 288.5	2.161 × 2.886 = 6.24
83	3.886	$(0.649)^2$ = .4212	6.739° = 6° 44.3′	99.862 × 3.886 = 388.1	3.916 × 3.886 = 15.22
84	4.886	$(0.815)^2$ = .6642	10.627° = 10° 37.6′	99.656 × 4.886 = 486.9	6.167 × 4.886 = 30.13
85	5.886	$(0.982)^2$ = .9643	15.429° = 15° 25.7′	99.276 × 5.886 = 584.3	8.930 × 5.886 = 52.56
+ 10.8 (S.C.)	5.994	$(1.000)^2$ = 1.000	16.000° = 16° 00.0′	99.223 × 5.994 = 594.7	9.257 × 5.994 = 55.49

edge = $\frac{1}{2}$ *width* − *offset,* and distance to *inner* edge = $\frac{1}{2}$ *width* + *offset.*

120. Spiralized Compound Curves

A transition spiral may be used to join the two branches of a compound curve, if the sharper curve is made still sharper, or is moved inward far enough to provide the proper clearance for the spiral. The function of the spiral is to give a gradual transition from one degree of curvature to the other.

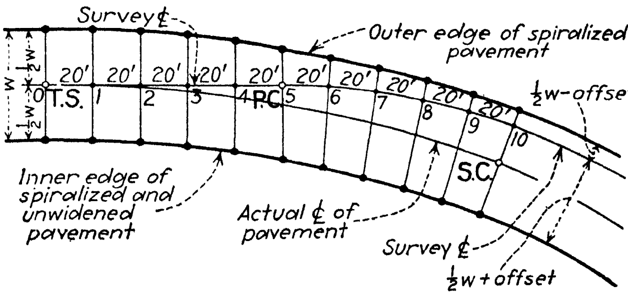


FIG. 56.—Modified 200-ft. spiral (see page 175).

In Fig. 57, page 179, the second curve (the sharper) is assumed to be moved radially inward at the P.C.C. a distance HJ $(= p)$; and the arc JW is laid off equal to the arc HQ. Then a spiral QNW, inserted between the two curves, will deviate inward from the circle QH at the same rate as it deviates outward from the circle WJ. The spiral therefore bisects HJ. Also, the offset HJ $(= p)$ and the spiral bisect each other, approximately.

The property that a spiral diverges from an osculating circular arc at the same rate both in angle and in offset, as from a tangent at the T.S., is useful here.

Let D_1 and D_2 = the degree-of-curve of the two branches of the compound curve;

l_s and θ_s = length and central angle of the spiral QW which joins the circular arcs;

l_2 and θ_2 = length and central angle of the completed spiral AW extending back to an imaginary T.S.;

l_1 and θ_1 = length and central angle of spiral AQ;
Δ_1 and Δ_2 = central angles of circular arcs QH and JW respectively;
p = offset HJ.

Since arcs QH and JW are equal,

$$\frac{100\Delta_1}{D_1} = \frac{100\Delta_2}{D_2}$$

Also, since spiral arc $(l_s) = 2 \times$ arc $QN = 2 \times$ arc NW, we have

$$l_s = \frac{200\Delta_1}{D_1} = \frac{200\Delta_2}{D_2} \text{ (very closely).}$$

For the completed spiral, we have

$$\frac{l_2}{l_1} = \frac{D_2}{D_1}, \qquad \text{or} \qquad \frac{l_2}{l_2 - l_1} = \frac{D_2}{D_2 - D_1}.$$

But $\quad l_2 - l_1 = l_s; \qquad$ hence $\qquad l_2 = l_s \times \dfrac{D_2}{D_2 - D_1}. \qquad (56)$

Similarly, we have $l_1 = l_s \times \dfrac{D_1}{D_2 - D_1}. \qquad (57)$

Since $\qquad l_2 = \dfrac{200\theta_2}{D_2} \qquad$ and $\qquad l_1 = \dfrac{200\theta_1}{D_1};$

we have $$\theta_2 = \frac{l_2 D_2}{200} = \frac{l_s D_2^2}{200(D_2 - D_1)}, \qquad (58)$$

and $$\theta_1 = \frac{l_1 D_1}{200} = \frac{l_s D_1^2}{200(D_2 - D_1)}. \qquad (59)$$

Then $$\theta_s = \theta_2 - \theta_1 = \frac{l_s(D_1 + D_2)}{200}. \qquad (60)$$

The initial step in locating the spiral is usually to decide upon the length (l_s), and then determine the offset (p) from the property of the spiral, that the offsets vary as the cube of the distance (very closely).

Then since the spiral QW (beginning at point W) diverges from the circular arc WJ at the same rate as from the imaginary tangent at the T.S., we have

$$\tfrac{1}{2}p = \left(\frac{\tfrac{1}{2}l_s}{l_2}\right)^3 \times y,$$

where y is found in Table 7 with θ_2 and l_2 as arguments.

It will be seen that the distance from point W to the unoffset D_2-curve is $p \cos \Delta_2$, as indicated in Fig. 57.

The spiral may be located by (a) *deflections* or (b) *offsets*, either method being slightly approximate for large differences in D_1 and D_2.

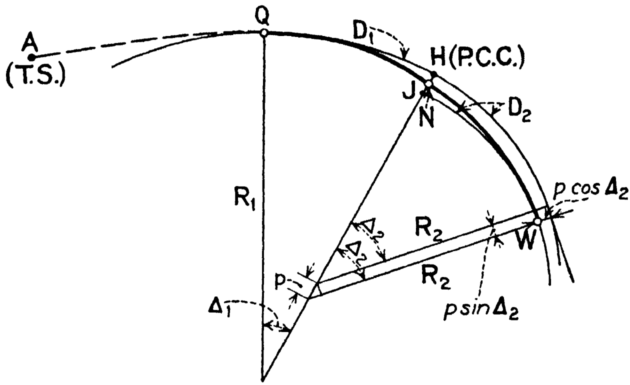


FIG. 57.—Spiralized compound curve.

a. Deflections. With transit at Q, vernier reading 0° 00′ on auxiliary tangent, the deflection to any point on the spiral $= \delta + \phi$; in which δ is deflection for circular curve of degree D_1 and ϕ is spiral deflection for the same distance as if measured from the T.S.

b. Offsets. The offset from H inward to the spiral $= \tfrac{1}{2}p$. Offsets inward from points on the circle QH and outward from corresponding points on the circle WJ vary practically as the *cube* of the ratio, $\dfrac{\text{distance from } Q \text{ or } W}{\tfrac{1}{2}l_s}$.

EXAMPLE. Given two circular curves compounded at H (P.C.C.); $D_1 = 6°$ and $D_2 = 10°$. It is required to insert a spiral between the two curves, the 10° curve being moved

inward at the P.C.C. a distance p depending upon the length of spiral selected.

SOLUTION. Assume a spiral of length 400 ft., extending 200 ft. on each side of the P.C.C. Then, using Eq. (60), $\theta_s = \frac{400(6+10)}{200} = 32°$. Also, $\theta_2 = \frac{400 \times (10)^2}{200 \times 4} = 50°$; and $\theta_1 = \frac{400 \times (6)^2}{200 \times 4} = 18°$.

As a check, $\theta_s = \theta_2 - \theta_1 = 50° - 18° = 32°$.

Using Eqs. (56) and (57), $l_2 = 400 \times \frac{10}{4} = 1{,}000$ ft., and $l_1 = 400 \times \frac{6}{4} = 600$ ft.

From Table 7 with θ_2 and l_2 as arguments, $y = 27.544 \times 10 = 275.44$. Then $\frac{1}{2}p = (200/1{,}000)^3 \times 275.44 = 2.20$ ft.

One point on the spiral is therefore determined by offsetting at the P.C.C. a distance $= \frac{1}{2}p = 2.20$ ft.

Assuming the transit set on the 6° curve (200 ft. back of P.C.C.), the deflections from auxiliary tangent to points 1, 2, 3, etc. are $\delta_1 + \phi_1$, $\delta_2 + \phi_2$, $\delta_3 + \phi_3$, etc., where δ is for a 6° curve and ϕ for a spiral as if run from the T.S.

Thus if point 1 is 100 ft. ahead, $\delta_1 = \frac{1}{2}D_1 = 3°\ 00'$ and $\phi_1 = \frac{1}{3}(50)(100/1{,}000)^2 = \frac{1}{6}° = 0°\ 10'$.

The deflections to various points on the 400-ft. spiral are listed below.

Distance (ft.)	Deflections $\delta + \phi$
100	3° 00′ + 0° 10.0′ = 3° 10.0′
200	6° 00′ + 0° 40.0′ = 6° 40.0′
250	7° 30′ + 1° 12.5′ = 8° 42.5′
300	9° 00′ + 1° 30.0′ = 10° 30.0′
350	10° 30′ + 2° 02.5′ = 12° 32.5′
400	12° 00′ + 2° 40.0′ = 14° 40.0′

121. Spiralized Reversed Curve

In Fig. 58, the two circular curves *HB* and *BF′*, shown by dotted lines, are reversed at *B* (the P.R.C.).

It is required to improve the alignment in every possible way without changing radically the existing location.

Assume point M_1 opposite vertex of the first curve (the sharper) to be fixed, but the first half of the second curve may be shifted slightly inward toward its center.

PROCEDURE. 1. At point B (P.R.C.), offset at right angles to tangent BV_1V_2 the distance $BN = \frac{1}{2}p$, where p is determined from a "spiral angle" ($= \frac{1}{2}\Delta_1$) and spiral arc ($= 2 \times$ arc BM_1).

2. Run in the half spiral NM_1 by offsets from the arc BM_1, or by deflections.

3. Extend a line through N parallel to BV_2 until it intersects the terminal tangent at V_2'. The distance V_2V_2' may be found either from field measurements or computations. The angle $\Delta_2' = \Delta_2 - \theta$.

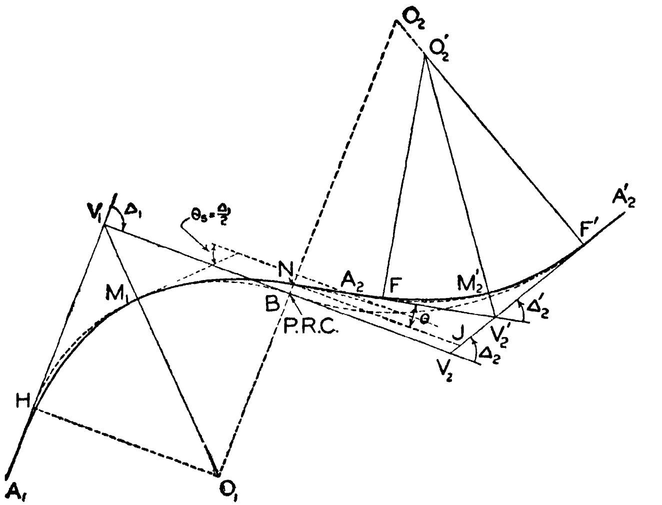


FIG. 58.—Spiralized reversed curve.

4. Run in new circular curve FF' connecting tangents NV_2' and $V_2'A_2'$, inserting spirals at each end if possible.

122. Inner and Outer Edges of Pavement

Assuming the center line to be a spiral, the inner and outer edges of the unwidened pavement will be parallel curves at radial distances of $\frac{1}{2}w$ from the center, where w = normal width of pavement. These outer and inner edge curves are strictly not true spirals, since they are laid out by offsetting radially from the center-line spiral.

A point on the outer or inner curve at an offset distance of $\frac{1}{2}w$ ft. is in the true radial direction if its distance to the next point on center spiral is $\sqrt{(\frac{1}{2}w)^2 + c^2}$ ft., where c is the center-spiral chord length, the chord being short enough practically to coincide with the arc.

The length of these outer and inner curves may be found as follows:

Let l_s'' = length of outer curve from point opposite the T.S. to radial line at S.C.;
l_s' = corresponding length of inner curve;
R'' = radius of curvature of outer edge at any point;
R' = radius of curvature of inner edge at any point.

For the center-line spiral,

$$R = \frac{R_c l_s}{l} \qquad \text{and} \qquad \theta = \frac{l^2}{2R_c l_s} \text{ (see Art. 112).}$$

Let dl'' = infinitesimal part of outer curve.

$$\text{Then } dl'' = R''d\theta = \left(R + \frac{1}{2}w\right)d\theta = \left(\sqrt{\frac{R_c l_s}{2}}\theta^{-\frac{1}{2}} + \frac{1}{2}w\right)d\theta.$$

$$\text{Integrating,} \qquad l'' = 2\sqrt{\frac{R_c l_s \theta}{2}} + \frac{1}{2}w\theta.$$

Let $\theta = \theta_s$,

hence $\qquad l_s'' = l_s + \frac{1}{2}w\theta_s.$

Similarly $\qquad l_s' = l_s - \frac{1}{2}w\theta_s.$

In the above formulas, θ_s is expressed in radians. (Divide degrees by 57.3 in order to change to radians.)

The formulas for l_s'' and l_s' give the exact length of curbing; and $l_s'' + l_s' = 2l_s$.

REVIEW PROBLEMS

Problem 1. If the central angle of a spiral arc is 15°, and the degree-of-curve of the osculating circle is 5°, what is the length of spiral? *Ans.:* 600 ft.

Problem 2. If the spiral angle is 18°, and the length of spiral is 500 ft., what is the deflection angle (ϕ) from the T.S. to a point 300 feet ahead?

Ans.: From formula, $\phi = \frac{\theta_s}{3}\left(\frac{l}{l_s}\right)^2 = 6(0.6)^2 = 2°\ 10'$;

from Table 10, $\phi = 2°\ 10'$.

Problem 3. The T.S. is at Sta. 148 + 90, the spiral angle is 12°, and the length of spiral is 1000 ft. What is the deflection angle (ϕ) from the T.S. to Sta. 152 + 00?

Ans.: 0° 23.1′ (Table 10).

Problem 4. Assuming a 4° 30′ circular curve to be laid out with its middle portion established inward as far as possible, what is the length of spiral which will be compounded with the unoffset circular curve at a point where the central angle of the spiral is 16° 00′? *Ans.:* 532.8 ft. (Table 12).

Problem 5. Using the data of Problem 4, what is the distance (a) from the P.C. to the T.S.? *Ans.:* 177.8 ft.

Problem 6. Given $\Delta = 20°\ 00'$, and $D = 1°$. Using a double-spiral curve compounded so that the middle point opposite vertex remains unoffset, (a) what is the length of each spiral branch, and (b) what is the offset from P.C. to spiral?

Ans.: (a) 1,499.6 ft. (Table 12); (b) 3.24 ft. (Table 12).

Problem 7. The T.S. is at Sta. 60 + 40, the length of spiral is 400 ft., and the spiral angle is 16°. With reference to the T.S. as origin, what are the coordinates of (a) the S.C., (b) Sta. 62 + 00?

Ans.: From Table 7: (a) $x = 396.9$ ft., $y = 37.03$ ft.;

(b) $x = 160.0$ ft., $y = 2.38$ ft.

Problem 8. The length of spiral is 300 ft., the spiral angle is 30°, the degree-of-curve of the osculating circle is 20° 00′, and $\Delta = 90°\ 00'$. What is the distance from the original circular curve to the T.S.? *Ans.:* $p \tan \frac{1}{2}\Delta + k = 161.60$ ft.

CHAPTER VI

CURVE SUPERELEVATION

123. Highways on straightaways are normally given a slight convex crown to take care of drainage. But if the same crown is continued around curves, a dangerous hazard exists for swift moving traffic, due to the *centrifugal force* which tends to cause skidding outward from the center of the curve.

This slipping tendency is resisted to a considerable extent by the adhesion or friction of the tires against the pavement. But *superelevating* or banking the curved roadway so that the surface becomes a plane, inclined toward the center, is a design feature of vital importance in resisting this centrifugal force and reducing the lurch experienced on entering curves.

The normal crown gives the inner half of pavement a partial banking effect, which is helpful to traffic on the inner lane; so much so, that those traveling in the reverse direction on the outer lane would tend to encroach and try to make their own transition, unless spirals are used and the entire pavement is superelevated around curves.

124. Theoretical Superelevation

a. Friction Not Considered. From principles stated in texts on mechanics, the centrifugal force exerted when bodies move rapidly around curved paths is given by the product of the *mass* times the *acceleration* toward the center of the circle; that is

$$\textit{Centrifugal force} = \frac{W}{g}\frac{v^2}{R},$$

where W = weight of body in pounds;

v = velocity in feet per second;

R = radius of curve in feet;

and g = acceleration due to gravity.

In Fig. 59, let θ represent the angle of inclination that is given to a curved roadway surface.

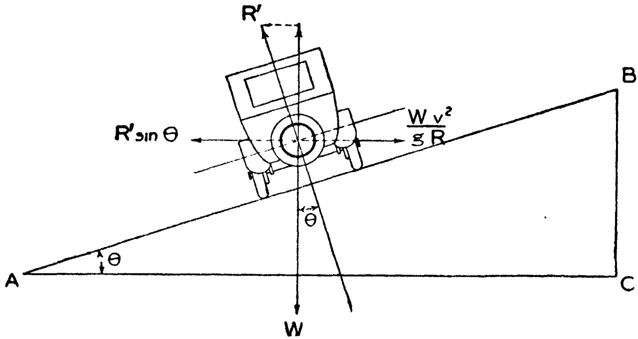


FIG. 59.

Considering the forces acting on a car, which is assumed to be moving at a uniform rate around the curve, we have:

For equilibrium horizontally and vertically

$$R' \sin \theta = \frac{W}{g}\frac{v^2}{R},$$

and

$$R' \cos \theta = W.$$

Hence, dividing the first equation by the second,

$$\tan \theta = \frac{v^2}{gR}. \tag{61}$$

Reducing v from feet per second to miles per hour (44 ft. per second = 30 m.p.h.), and substituting 32.2 for g, Eq. (61) becomes

$$\tan \theta = \text{transverse slope} = \frac{V^2}{15R} \text{ (closely)}. \tag{62}$$

Hence, the theoretical slope varies as the *square* of the velocity and *inversely* as the radius.

Letting $R = 5{,}730/D$, Eq. (62) shows that for 30 m.p.h., transverse slope $= 0.01D$. Thus for a 10° curve, the rate of superelevation $= 0.10$ ft. per foot of width of pavement; and the full theoretical superelevation for a 20-ft. pavement is 2.0 ft.

On the basis of 60 m.p.h., the slope $= 0.04D$, etc.

Owing to slow-moving traffic and slippery conditions on ice-coated pavements, the full theoretical rate of superelevation required for high velocities cannot be realized in practice. In fact, this is not necessary since friction offers considerable resistance. Experience has shown that one-half the theoretical superelevation is generally satisfactory.

The maximum practical superelevation is limited to about 0.10 to 0.13 ft. per foot of width. But the rate is not usually over 0.1 ft. per foot. For flat curves a smaller rate is customary.

The table on page 187 was compiled from computations based on Eq. (62).

b. Friction Considered. If f = coefficient of friction, then the frictional force, for the moderate slopes that occur in practice, is fW. Also, the component of the weight down the slope, $W \sin \theta$, is very closely equal to $W \tan \theta$. Hence we have

$$\textit{Centrifugal force} = \frac{Wv^2}{gR} = W \tan \theta + fW.$$

Dividing by W and solving gives

$$v = \sqrt{gR(\tan \theta + f)} \tag{63}$$

Letting V = miles per hour, $\tan \theta = s$, and $g = 32.2$, Eq. (63) becomes

$$V = \sqrt{15R(s + f)} \tag{64}$$

The value of f is taken to be about 0.25 to 0.30.

Thus, if f is 0.3 and the rate of superelevation is 1 in. per foot (0.083 ft. per foot), then it is just possible for a car to go around a 13° curve at 50 m.p.h. without skidding.

THEORETICAL SUPERELEVATION
(Transverse slope per foot of width)

$$e = \frac{V^2}{15R}$$

Deg. of curve	Velocity (miles per hour)								Radius (ft.)
	10	20	30	40	50	60	70	80	
0° 34′	.....	.....	0.006	0.011	0.017	0 024	0.033	0.043	**10,000**
0° 38′	.....	.....	0.007	0.012	0.018	0.027	0.036	0.047	**9,000**
0° 43′	.....	.....	0.007	0.014	0.021	0.030	0.042	0.055	**8,000**
0° 49′	.....	.....	0 009	0.015	0.024	0.034	0.047	0.061	**7,000**
0° 57′	.....	.....	0 010	0.018	0.028	0.040	0.054	0.071	**6,000**
1° 00′	.....	0.005	0.010	0.019	0.029	0.042	0.057	0.074	5,730
1° 09′	.....	0.005	0.012	0.021	0.033	0.048	0.065	0.085	**5,000**
1° 26′	.....	0.007	0.015	0.027	0.042	0.060	0.082	0.107	**4,000**
1° 55′	.....	0.009	0.020	0.036	0.056	0.080	0.109	0.142	**3,000**
2° 00′	.....	0.009	0.021	0.037	0.058	0.084	0.114	0.149	2,865
2° 18′	.....	0.011	0.024	0.043	0.067	0.096	0.131	0.171	**2,500**
2° 52′	.....	0.013	0.030	0.053	0.083	0.120	0.163	0.213	**2,000**
3° 00′	.....	0.014	0.031	0.056	0.087	0.126	0.171	0.223	1,910
3° 49′	.....	0 018	0.040	0.071	0.111	0.160	0.218	0.284	**1,500**
4° 00′	0.005	0.019	0.042	0.074	0.116	0.168	0.228	0.298	1,432
5° 00′	0 006	0.023	0.052	0.093	0.145	0.209	0.285	0.372	1,146
5° 44′	0.007	0.027	0.060	0.107	0.167	0.240	0.327	0.427	**1,000**
6° 00′	0.007	0.028	0.063	0.112	0.175	0.251	0.342	0.447	955
6° 22′	0.007	0.030	0.067	0.118	0.185	0.267	0.363	0.474	**900**
7° 00′	0.008	0.033	0.073	0.130	0.204	0.293	0.399	0.521	819
7° 10′	0.008	0.033	0.075	0.133	0.208	0.300	0.408	0.533	**800**
8° 00′	0.009	0.037	0.084	0.149	0.233	0.335	0.456	0.596	716
8° 11′	0.009	0.038	0.086	0.152	0.238	0.343	0.467	0.609	**700**
9° 00′	0.010	0.042	0.094	0.168	0.262	0.377	0.513	0.670	637
9° 33′	0.011	0.044	0.100	0.178	0.278	0.400	0.544	0.711	**600**
10° 00′	0.012	0.046	0.105	0.186	0.291	0.419	0.570	.....	573
11° 00′	0.013	0.051	0.115	0.205	0.320	0.461	0.627	.....	521
11° 28′	0.013	0.053	0.120	0.213	0.333	0.480	0.653	.....	**500**
12° 00′	0.014	0.056	0.126	0.223	0.349	0.503	0.684	.....	477
13° 00′	0.015	0.060	0.136	0.242	0.378	0.545	.....	.....	441
14° 00′	0.016	0.065	0.147	0.261	0.407	0.586	.....	.....	409
14° 19′	0.017	0.067	0.150	0.267	0.417	0.600	.....	.....	**400**
15° 00′	0.017	0.070	0.157	0.279	0.436	0.628	.....	.....	382
16° 00′	0.019	0.074	0.168	0.298	0.465	0.670	.....	.....	358
17° 00′	0.020	0.079	0.178	0.316	0.494	.....	.....	.....	337
18° 00′	0.021	0.084	0.188	0.335	0.524	.....	.....	.....	318
19° 00′	0.022	0.088	0.199	0.354	0.553	.....	.....	.....	302
19° 06′	0.022	0.089	0.200	0.356	0.556	.....	.....	.....	**300**
20° 00′	0.023	0.093	0.209	0.372	0.582	.....	.....	.....	286
21° 00′	0.024	0.098	0.220	0.391	0.611	.....	.....	.....	273
22° 00′	0.026	0.102	0.230	0.410	0.640	.....	.....	.....	260
23° 00′	0.027	0.107	0.241	0.428	0.669	.....	.....	.....	249
24° 00′	0.028	0.112	0.251	0.447	0.698	.....	.....	.....	239
25° 00′	0.029	0.116	0.262	0.465	0.727	.....	.....	.....	229
28° 39′	0.033	0.133	0.300	0.533	0.833	.....	.....	.....	**200**

Oregon has adopted as the average superelevation for flat curves a value of 0.075 ft. per foot and a frictional coefficient of 0.3. Using these values, the effect of frictional

resistance is equal to twice that of superelevation. However, the importance of superelevation should not be minimized, since proper superelevation will overcome the lurch or side sway of both vehicle and passengers, and at moderate speeds will balance the centrifugal force acting on them.

Curves sharper than 0° 30′ or 1° 00′ are superelevated.

125. Critical Speeds

According to Oregon[1] practice, three speeds are considered:

1. *Critical speed,* the maximum that can be attained with the standards used and beyond which only the most skillful racing drivers can operate without extreme hazards.

2. *Designed speed,* 80 per cent of the critical speed and a speed that is safe for skillful drivers.

3. *Recommended safe speed,* which takes into account normal traffic conditions and the limitations of the ordinary driver. Hence, a speed somewhat less than the designed speed.

The critical speed of an automobile on a highway is (quoting Mr. Baldock) controlled by the following factors: (1) the ability of the operator to function properly—the human equation, particularly in emergencies; (2) the ability of the mechanism of the vehicle to operate at high speeds without undue hazard; (3) the stopping distance or the distance traveled during the reaction time of the operator plus the braking distance; (4) the curvature, with which are involved easement, superelevation, and tractive resistance—all of which are brought into the equation for centrifugal force; (5) the horizontal sight distance on curves, which, of course, varies with the curvature, the position of the vehicle on the road, and the distance from the line of travel to the sight obstruction; (6) the sight distance over vertical curves; (7) the sight distance required in passing vehicles

[1] Baldock, R. H., "Highway Design for Speeds Up to 100 Miles per Hour," *Engineering News-Record,* May 23, 1935, p. 732.

at varying speeds; and (8) the gradient used in the mountain sections.

The critical speed permitted by one factor for a certain standard will not necessarily govern for another factor, so, in order to correlate the standards, it is necessary to consider the effect on speeds of all the factors with relation to the standards used for both horizontal and vertical curvatures.

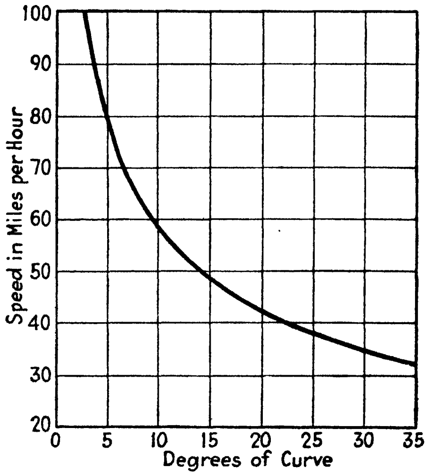


FIG. 60.—Limitations on critical speeds imposed by curves up to 35 deg.

126. Theoretical Length of Transition Spirals

To assume a constant length of transition spiral for different radii of curve is wrong in theory as was demonstrated in 1909 by Shortt and Spiller of England.

Letting l_s = length of spiral and v = velocity in feet per second, then the corresponding time (t) in seconds for traveling the distance (l_s) at the specified speed (v) is l_s/v. If R is the radius of curve, the acceleration toward the center is v^2/R, and the rate of gain of acceleration along the spiral of constantly decreasing radius is v^2/R divided by l_s/v; that is, v^3/l_sR.

Letting C equal the maximum rate of change of acceleration that will pass without noticeable discomfort, then $C = v^3/l_sR$, from which

$$l_s = \frac{v^3}{CR}. \tag{65}$$

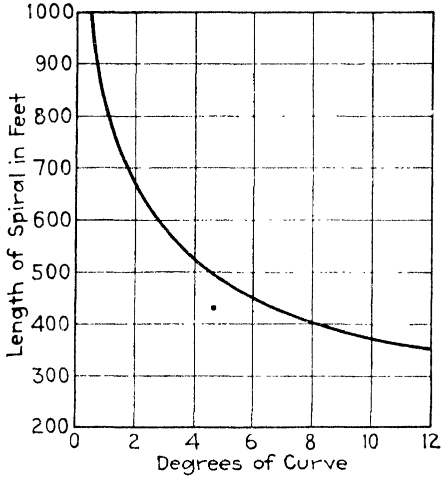


FIG. 61.—Theoretical lengths of spirals required for critical speeds on curves of trunk highways up to 12 deg.

If $C = 3$ ft. per second per second, as used by the Oregon Highway Commission, then

$$l_s = \frac{1.0577V^3}{R}, \tag{66}$$

where V is speed in miles per hour, and l_s and R are in feet.

Figure 61 is based on Eq. (66) and the same critical speeds as represented in Fig. 60.

127. Superelevation Methods

While changing in successive stages of warped surfaces from a normal crown to a plane, rotation may take place about the center line as an axis. The effect is to lower the

inside edge of the pavement and raise the outside edge, the grade of the center line remaining unchanged as shown in Fig. 62 (page 192).

Another method consists in revolving about the inner edge as an axis, so that it retains the normal grade throughout, while the grade of the center line is raised, as shown in Fig. 63 (page 193).

Care should be exercised to see that the profile grade is designed to avoid drainage difficulties which might result incidental to superelevation.

When banking unwidened pavements on curves flatter than 6°, some states leave the crown in while rotating about the center (or inner edge). This procedure enables the contractor to use his standard crown finishing machine throughout, but the resulting pavement is not properly superelevated into an inclined plane, and therefore this practice should not be continued.

The transition from a convex cross section on tangents to a plane section on curves should be developed so as to avoid angular breaks or too sudden changes. The length of spiral runoff is apt to be too short, thus producing a rate of change that is too high, resulting in what appears to be a sag at the beginning of the superelevation and a hump at the point of full superelevation. The angular breaks should be rounded off with vertical curves.

Construction methods for securing the desired superelevation vary somewhat with the different types of surfaces, thus: (1) Portland cement concrete pavements, if laid in strips 10 to 12 ft. in width, can be finished satisfactorily throughout the warping process by means of forms or headers set true to grade. (2) On bases and pavements laid full width and requiring the use of mechanical finishing machines, the slight transverse warping can be accomplished by adjusting the finishing machine to the cross section and forms at short intervals. Then hand finishing or rolling may be used to smooth out irregularities. (3) On crushed stone bases and pavements and other types finished by blading, rolling, etc., and on unsurfaced roads, the

roadbed can be shaped to the exact transition elevations as defined by construction stakes, etc.

The profile of the transition runoff from zero to full superelevation may consist of a pair of vertical curves, or a

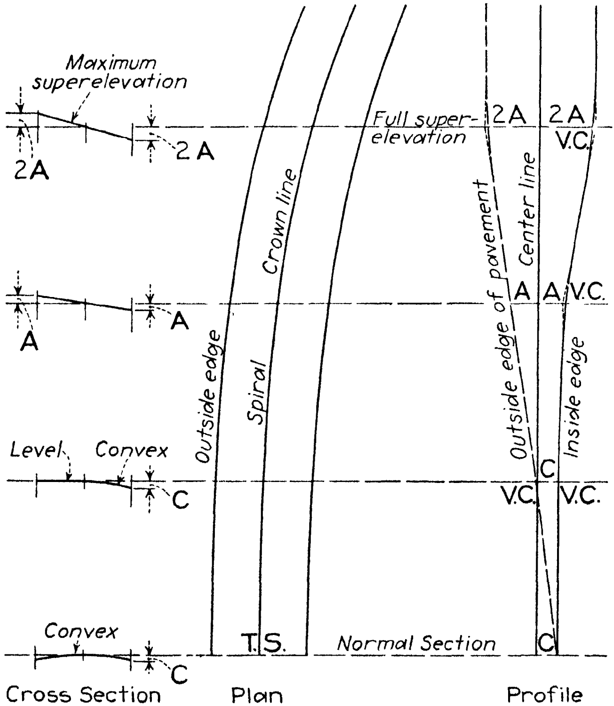


FIG. 62.—Plan showing development of transition from convex to plane section sloping inward; where C equals crown, and 2A equals maximum superelevation.

straight runoff with vertical curves at each end. The superelevation is started on the tangent, or on the spiral, at a distance of 100 or more ft. from the original P.C. (or P.T.).

Figure 62 indicates a method of development from the normal convex section to a plane surface, sloping from the

outside of the curve downward toward the inside of the same at any desired rate of slope. Beginning at (or near) the T.S. with a normal convex section, the outer edge of pavement is raised gradually (*i.e.*, uniformly all the way except perhaps for vertical curves at the angular breaks) until a one-slope section is obtained. The inner half of the pavement continues with the normal crown to a point where the outer half becomes level, thereafter flattening until the slope of the outer half equals either the average slope of the normal section or one-half the final slope, both halves being now in one sloping plane. Beyond this point, the entire

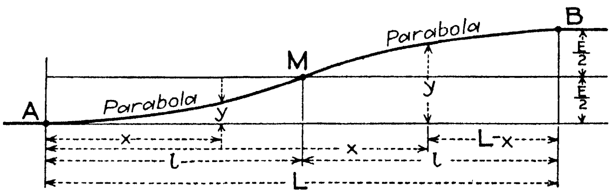


FIG. 63.

pavement is rotated about the center line, the outer edge is gradually raised, and the inner half is gradually lowered until the maximum superelevation is reached at the S.C., or at some preceding section (as indicated in Fig. 66).

Where a curve is widened, the grade for the center of *construction* will be *lower* than the staked grade by the amount of one-half of the extra width multiplied by the rate of transverse slope at the point considered. Also the grade for the inner edge will be lower than that for the normal inner edge by the amount of the extra width multiplied by the rate of slope at the point considered.

In Fig. 63, rotation takes place about the inner edge, and the superelevation transition consists of a reversed parabolic vertical curve between two parallel grades. The length ($L = 2l$) is equal to the superelevation transition distance, and the vertical distance (E) between the parallel grades equals the total rise at the point of maximum superelevation

of the outer edge above the inner edge. Half of the total rise above the regular grade occurs at the midpoint (M).

The total rise (y) for any point between A and M (Fig. 63) is given by the formula:

$$y = \left(\frac{x}{l}\right)^2\left(\frac{1}{2}E\right),$$

where x is measured from point A.

The total rise (y) for any point between M and B is given by the formula:

$$y = E - \left(\frac{L - x}{l}\right)^2\left(\frac{1}{2}E\right),$$

where x is measured from point A.

The example given below illustrates the foregoing method, which is similar to that specified by the Missouri State Highway Commission.

EXAMPLE. Given $\Delta = 40° 36'$, $D = 4°$, and a normal width of pavement = 20 ft. Letting the spiral angle (θ_s) = 15°, a spiral length of (l_s) = 562.0 ft. (Table 12) is compounded with the unoffset circular curve at Sta. 86 + 87.2. The T.S. is at Sta. 81 + 25.2.

Assuming full superelevation to occur near the ⅔-point along the spiral (at Sta. 85 + 00), a length of 300 ft. will be used for the superelevation transition. A maximum rate of superelevation of 0.056 will be assumed, giving a total maximum superelevation (E) = 20 × 0.056 = 1.12 ft.

From Sta. 81 + 25.2 (the T.S.) to Sta. 82 + 00, the normal crown continues; from Sta. 82 + 00 to Sta. 85 + 00, the superelevation increases to a maximum, which remains constant as far as Sta. 91 + 39.4; from Sta. 91 + 39.4 to Sta. 94 + 39.4, the superelevation decreases to a normal convex crown.

Using the foregoing parabolic formulas, we have:

$$y = \left(\frac{x}{l}\right)^2\left(\frac{1}{2}E\right) = \left(\frac{x}{150}\right)^2(0.56); \qquad \text{and}$$

$$y = E - \left(\frac{L - x}{l}\right)^2\left(\frac{1}{2}E\right) = 1.12 - \left(\frac{300 - x}{150}\right)^2(0.56).$$

The superelevation computations are recorded in Tables A and B below.

TABLE A.—INCREASING SUPERELEVATION

Station	x	x/l	$(x/l)^2$	Superelevation	
82 + 00	00	0.000	0.000		0.00
+ 25	25	0.167	0.028		0.02
+ 50	50	0.333	0.111		0.06
+ 75	75	0.500	0.250		0.14
83 + 00	100	0.667	0.444		0.25
+ 25	125	0.833	0.694		0.39
+ 50	150	1.000	1.000		0.56
	$L - x$	$\frac{L-x}{l}$	$\left(\frac{L-x}{l}\right)^2$		
+ 75	125	0.833	0.694	0.39	0.73
84 + 00	100	0.667	0.444	0.25	0.87
+ 25	75	0.500	0.250	0.14	0.98
+ 50	50	0.333	0.111	0.06	1.06
+ 75	25	0.167	0.028	0.02	1.10
85 + 00	00	0.000	0.000	0.00	1.12

TABLE B.—DECREASING SUPERELEVATION

Station	$L - x$	$\frac{L-x}{l}$	$\left(\frac{L-x}{l}\right)^2$	Superelevation	
91 + 39.4	0.0	0.000	0.000	0.00	1.12
+ 50.0	10.6	0.071	0.005	0.00	1.12
+ 75.0	35.6	0.237	0.056	0.03	1.09
92 + 00.0	60.6	0.404	0.163	0.09	1.03
+ 25.0	85.6	0.571	0.326	0.18	0.94
+ 50.0	110.6	0.737	0.543	0.30	0.82
+ 75.0	135.6	0.904	0.817	0.46	0.66
+ 89.4	150.0	1.000	1.000	0.56	0.56
	x	x/l	$(x/l)^2$		
93 + 00.0	139.4	0.929	0.863		0.48
+ 25.0	114.4	0.763	0.582		0.29
+ 50.0	89.4	0.596	0.355		0 20
+ 75.0	64.4	0.429	0.184		0.10
94 + 00.0	39.4	0.263	0.069		0.04
+ 25.0	14.4	0.096	0.009		0.01
+ 39.4	0.0	0.000	0.000		0.00

CHAPTER VII

WIDENING CURVED PAVEMENTS

128. It is advisable to widen pavements on curves for two reasons: (1) The rear wheels of a motor vehicle follow in a different track from the front wheels when rounding curves, thus occupying more space laterally than the width of the car. Hence to provide the same clearance between vehicles on curves as on tangents, pavements on curves must have an extra width. (2) There is a psychological demand for more clearance to divert a fast-moving car safely around the turn.

Mr. James S. Voshell of the U. S. Bureau of Public Roads has proposed the following formula; which takes into account the foregoing requirements.

$$f = 2(R - \sqrt{R^2 - L^2}) + \frac{35}{\sqrt{R}}, \qquad (67)$$

in which f = extra widening, L = length of wheel base (20 ft. being recommended), and R = radius of curve in feet.

The Bureau of Public Roads recommends 2 to 3 ft. extra widening for curves of 500 to 1,000-ft. radius, and 3 to 6 ft. for radii below 500 ft.

129. Methods of Widening

Assuming the inner edge of widened pavement to be a modified spiral, two cases will be treated here:

Case 1—*When the unwidened center line is a spiral,* and

Case 2—*When the unwidened center line is a circular curve.*

For either case, widening occurs on the inner side, and the inner edge of widened pavement is a modified spiral.

CASE 1

130. Length of Inner Edge of Widened Pavement

The inner edge of widened pavement is assumed to be a curve whose radial distance from the center-line spiral

increases gradually from $\frac{1}{2}w$ at the T.S. to $\frac{1}{2}w + f$ at the S.C. or some preceding section. The widening, therefore, varies directly as the distance l along the surveyed center line, and, according to the formula

$$\bar{R} = R' - \frac{l}{l_s}f = R - \frac{1}{2}w - \frac{l}{l_s}f,$$

where R = variable radius of surveyed center line of pavement;
R' = variable radius of unwidened inner edge of pavement;
$\bar{R}$ = variable radius of widened inner edge of pavement.

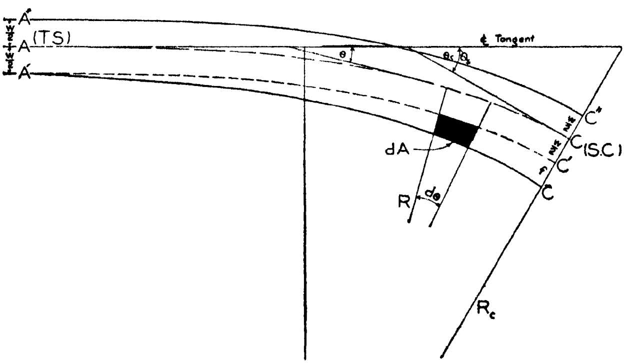


FIG. 64.

In Fig. 64, let $d\bar{l}$ = differential part of inner curve, then $d\bar{l} = \bar{R}d\theta = \left(R - \frac{1}{2}w - \frac{l}{l_s}f\right)d\theta.$

Expressing l in terms of θ (see Art. 112) and integrating between the limits of 0 and θ_s, we have

$$\bar{l} = l_s - (\tfrac{1}{2}w + \tfrac{2}{3}f)\theta_s \tag{68}$$

= length of inner edge of widened pavement from point opposite the T.S. to radial line at the S.C.

If the full extra widening (f) takes place ahead of the S.C. at some point (P_1) whose distance from the T.S. is l_1, as measured along the spiral; then the corresponding central angle (θ_1) is given by the formula:

$$\theta_1 = \left(\frac{l_1}{l_s}\right)^2 \theta_s.$$

It follows by comparison with Eq. (68) that the length of inner edge of widened pavement from a point opposite the T.S. to a radial line at point P_1 is

$$\bar{l} = l_1 - (\tfrac{1}{2}w + \tfrac{2}{3}f)\theta_1. \tag{68a}$$

Equations (68) and (68*a*) would be useful in case the length of curbing is desired.

The corresponding length of outer edge of pavement is $l'' = l_s + \frac{1}{2}w\theta_s$, or $l'' = l_1 + \frac{1}{2}w\theta_1$.

For the unwidened pavement, $l'' + l' = 2l_s$.

The angles θ_s and θ_1, in the foregoing expressions, are measured in *radians* (which equal *degrees* divided by 57.3).

EXAMPLE. Given $l_s = 400$ ft., $w = 20$ ft., $\theta_s = 15° = 0.2618$ radian, and $f = 3.0$ ft.

It is required to find $\bar{l}$ and l'', the inner and outer edges respectively of the pavement extending from the T.S. to radial lines at the S.C.

SOLUTION. $\bar{l} = 400 - (10 + 2)(0.2618) = 396.9$ ft., and $l'' = 400 + (10)(0.2618) = 402.6$ ft.

131. Extra Area Due to Widening Pavement

a. The Spiral Sections on Each Side. In Fig. 64, it is required to find a formula for the extra widened area (A_s) between the curves $A'C'$ and $A'\bar{C}$.

As stated in Art. 130, the widening is assumed to begin at the T.S. and increase gradually up to f feet at the S.C. That is, the widening varies directly as the distance from T.S. to S.C.

Using the notation of Art. 130, *differential area* (blackened area) $= dA_s =$ difference between two sectors of radii

R' and $\bar{R} = \frac{1}{2}R'^2 d\theta - \frac{1}{2}\bar{R}^2 d\theta = \frac{1}{2}(R' + \bar{R})(R' - \bar{R})d\theta =$
$\frac{1}{2}\left(2R - w - \frac{l}{l_s}f\right)\left(\frac{l}{l_s}f\right)d\theta.$

Expressing l in terms of θ (see Art. 112), integrating between the limits 0 and θ_s, and reducing, we get

$$A_s = \tfrac{1}{2}l_s f - (\tfrac{1}{3}fw + \tfrac{1}{4}f^2)\theta_s.$$

Multiplying by 2 to include both spiral sections, dividing by 9 to give results in square yards, and expressing the spiral

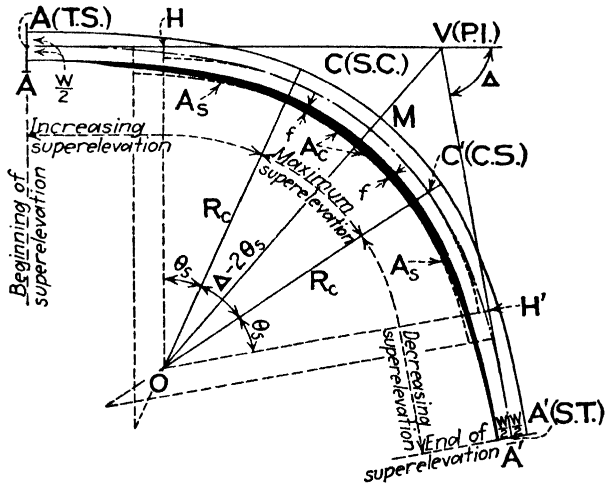


FIG. 65.

angle θ_s in degrees, the formula for area of *both spiral sections* (see Fig. 65) $= 2A_s =$

$$\frac{l_s f}{9} - \left[\frac{2}{27}fw + \frac{1}{18}f^2\right]\frac{\pi}{180}\theta_s^\circ \text{ (square yards).} \qquad (69)$$

Finally, $$2A_s = \frac{l_s f}{9} - B, \qquad (70)$$

where B is given in Table 15 for various values of θ_s and f, and for $w = 16, 18, 20$ and 22 ft.

EXAMPLE 1. Given $l_s = 400$ ft., $\theta_s = 20°$, $f = 2$ ft., and $w = 20$ ft. It is required to find the extra area due to widening.

SOLUTION. Using Eq. (70), and referring to Table 15 (part 3), the extra area due to widening $= \frac{400 \times 2}{9} - 1.11 =$ 87.77 sq. yd. If the full extra widening (f) takes place ahead of the S.C., as indicated in Fig. 66, then replacing l_s and θ_s by l_1 and θ_1, Eqs. (69) and (70) become

$$2A_s = \frac{l_1 f}{9} - \left[\frac{2}{27}fw + \frac{1}{18}f^2\right]\frac{\pi}{180}\theta_1{}^\circ \text{ (square yards)} \quad (69A)$$

and

$$2A_s = \frac{l_1 f}{9} - B. \quad (70A)$$

where the proper values of B are found in Table 15 by using θ_1 instead of θ_s as argument.

b. Central Circular Section. In Fig. 65, the area (A_c) of the circular section due to the constant extra width of f ft. equals the difference between two sectors of central angle $(\Delta - 2\theta_s)$ and radii R' and $(R' - f)$, where $R' = R - \frac{1}{2}w$ (R being the center-line radius); or, more simply, the area $=$ (length of arc through middle of strip) $\times f =$ $(R' - \frac{1}{2}f)(\Delta - 2\theta_s)(f) = (R - \frac{1}{2}w - \frac{1}{2}f)(f)(\Delta - 2\theta_s) =$ $L_c f - \left(\frac{\Delta - 2\theta_s}{2}\right)(wf + f^2)$ sq. ft., where L_c is length of nominal center-line curve from S.C. to C.S., and the angle is expressed in radians.

Area per *foot* of center-line curve $=$

$$f - \frac{\Delta - 2\theta_s}{2L_c}(wf + f^2) = f - \frac{1}{2R}(wf + f^2) =$$
$$f - \frac{D}{200}(wf + f^2) \text{ (sq. ft.)},$$

where D is expressed in radians.

Dividing by 9, and expressing D in degrees, we have

$$\text{Area per } \mathrm{℄} \text{ linear foot} = \frac{f}{9}\left[1 - \frac{D^\circ}{200}(w + f)\frac{\pi}{180}\right] \text{sq. yd.} \quad (71)$$

Table 16 is based on Eq. (71).

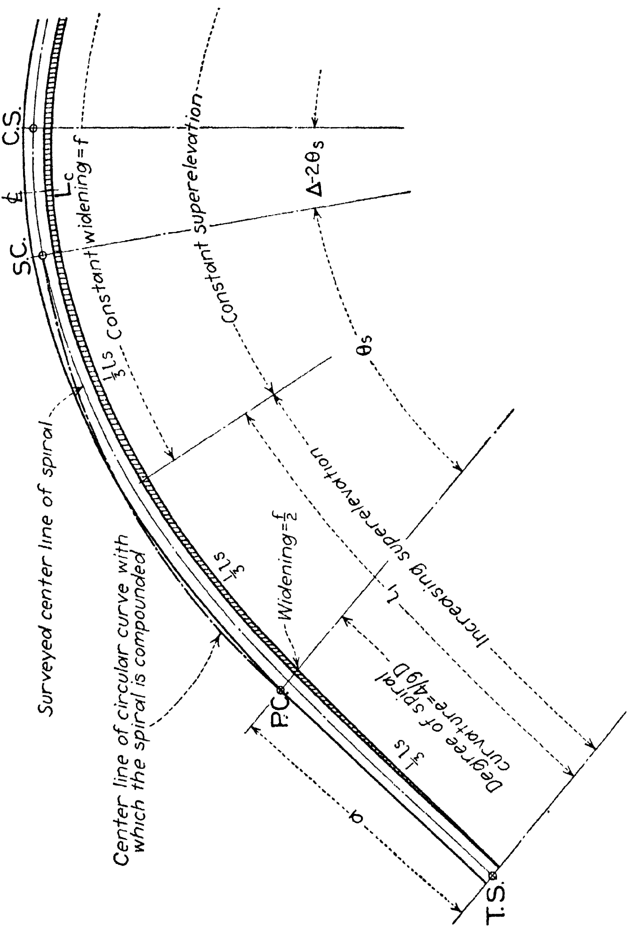


FIG. 66.—Plan of a superelevated and widened pavement whose surveyed center line consists of a spiral compounded with the unoffset circular curve.

EXAMPLE 2. Referring to the previous Example 1, suppose $\Delta = 52° \ 12'$, $\theta_s = 20°$, $D = 10°$, $f = 2$ ft., and $w = 20$ ft. What is the extra area around the central circular arc?

SOLUTION. From Table 16, the extra area per foot of center-line curve = 0.218 sq. yd.

Since $L_c = \dfrac{100(\Delta - 2\theta_s)}{D} = \dfrac{100 \times 12.2}{10} = 122.0$ ft., the total extra area = $0.218 \times 122.0 = 22.6$ sq. yd.

132. Laying Out Inner Edge of Widened Pavement

The inner edge of widened pavement is located very easily by means of radial offsets from the surveyed center-line spiral.

These offsets vary directly with the center-spiral-arc distance from the T.S. Thus, if the widening at the S.C. is f ft., that at the mid-point is $\frac{1}{2}f$ ft., etc.

EXAMPLE. Given $l_s = 400$ ft., $f = 2.5$ ft., and T.S. at Sta. 37 + 88.7. It is required to find the extra widening at 50-ft. stations.

SOLUTION. Multiplying the "distance ratios" by the value of f gives the widening. Thus:

$$f = 2.5 \text{ ft.}$$

Sta.	$\dfrac{l}{l_s}$	Extra width (ft.)
37 + 88.7 (T.S.)	0.000	0.00
38 + 00	0.028	0.07
+ 50	0.153	0.38
39 + 00	0.278	0.70
+ 50	0.403	1.01
40 + 00	0.528	1.32
+ 50	0.653	1.63
41 + 00	0.778	1.95
+ 50	0.903	2.26
41 + 88.7 (S.C.)	1.000	2.50

133. Area of Unwidened Pavement

The area of each unwidened spiral section is $l_s w$, since the outer and inner curves are equidistant radially from the center line. The area of intermediate portion $= L_c w$.

CASE 2

134. Extra Widening When Surveyed Center Line Is Unspiralized Circular Curve

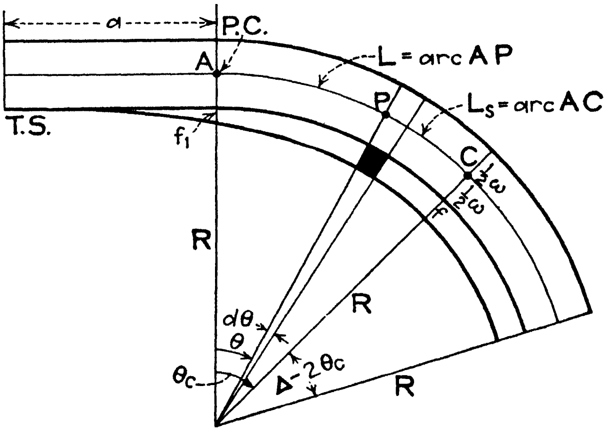


FIG. 67.

In Fig. 67, let a = distance from T.S. of inner curve to P.C.;

L_s = length of center-line circular arc AC, extending from the P.C. to the section of full widening (f), and subtending an angle θ_c;

f_1 = widening at the P.C. (if $a = L_s$, $f_1 = \frac{1}{2}f$, that is, the widening varies as the distance from the T.S.);

R = radius of circular arc AC;

$$R' = R - \frac{1}{2}w; \qquad \text{and} \qquad R = R' - \left(f_1 + (f - f_1)\frac{L}{L_s}\right),$$

in which L = any arc AP.

The widened area extending from the T.S. to the P.C. is that of a triangle or $\frac{1}{2}af_1$.

For the circular part, we have *differential area* = difference between two sectors = $\frac{1}{2}R'^2d\theta - \frac{1}{2}\bar{R}^2d\theta$.

Substituting, integrating between the limits 0 and θ_c, and simplifying, we have

Extra area of widening (opposite AC) =

$$\frac{1}{2}L_s(f_1 + f) - \left[\frac{1}{4}(f_1 + f)w + \frac{f_1f + f^2 + f_1^2}{3}\right]\theta_c. \quad (72)$$

If $f_1 = \frac{1}{2}f$, Eq. (72) becomes,

$$\tfrac{3}{4}L_sf - (\tfrac{3}{8}fw - \tfrac{7}{12}f^2)\theta_c. \quad (73)$$

Multiplying by 2 to include both sides, dividing by 9 to give results in square yards, and expressing the angle θ_c in degrees, we have

Extra area (when $f_1 = \frac{1}{2}f$) =

$$\frac{1}{18}af + \frac{1}{6}fL_s - \left(\frac{1}{12}fw - \frac{7}{54}f^2\right) \times \frac{\pi}{180}\theta_c{}^\circ \text{ sq. yd.} \quad (74)$$

To the above expression will be added the extra area around the central part of curve (see Table 16).

REVIEW PROBLEMS

Problem 1. Given $\Delta = 40° 36'$, $\theta_s = 12°$, $D_c = 6°$, l_s = 400 ft., normal width (w) of pavement = 20 ft., and extra width (f) = 2.0 ft.

It is required to find the total extra area due to widening.

Ans.: Two spiral sections = 88.2 sq. yd.
Central circular section = 60.9 sq. yd.

Total extra area = 149.1 sq. yd.

Problem 2. Given $\Delta = 50° 12'$, $\theta_s = 20°$, $D = 8°$, normal width (w) of pavement = 20 ft., extra width (f) = 2.0 ft., and the spiral is compounded with the unoffset circular curve, giving l_s = 374.4 ft. (Table 12).

It is required to find the total extra area due to widening, if full widening takes place at a point 200 ft. from the T.S. $\left(\text{that is, } l_1 = 200 \text{ ft., and } \theta_1 = \left(\frac{200}{374.4}\right)^2 \times 20° = 5° 42'\right)$.

Ans.. Two approach sections = 44.1 sq. yd.
Central circular section = 104.3 sq. yd.

Total extra area = 148.4 sq. yd.

Problem 3. Assuming the circular curve to be unspiralized, owing to limited tangent runoff distances, as may happen in mountainous country, we have $\Delta = 80° 42'$, $D = 20°$, $a =$ 60 ft., $L_s = 60$ ft., $\theta_c = 12°$, normal width (w) of pavement = 20 ft., and extra width (f) = 3.0 ft.

It is required to find the total extra area due to widening.

Ans.: End portions = 39.2 sq. yd.
Circular portion = 90.7 sq. yd.

Total extra area = 129.9 sq. yd.

Problem 4. Using the data of Problem 2, suppose the T.S. is at Sta. 145 + 62.7. If the widening starts at Sta. 146 + 00, and full widening occurs at Sta. 148 + 00, what is the approximate extra area due to widening?

Ans.: Two approach sections = 44.4 sq. yd.
Central circular section = 88.8 sq. yd.

Total extra area = 133.2 sq. yd.

Problem 5. Given $\Delta = 40° 12'$, $D_c = 10°$, $\theta_s = 10°$, $l_s =$ 200 ft., $w = 20$ ft., and $f = 2.0$ ft. The maximum rate of superelevation = 0.100, giving maximum superelevation of outer above inner edge = $22 \times 0.10 = 2.20$ ft.

The point of beginning of both widening and superelevation is at Sta. 74 + 05.2 (the T.S.), and the point of full widening and superelevation is at Sta. 76 + 05.2 (the S.C.).

Tables A and B on page 206 give widening and superelevation data both from the tangent to the curve and from the curve to the tangent.

TABLE A.—INCREASING SUPERELEVATION

Station	x	x/L	Extra width	x/l	$(x/l)^2$	Super-elevation
74 + 05.2 T.S.	0.0	0.000	0.00	0.000	0.000	0.00
+ 25	19.8	0.100	0.20	0.200	0.040	0.04
+ 50	44.8	0.224	0.45	0.448	0.201	0.22
+ 75	69.8	0.349	0.70	0.698	0.487	0.54
75 + 00	94.8	0.474	0.95	0.984	0.899	0.99
+ 05.2	100.0	0.500	1.00	1.000	1.000	1.00
	x	$\frac{x}{L}$		$\frac{L-x}{l}$	$\left(\frac{L-x}{l}\right)^2$	
+ 25	119.8	0.599	1.20	0.802	0.643	1.29
+ 50	144.8	0.724	1.45	0.552	0.305	1.66
+ 75	169.8	0.849	1.70	0.302	0.091	2.10
76 + 00	194.8	0.974	1.95	0.052	0.003	2.20
+ 05.2 S.C.	200.0	1.000	2.00	0.000	0.000	2.20

TABLE B.—DECREASING SUPERELEVATION

Station	x	$\frac{x}{L}$	Extra width	$\frac{L-x}{l}$	$\left(\frac{L-x}{l}\right)^2$	Super-elevation
78 + 07.2 C.S.	200.0	1.000	2.00	0.000	0.000	2.20
+ 25	182.2	0.911	1.82	0.178	0.032	2.17
+ 50	157.2	0.786	1.57	0.428	0.183	2.00
+ 75	132.2	0.661	1.32	0.678	0.460	1.69
79 + 00	107.2	0.536	1.07	0.928	0.861	1.25
+ 07.2	100.0	0.500	1.00	1.000	1.000	1.10
	x	x/L		x/l	$(x/l)^2$	
+ 25	82.2	0.411	0.82	0.822	0.676	0.74
+ 50	57.2	0.286	0.57	0.572	0.327	0.36
+ 75	32.2	0.161	0.32	0.322	0.104	0.11
80 + 00	07.2	0.004	0.07	0.072	0.005	0.01
+ 07.2 S.T.	00.0	0.000	0.00	0.000	0.000	0.00

CHAPTER VIII

MISCELLANEOUS

135. Duties and Responsibilities of the Highway Engineer

A large part of the highway work in the United States is done through organized State Highway Departments in cooperation with the U. S. Bureau of Public Roads.

Among the functions of the highway organizations may be mentioned the following:

a. Preparation of plans, specifications, and estimates and the letting of contracts for the improvement of roads and streets, including bridges, culverts, and all drainage structures.

b. Supervision of the construction of roads, culverts, and bridges.

c. Maintenance of highways.

d. Testing of road and bridge material and investigating new sources of materials.

The onus of carrying out the location, construction, and maintenance of highways to a successful completion rests largely upon the engineer.

The following duties and responsibilities have been laid down by the Maryland, Missouri, and other Roads Commissions as applying particularly to the resident engineer or inspector:

1. *Familiarity with the Work.* Make a careful study of the plans, specifications and special provisions applying to the work and be sure that you understand them fully. Study each note and dimension so that there will be no doubt in your mind about any detail. You should make no changes in plans without previous authorization.

If there are any doubtful points submit them to the district engineer for interpretation. If there are any discrepancies, call his attention to them at once.

Each clause of the specifications is included for a definite purpose and should not be overlooked or disregarded. Practically every provision has been tested by experience, and the present specifications are the result of much study and many revisions. But specifications of themselves are of no value. The work which they are intended to control must be intelligently supervised and executed to produce a structure such as was intended. No set of specifications can be written to cover completely all conditions which may arise. Experience, good judgment, and honesty must dictate the answer to doubtful questions.

Study the location and learn the details of the proposed structure so thoroughly as to be able to make necessary interpretations and to answer questions which are certain to arise daily.

Before construction commences, go over the project in detail with the contractor and get acquainted with his plan of procedure; equipment he will employ; his plans for handling and storing materials; and his sequence of operations, so you can arrange your work accordingly.

2. *Relations with the Contractor.* In his dealings with the contractor, the inspector should follow a friendly, business-like and ethical course. He should see that the contractor, as well as the state, is given a fair deal, and that the work is performed in strict accordance with the specifications.

The inspector should not become too familiar with the contractor or his representatives, but should maintain a natural reserve and impartial attitude, which in turn will command the contractor's respect. The inspector should be exacting at the beginning of the job; for methods resulting in unsatisfactory work are more easily corrected at the start than after they have been permitted for some time.

The inspector should not try to direct the contractor's method of doing the work; but it is the inspector's duty to see that the results meet the specifications. He should use every effort to correct any tendency on the part of the contractor to slight the work. Inspectors should report to the district engineer, promptly and fully, any attempt of the contractor not to comply with the plans or specifications or with written orders.

Orders should not be given directly to a laborer, but all matters should be taken up with the superintendent or foreman; and then the inspector should see that they are carried out.

All important orders given the contractor and his representatives should be in writing. In case of emergencies which prohibit waiting for written orders, all important verbal orders or instructions should thereafter be confirmed in writing. The inspector should retain a copy of all written orders. This enables the inspector to defend his acts in case of later contention or dispute.

3. *Relations with the Public.* Give particular attention to the enforcement of that part of the specifications concerning public convenience and safety. Before construction starts, go over all detours, making certain that they are in a safe and passable condition; that they are properly posted; and that all barricades and lights are in place, so that there will be no reasonable complaint on the part of the public.

Be courteous and agreeable in all your dealings with the public, remembering that you are an employee of the public as well as of the State Roads Commission; and that the Commission will be judged by your conduct.

Do not discuss the details of your work with the public, nor discuss with it shortcomings of the contractor or of his organization or methods.

If provided with an office, see that it is kept neat and clean.

4. *Proper Inspection Essential.* The inspector should be on the job at all times while construction operations are in progress. The first concrete batch of the day is the one most likely to be bad. Be there to see it mixed. The last one of the day is most likely to be carelessly spaded. Be there to see it poured.

Organize the work of your assistants so that each man's duties will be clear cut, and hold him responsible for properly carrying out the part of the work you have assigned to him.

Competent inspection is one of the most important elements entering into the construction of any engineering project. The most careful and accurate plans and specifications may be prepared, and the structure may be accurately laid out in the field; but if faulty materials or poor workmanship are incorporated in the construction, the purpose of all the preceding effort is wasted. Proper inspection is a man-sized job. It requires constant vigilance, diplomacy and firmness.

The work demanded by the Standard Specifications and by special provisions is, when finished, complete, uniform, of good quality, and in appearance pleasing to the eye. Whether it be a graded section, a culvert, a bridge, a concrete slab, or any

other item, watch your lines, grades, surface, quality, and the method of obtaining them.

136. Care of Field Equipment

The field equipment and particularly the transits and levels are deserving of good care not only because they are delicate and expensive, but because an accurate set of notes cannot be made with instruments in bad condition.

When transporting surveying instruments in automobiles or trucks, they must be carefully handled to avoid damage. When the party is driven to and from work, the best way to carry an instrument is on the lap of a member of the party either in or out of the box, but under no circumstances should a boxed instrument be placed in a vehicle so that it can bounce around. If transported in a case, it should be cushioned with coats, burlap, blankets, or other suitable materials; and be frequently inspected to make certain it is riding satisfactorily. In transporting level rods and range poles long distances, they should be wrapped to preserve the paint and graduations.

If it becomes necessary to ship an instrument by rail, the instrument case should be inclosed in an outer packing box and the space between (on bottom, sides and top) should be carefully surrounded by some shock-absorbing material. The top of the tripod should be carefully wrapped to protect the threads.

If an instrument head sticks to the tripod, rough handling, such as by use of pipe wrenches or hammer and chisel, should never be employed. If it can not be unscrewed by hand, it can usually be expanded enough to permit easy removal, by applying a hot towel to the base plate. Or it may be released in the following manner: First set up the instrument; then with two tripod legs in this position, raise the third leg upward until it touches the instrument head; now try to unscrew the head; if still tight, raise or lower the same leg, and at some place in this process the strain will be released and the head will come off easily. Trouble of this sort is avoided by keeping the threads always clean, and not screwing the head on the tripod too tightly.

When carrying an instrument mounted on a tripod through brush or through doors of a building or other contracted spaces, the tripod should be carried under the arm with the instrument forward, and not over the shoulder; thus avoiding the possibility of the instrument being accidentally struck. When setting up the instrument, the tripod should not be set too high, as it may easily be tipped over. An instrument should never be left unattended or out of sight, where it is likely to be knocked over by vehicles, cattle, or pedestrians.

In handling an instrument, care should be taken to grip it at such points that no strain will be put on any of the adjustments. Before picking up an instrument which is on the tripod, always loosen the spindle clamp; and, in the case of a transit, tighten the compass needle.

Always set up an instrument with the tripod head as nearly level as possible, so that the leveling screws will show about the same amount of thread. In leveling up, the leveling screws should be set firm but not tight. In tightening the clamp screws it is necessary only to screw them home lightly, as the design of the clamps makes them grip very firmly. If it becomes necessary to lubricate a thread or bearing, only the best grade of *watch oil* should be used and only a small amount of that.

A hood should be placed over the transit when it is exposed to wet weather or dust storms. Wet tapes should always be wiped dry before being put away or reeled up. Steel tapes should be rubbed with an oily cloth after cleaning to prevent rusting. All instruments should be kept clean and in good adjustment at all times. When in constant use, the adjustments should be checked once a week, or oftener if necessary.

137. Adjustments of the Instruments

Every adjustment consists of (1) the *test* and (2) the *correction.*

Repeat each as often as seems necessary.

In using two opposing screws, one should always be slightly loosened before the other is tightened.

The adjustments should be made in the order given below. Before using an instrument, however, and especially before attempting to adjust it, the observer should focus the eyepiece upon the wires.

The Transit

The level tubes should be parallel to the vernier plate.

Test. After setting up, bring the two levels each parallel to a pair of leveling screws. Bring both bubbles to the center of the level tubes by means of the leveling screws. Turn the instrument 180° in azimuth. The bubbles should remain centered; if they do not—

Correction. Bring them halfway back to the center by means of the leveling screws, and the remaining half with an adjustment pin working in the capstan nuts of the bubble tube, by loosening one nut and tightening the opposite one. Adjust the bubbles one at a time. Tighten all screws.

The vertical wire should be truly vertical.

Test. The vertical wire should cover a point upon which the transit has been set as the telescope is moved slightly up and down. If it does not—

Correction. Loosen the four capstan-headed screws and rotate the reticle until the wire stands the test, then tighten all four adjusting screws.

The line of sight should be perpendicular to the horizontal axis.

Test. Set up and level the instrument. Sight on point *A* a few hundred feet away, plunge the telescope and set another point *B* on the line of sight and about the same distance away on the opposite side of the transit. Turn the instrument in azimuth and again sight at *A* (with telescope inverted). Plunge the telescope; point *B* should be in the line of sight. If it is not—

Correction. Set a point *C* in line of sight opposite *B*. Set point *D* one-fourth of the way from *C* to *B*. Move the vertical cross wire over until it intersects point *D*, by means of the screws on side of telescope.

The horizontal axis of the telescope should be perpendicular to the vertical axis of the instrument.

Test. Set up transit and sight the vertical cross wire on a well-defined high point A. Depress the telescope and set a point B on the ground. Reverse telescope, turn in azimuth and again sight on A. Depress telescope and the line of sight should cover B. If it does not—

Correction. Set point C in line of sight opposite B. Set point D half way between C and B. Adjust the movable end of horizontal axis until line of sight intersects point D.

If there is a level under the telescope it should be parallel to the line of sight.

Test. Use the "peg method." Set up halfway between two driven stakes or other solid points, and with bubble in center take a rod reading on each point. The difference of rod readings gives the true difference in elevation. Set up by the side of one of the points just far enough away to allow the telescope to swing clear of a vertical rod held on the near point. Take a rod reading on the near point by sighting through the large end of the telescope. Then, knowing the difference in elevation, compute the correct reading for the far point. With the bubble centered, take a rod reading on the far point. This should agree with the computed reading. If it does not—

Correction. Set wire on the correct reading and bring bubble to center by the adjusting screws at the end of the tube.

The Y-Level

The line of sight should coincide with the axis of the rings.

Test. Remove pins and pull clips back so that the telescope is free to turn in the *Y's*. Sight the intersection of the cross hairs at some well-defined point several hundred feet away, using the leveling screws for vertical motion and the clamp and tangent screw for horizontal motion. When the telescope is rotated halfway around in its *Y's*, the intersection of cross hairs should still be on the point. If not—

Correction. Bring the horizontal cross hair halfway back to its first position by means of the upper and lower adjusting screws of the cross hair ring.

The line of sight should be parallel to the level tube.

Test. Level over both pairs of screws, then clamp over one of them. With clips open, center the bubble accurately. Lift telescope out of its Y's, turn it end for end and carefully replace it. The bubble should come exactly back to the center of tube. If it does not—

Correction. Correct half the deviation by the two leveling screws and the remainder by turning the nuts at one end of tube.

The level tube and telescope are now in parallel planes. To make sure that they are parallel to each other—

Test. Bring bubble to center of tube and rotate telescope above 20° in its *Y's*, and the bubble should remain centered. If it does not—

Correction. Bring it to the center by adjusting lateral motion screws at the side of level tube.

The axis of bubble tube should be perpendicular to the vertical axis of the instrument.

Test. Level the instrument as closely as possible over both pairs of leveling screws, then exactly over one pair. Turn the instrument 180° in azimuth. The bubble should remain centered. If it does not—

Correction. Bring it half way back by the two level screws, and the remainder by the large nuts at end of level bar.

The Dumpy Level

The axis of the bubble tube should be perpendicular to the vertical axis of instrument.

The *test* and the *correction* are the same as for the plate level adjustment of the transit.

The line of sight should be parallel to the level tube:

The *test* for this adjustment is the same as the "peg method" for level under the telescope of transit.

The *correction,* however, is made by adjusting the cross-hair ring.

138. Distances on Slopes

All distances on a survey are to be measured horizontally. In some cases it is more convenient to make measurements

on the slope and compute the correct horizontal distances. The following formula is sufficiently accurate for such computations.

Let r = distance measured on slope;
y = difference of elevation;
x = true horizontal distance.

When y is small in comparison with r or x,

$$r - x = \frac{y^2}{2x} \text{ (approx.)}.$$

Proof. From Fig. 68, $r^2 - x^2 = y^2$.

Factoring, $(r + x)(r - x) = y^2$;

and solving, $r - x = \dfrac{y^2}{r + x}$.

Hence $$r - x = \frac{y^2}{2x} \text{ (approx.)}. \qquad (75)$$

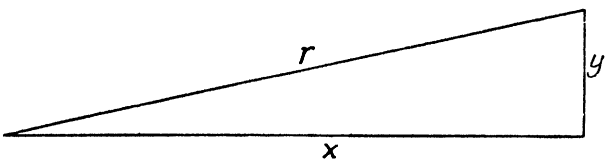


FIG. 68.

Thus, for a 4 per cent grade; $r - x = (4)^2/200 = 0.08$, which means that a distance of 100.08 ft. on the slope is equivalent to 100.00 ft. horizontally.

The table shown on page 216 is useful.

139. Stadia Measurements

The stadia method of surveying is particularly advantageous in obtaining data for topographic maps of an area embracing sites for bridges, railroad crossing eliminations, etc.

The telescope of the transit is usually furnished with two horizontal stadia hairs in addition to the regular cross hairs.

Taking *stadia measurements* consists of observing through the telescope the apparent interval between the two stadia hairs on a vertical rod held at a distant point, the rod being

DISTANCES ON SLOPES

Per cent grade	Vertical angle	Slope distance per 100 ft. horizontal	Horizontal distance per 100 ft. slope
1.0	0° 35′	100.00	99.99
1.5	0° 52′	100.01	99.99
2.0	1° 09′	100.02	99.98
2.5	1° 26′	100.03	99.97
3.0	1° 43′	100.04	99.96
3.5	2° 00′	100.06	99.94
4.0	2° 18′	100.08	99.92
4.5	2° 35′	100.10	99.90
5.0	2° 52′	100.13	99.87
5.5	3° 09′	100.15	99.85
6.0	3° 26′	100.18	99.82
6.5	3° 43′	100.21	99.79
7.0	4° 00′	100.24	99.76
7.5	4° 17′	100.28	99.72
8.0	4° 35′	100.32	99.68
8.5	4° 52′	100.36	99.64
9.0	5° 09′	100.41	99.60
9.5	5° 26′	100.45	99.55
10.0	5° 43′	100.50	99.50
10.5	6° 00′	100.55	99.45
11.0	6° 17′	100.60	99.40
11.5	6° 34′	100.66	99.34
12.0	6° 51′	100.72	99.29

graduated in feet and tenths. The interval thus determined, called the *stadia reading*, is directly proportional to the distance from the instrument. Thus, the distance from the instrument to any given point is determined by observing the stadia interval on the rod held at the given point.

The horizontal distance (D) from the center of the instrument to the rod is

$$D = Ks + (f + c), \qquad (76)$$

in which f is a constant for a given instrument; c is distance from vertical axis of instrument to the objective (slightly variable, but for practical purposes may be considered constant); $K = f/i$, where i is the distance between stadia hairs; and s is the interval on the rod apparently intercepted by the stadia hairs.

The value of $(f + c)$ is determined by the manufacturer and placed on the inside of the instrument box. Usually $(f + c)$ is taken to be 1 ft.

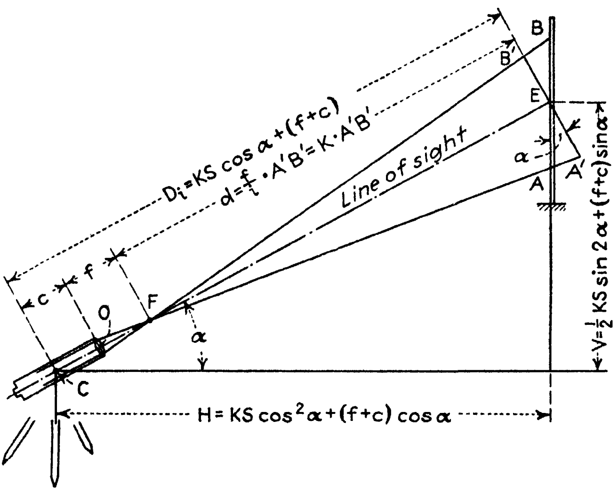


FIG. 69.

The nominal value of the *stadia interval factor* K $(= f/i)$ is usually 100. This interval factor may be determined in the field, thus: Set up the transit where a horizontal sight can be obtained, and with a tape, lay off from a point $(f + c)$ ft. in front of the instrument, distances of 100 ft., 200 ft., etc., then a rod is held at these points in turn. For each point, divide the distance from the principal focus $(f + c)$ by the stadia interval read on the rod. The results should consistently give the factor K.

Inclined Sights. In Fig. 69, AB represents the stadia interval (s) on the vertical rod. The *horizontal component* (H) of the inclined distance (D_i) is

$$H = Ks \cos^2 \alpha + (f + c) \cos \alpha, \tag{77}$$

which is the general expression for finding the horizontal distance from center of instrument to rod, when the line of sight is inclined.

The *vertical component* (V) of the inclined distance (D_i) is

$$V = Ks \cos \alpha \sin \alpha + (f + c) \sin \alpha. \tag{78}$$

Table 24 gives for each 02′ of vertical angle up to 30° the horizontal distances and differences in elevation for $Ks = 100$ ft.

For any other value of Ks, the tabular values are multiplied by the value in hundreds of feet. Table 24 also gives the horizontal distances and differences in elevation for several values of $(f + c)$—indicated as c in table.

A rapid and convenient means of calculating horizontal distances and differences in elevation is to use a stadia slide rule of which there are several patterns.

Permissible Approximations. Owing to unequal refraction and accidental inclination of the rod, observed stadia intervals are, in general, slightly too large. To offset the systematic errors from these sources the $(f + c)$ constant is generally neglected except on surveys of more than ordinary precision.

Hence, ordinarily for consistent precision Eqs. (77) and (78) may properly be reduced to the forms

$$H = Ks \cos^2 \alpha; \tag{79}$$

and

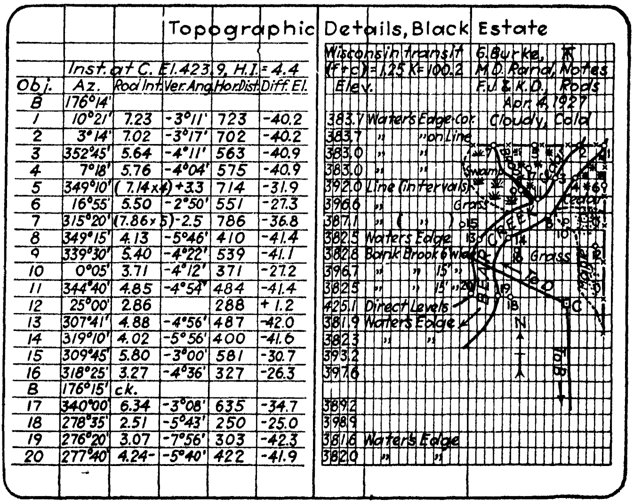
$$V = \tfrac{1}{2} Ks \sin 2\alpha. \tag{80}$$

Differences in elevation obtained by Eq. (80) will be correct to the nearest 0.1 ft. for angles less than 25°.

Use of the Stadia. In all transit surveys where a high precision is not required, the stadia method is very rapid and convenient. This applies both to plane and topographic surveying. When the plane table is used, stadia

observations are made with the telescope alidade in the same manner as with the transit, but calculations of horizontal distance and differences in elevation are made in the field and are immediately plotted instead of being recorded in the form of notes.

For certain reconnaissance or preliminary surveys, rough surveys for the location of boundaries, and detailed surveys for maps, where only the horizontal position of objects and



Topographic Details, Black Estate

Inst. at C. El. 423.9, H.I. = 4.4

Wisconsin transit (f+c) = 1.25 K = 100.2

G. Burke, ⊼ M.D. Rand, Notes F.J. & K.D. Rods Apr. 4, 1927 Cloudy, Cold

Obj.	Az.	Rod Int.	Ver. Ang.	Hor. Dist.	Diff. El.	Elev.	
B	176°14'						
1	10°21'	7.23	-3°11'	723	-40.2	383.7	Water's Edge Cor.
2	3°14'	7.02	-3°17'	702	-40.2	383.7	" " on Line
3	352°45'	5.64	-4°11'	563	-40.9	383.0	" "
4	7°18'	5.76	-4°04'	575	-40.9	383.0	" "
5	349°10'	(7.14×4)	+3.3	714	-31.9	392.0	Line (Intervals)
6	16°55'	5.50	-2°50'	551	-27.3	396.6	" Grass
7	315°20'	(7.86×5)	-2.5	786	-36.8	387.1	" (")
8	349°15'	4.13	-5°46'	410	-41.4	382.5	Water's Edge
9	339°30'	5.40	-4°22'	539	-41.1	382.8	Bank Brook 6' wide
10	0°05'	3.71	-4°12'	371	-27.2	396.7	" " 15' "
11	344°40'	4.85	-4°54'	484	-41.4	382.5	" " 15' "
12	25°00'	2.86		288	+1.2	425.1	Direct Levels
13	307°41'	4.88	-4°56'	487	-42.0	381.9	Water's Edge
14	319°10'	4.02	-5°56'	400	-41.6	382.3	" "
15	309°45'	5.80	-3°00'	581	-30.7	393.2	
16	318°25'	3.27	-4°36'	327	-26.3	397.6	
B	176°15'	ck.					
17	340°00'	6.34	-3°08'	635	-34.7	389.2	
18	278°35'	2.51	-5°43'	250	-25.0	398.9	
19	276°20'	3.07	-7°56'	303	-42.3	381.6	Water's Edge
20	277°40'	4.24-	-5°40'	422	-41.9	382.0	" "

Fig. 70.

lines is desired, the transit-stadia method is sufficiently accurate and more rapid than corresponding surveys with the transit and tape.

In topographic surveying, the field procedure of locating points consists of observing directions, usually by azimuths, distances by stadia, and differences in elevation by the vertical angle. This method may be employed merely for the collection of details, the horizontal and vertical control being established by other means, or it may be utilized for establishing control as well as details.

Figure 70 is a page of notes showing observations taken from station C of a traverse, the elevation of the station having been previously determined as 423.9. The H.I. is 4.4 ft. The transit is oriented by sighting to B, the azimuth of the line CB being set off on the horizontal circle prior to taking the sight.

In measuring vertical angles it is customary, when practicable, to sight at a rod reading equal to the height of instrument above the station over which the transit is set.

Determination of the True Meridian

140. Direct Solar Observation

The determination of the true azimuth of a line by a direct solar observation requires two field measurements; namely, the true altitude of the sun and the horizontal angle between the sun and the line, both being determined simultaneously at a known instant of time.

By spherical trigonometry the following equation may be derived for the PZS triangle shown in Fig. 71.

$$\cos Z = \frac{\sin \delta}{\cos h \cos \phi} - \tan h \tan \phi, \tag{81}$$

in which Z = true azimuth of the sun measured from the north either east or west as the case may be;

ϕ = latitude of the observer, which may be easily obtained to the nearest 30″ from a reliable map of the given territory;

δ = declination of the sun at the instant of observation; obtained from a table (similar to Table A) given in the American Ephemeris and Nautical Almanac published each year by the Government Printing Office;

h = true altitude of the sun at the instant of observation.

Having obtained the true azimuth of the sun by Eq. (81) it may be combined with the measured horizontal angle

between the given line and the sun so as to obtain the true azimuth of the line.

The observed altitude of the sun as obtained in the field is not the true altitude. However, the true altitude may be obtained by correcting the observed altitude for two effects, one called parallax and the other called refraction. The parallax correction is that part of the total correction that reduces the observed altitude determined at the earth's surface to that value which would be obtained referred

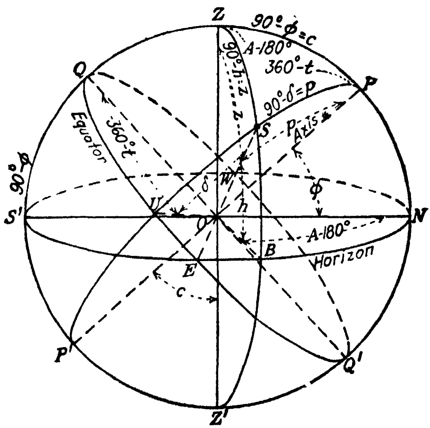


FIG. 71.

to the earth's center. The parallax correction is always *added* to the observed altitude to obtain the true altitude. The refraction correction is due to the bending of light waves in passing through a medium of varying density. Light waves from outside the earth are always bent downward, thus making a body appear higher than it actually is. Thus the refraction correction is always *subtracted* from the observed altitude. For the sun, the parallax and refraction corrections have been combined and recorded in Table B. It should be noted that the resultant correction given in Table B should be subtracted from the observed altitude.

To avoid uncertainty as to the value of the refraction correction at low altitudes and also to eliminate the low precision of Eq. (81) for high altitudes the best results are obtained by making a solar observation between the hours of 8 to 10 A.M. and 2 to 4 P.M.

Field Procedure:

1. Set up the transit at a convenient station, *P*, for viewing the sun and establish a forward point, *Q*, in a line *PQ* whose azimuth is to be determined.

2. With vernier *A* set at zero on the horizontal circle, and the reading of vernier *B* noted, sight along the given line *PQ* with the telescope in the direct or normal position. The lower motion is then clamped. Loosen the upper motion and take a sight at the sun so as to bring the vertical and horizontal cross hairs tangent to the edges (or limbs) of the sun's disc. If the observation is made in the morning the sun is brought to tangency in the upper left-hand quadrant of the field of view; or in the upper right-hand quadrant if the observation is in the afternoon. At the instant of tangency the time is observed. As quickly as convenient take the vertical and horizontal circle readings for all the verniers. Then loosen the upper motion and plunge the telescope. A second sight is then taken on the sun, this time with the sun in the diagonally opposite quadrant from that used for the first sight. Note again the time of tangency and read the vertical and horizontal circles as before.

3. Loosen the upper motion and complete the field work by sighting along the line *PQ* and reading the verniers of the horizontal circle.

4. The mean of the two observed altitudes is taken to be the altitude of the sun's center at the mean of the two observed times. Similarly the mean of the two horizontal angles is taken as the horizontal angle to the sun's center at the mean of the observed times. Any error of the watch should be noted.

If the transit is not equipped with a colored eyepiece the position of the sun may be determined by focusing the

sun's image on a card held a short distance in the rear of the eyepiece.

For a morning observation the horizontal cross hair is first sighted a short distance above the sun's lower edge. Then at the same time that the horizontal cross hair is approaching tangency due to the sun's motion the vertical cross hair is kept continuously on the sun's western edge or limb by means of the upper motion tangent screw. For the second observation, in the lower right hand quadrant, the vertical cross hair is set a short distance to the right of the sun's eastern limb. The vertical cross hair will then approach tangency due to the sun's motion while the horizontal cross hair is kept tangent by the vertical

Table A (*American Ephemeris*, 1936, p. 10)
Sun 1936
For 0^h Greenwich Civil Time

Date	Day of week	Apparent declination	Diff. (sec.)
Aug. 1	Sa.	+18° 07′ 52.5″	
			907.1
2	Su.	17 52 45.4	
			924.6
3	Mo.	17 37 20 8	
			941.7
4	Tu.	17 21 39.1	
			958.7
5	We.	17 05 40.4	
			975.3
6	Th.	16 49 25.1	
			991.7
7	Fr.	16 32 53.4	
			1007.9
8	Sa.	16 16 05.5	
			1023.6
9	Su.	15 59 01.9	
			1039.2
10	Mo.	15 41 42.7	

TABLE B.—CORRECTION FOR PARALLAX AND REFRACTION TO BE SUBTRACTED FROM THE OBSERVED ALTITUDE OF THE SUN

For Polaris Observations add 0.2′ for altitudes to 15°, inclusive; and 0.1′ for altitudes to 70°, inclusive.

To be used when making latitude observations and determinations of time.

Apparent altitude (deg.)	Temperature, Centigrade						
	−10°	0°	+10°	+20°	+30°	+40°	+50°
	′	′	′	′	′	′	′
5	10.5	10.1	9.7	9.4	9.1	8.8	8.5
6	9.0	8.6	8.3	8.0	7.8	7.5	7.[illegible]
7	7.8	7.5	7.3	7.0	6.8	6.6	6.4
8	6.9	6.7	6.4	6.2	6.0	5.8	5.6
9	6.2	6.0	5.7	5.5	5.4	5.2	5.0
10	5.6	5.4	5.2	5.0	4.8	4.7	4.5
11	5.1	4.9	4.7	4.5	4.4	4.2	4.1
12	4.7	4.5	4.3	4.2	4.0	3.9	3.8
13	4.3	4.1	4.0	3.8	3.7	3.6	3.5
14	4.0	3.8	3.7	3.6	3.4	3.3	3.2
15	3.7	3.6	3.4	3.3	3.2	3.1	3.0
16	3.5	3.3	3.2	3.1	3.0	2.9	2.8
17	3.2	3.1	3.0	2.9	2.8	2.7	2.6
18	3.0	2.9	2.8	2.7	2.6	2.5	2.5
19	2.9	2.8	2.7	2.6	2.5	2.4	2.3
20	2.7	2.6	2.5	2.4	2.3	2.3	2.2
21	2.6	2.5	2.4	2.3	2.2	2.1	2.1
22	2.4	2.3	2.3	2.2	2.1	2.0	2.0
23	2.3	2.2	2.1	2.1	2.0	1.9	1.9
24	2.2	2.1	2.0	2.0	1.9	1.8	1.8
25	2.1	2.0	1.9	1.9	1.8	1.7	1.7
26	2.0	1.9	1.9	1.8	1.7	1.7	1.6
27	1.9	1.8	1.8	1.7	1.6	1.6	1.5
28	1.8	1.8	1.7	1.6	1.6	1.5	1.5
29	1.8	1.7	1.6	1.6	1.5	1.5	1.4
30	1.7	1.6	1.6	1.5	1.4	1.4	1.4
32	1.6	1.5	1.4	1.4	1.3	1.3	1.3
34	1.4	1.4	1.3	1.3	1.2	1.2	1.2
36	1.3	1.3	1.2	1.2	1.1	1.1	1.1
38	1.2	1.2	1.1	1.1	1.0	1.0	1.0
40	1.1	1.1	1.0	1.0	1.0	0.9	0.9
42	1.1	1.0	1.0	0.9	0.9	0.9	0.8
44	1.0	0.9	0.9	0.9	0.8	0.8	0.8
46	0.9	0.9	0.8	0.8	0.8	0.8	0.7
48	0.9	0.8	0.8	0.8	0.7	0.7	0.7
50	0.8	0.8	0.7	0.7	0.7	0.6	0.6
55	0.7	0.6	0.6	0.6	0.6	0.5	0.5
60	0.5	0.5	0.5	0.5	0.5	0.4	0.4
65	0.4	0.4	0.4	0.4	0.4	0.3	0.3
70	0.3	0.3	0.3	0.3	0.3	0.3	0.3
75	0.2	0.2	0.2	0.2	0.2	0.2	0.2
80	0.2	0.2	0.2	0.2	0.1	0.1	0.1
85	0.1	0.1	0.1	0.1	0.1	0.1	0.1
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0

circle tangent screw. This procedure is such that the final setting for either observation is accomplished by the movement of only one tangent screw.

EXAMPLE. *Azimuth by Direct Solar Observation.*

Chapel Hill, N. C., Aug. 6, 1936
Lat. of observer = 35° 54.6′
Temperature = 30 deg. C.

Direction of turn	Object sighted	Watch time (E.S.T.)	Vertical circle	Horizontal circle	
				Ver. A	Ver. B
	Sta. *Q*	....	....	0° 00′	180° 00′
Counter-	☉	$9^h\ 5^m\ 40^s$	43° 27.0′	144° 35′	324° 35′
clockwise	☉	$9^h\ 8^m\ 20^s$	43° 32.0′	323° 33′	143° 33′
	Sta. *Q*	....	....	179° 59′	359° 59′

Mean $9^h\ 7^m\ 00^s$ 43° 29.5′ 144° 04.5′

Watch time of observation = $9^h\ 7^m\ 0^s$ (E.S.T.)
Watch correction (slow) = $3^m\ 0^s$

Time of observation (75th meridian) = $9^h\ 10^m\ 0^s$ (E.S.T.)
Correction for 75° Long. = $5^h\ 0^m\ 0^s$
Time of observation = $14^h\ 10^m\ 0^s$ (G.C.T.) = 14.167 hr.
From Table A,

δ at 0^h G.C.T. = +16° 49′ 25.1″

$$\text{Change} = \frac{14.167}{24} \times 991.7'' = -\ 0°\ 09'\ 45.0''$$

δ at instant of observation = +16° 39′ 40.1″

Observed h = 43° 29.5′
Corr. for parallax and refraction (see Table B) = 0° 01.0′

True altitude = 43° 28.5′

Hence, h = 43° 28.5′ $\sin \delta = 0.28671$
ϕ = 35° 54.6′ $\cos h = 0.72567$
δ = 16° 39′ 40″ $\cos \phi = 0.80994$
$\tan h = 0.94819$
$\tan \phi = 0.72414$

$$\cos Z = \frac{\sin \delta}{\cos h \cos \phi} - \tan h \tan \phi \tag{75}$$
$$= \frac{0.28671}{0.72567 \times 0.80994} - 0.94819 \times 0.72414$$
$$= -0.19881.$$

Then $Z = (180° - 78° \ 32.0')$ = Azimuth of sun east of north.
Hence, azimuth of $PQ = 144° \ 04.5' - 78° \ 32.0'$
$= 65° \ 32.5'$ (from the south)

141. Observations on Polaris at Elongation

As a result of the rotation of the earth about its axis, Polaris, like other stars, appears to move in a circle about

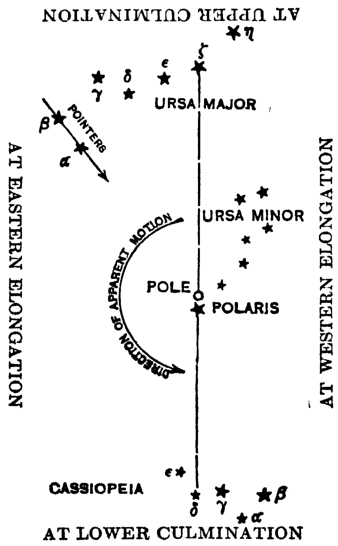


FIG. 72.

the pole of the heavens. It is said to be at culmination when it is in the vertical plane defined by the observer and the pole (upper culmination when it is above the pole) and at elongation when it reaches its extreme easterly and westerly positions with respect to the pole. At culmination its apparent motion is nearly horizontal, from east to west

at upper culmination and from west to east at lower culmination. At elongation its apparent motion is nearly vertical, upward at eastern elongation and downward at western elongation.

The azimuth of Polaris is the angle at the observing station between the vertical plane through the pole and the one through the star. For the short time before and after elongation there is practically no change in the azimuth of Polaris, and that time is usually selected for observing it for the purpose of determining the true meridian.

Observations of Polaris at Elongation

With a surveyor's transit the true meridian may be determined by observing Polaris at elongation as follows:

For observing, select a station which affords a good view of the northern sky and with the ground clear for at least 100 yards to the north.

If the station is to be used for determining the magnetic declination, care should be taken to have it well removed from electric car lines, buildings, and other possible sources of disturbance. If a meridian line is to be established for future use, the ends should be placed so that they may be protected from disturbance.

About 30 minutes before the time of elongation of Polaris, derived from Table C, set up the transit with its vertical axis exactly over the station mark and carefully level the instrument. It is essential that the transverse axis of the telescope be horizontal. This should be tested in the daytime by pointing on the vertical edge of a house and noting whether the vertical cross wire continues to coincide with the edge of the house as the telescope is turned in altitude.

Illuminate the cross wires by means of a pocket flashlight directed obliquely into the object end of the telescope by an assistant.

Point the telescope at the star and clamp the horizontal circle. Keep the star covered by the vertical cross wire by means of the tangent screw of the vernier plate, until a point is reached where it appears to move up or down along the

wire without moving away from it, thus indicating that elongation has been reached.

Depress the telescope to the horizontal position; about 100 yards north of the instrument drive a stake and mark a point on its top exactly coincident with the vertical wire of the telescope. This will require a second assistant and light. Turn the vernier plate 180° and again set the vertical wire on the star, clamp the horizontal circle, depress the telescope, and mark another point on the stake. The point midway between the two marks, with the point under the instrument, will define on the ground the vertical plane through Polaris at its eastern or western elongation, as the case may be.

Near elongation the azimuth of the star changes very slowly, not more than 0.1′ in the 10 minutes before or after elongation in the United States, so that there is plenty of time to make the second pointing after reversal, if there is no unnecessary delay.

By daylight lay off the proper angle taken from Table D to the east for western elongation and to the west for eastern elongation, and place a suitable marker to mark the north end of the meridian line of which the station marker will be the south end. The angle should be measured both before and after reversal, as in the case of the star.

For any year after 1935, *the values given by Tables C and D may be found* in *The American Ephemeris* or (in a condensed form) in the *K. and E. Solar Ephemeris.*

TABLE C.—POLARIS FOR THE MERIDIAN OF GREENWICH LATITUDE 40°N.

Civil Date and Civil Time

1935		East elongation	Upper culmination	West elongation	Lower culmination
		h m	h m	h m	h m
Jan.	1	1 02.4 P.M.	6 57.9 P.M.	0 57.3 A.M.	6 59.9 A.M.
	15	0 07.1 P.M.	6 02.6 P.M.	11 58.1 P.M.	6 04.5 A.M.
Feb.	1	11 00 0 A.M.	4 55 5 P.M.	10 51.0 P.M.	4 57.4 A.M.
	15	10 04 7 A.M.	4 00 2 P.M.	9 55.7 P.M.	4 02.1 A.M.
Mar.	1	9 09.5 A.M.	3 04 9 P.M.	9 00.4 P.M.	3 06.8 A.M.
	15	8 14 2 A.M.	2 09.7 P.M.	8 05 2 P.M.	2 11.6 A.M.
April	1	7 07 3 A.M.	1 02 8 P.M.	6 58.3 P.M.	1 04 7 A.M.
	15	6 12 2 A.M.	0 07.7 P.M.	6 03 2 P.M.	0 09.6 A.M.
May	1	5 09.4 A.M.	11 04.9 A.M.	5 00.4 P.M.	11 02.9 P.M.
	15	4 14.4 A.M.	10 09 9 A.M.	4 05.4 P.M.	10 07 9 P.M.
June	1	3 07 9 A.M.	9 03 4 A.M.	2 58 9 P.M.	9 01.4 P.M.
	15	2 13.1 A.M.	8 08 6 A.M.	2 04.1 P.M.	8 06.6 P.M.
July	1	1 10 4 A.M.	7 05 9 A.M.	1 01.4 P.M.	7 03.9 P.M.
	15	0 15 7 A.M.	6 11.2 A.M.	0 06.7 P.M.	6 09.2 P.M.
Aug.	1	11 05 3 P.M.	5 04.7 A.M.	11 00 2 A.M.	5.02.7 P.M.
	15	10 10 5 P.M.	4 09.9 A.M.	10 05 3 A.M.	4 07.9 P.M.
Sept.	1	9 04 0 P.M.	3 03.4 A.M.	8 58 9 A.M.	3 01.4 P.M.
	15	8 09.1 P.M.	2 08.5 A.M.	8 04.0 A.M.	2 06 5 P.M.
Oct.	1	7 06.4 P.M.	1 05.8 A.M.	7 01.3 A.M.	1 03.8 P.M.
	15	6 11 5 P.M.	0 10 9 A.M.	6 06.4 A.M.	0 08.9 P.M.
Nov.	1	5 04 6 P.M.	11 00 1 P.M.	4 59.5 A.M.	11 02.0 A.M.
	15	4 09 5 P.M.	10 05.0 P.M.	4 04 4 A.M.	10 06.9 A.M.
Dec.	1	3 06.5 P.M.	9 02.0 P.M.	3 01.4 A.M.	9 03.9 A.M.
	15	2 11.3 P.M.	8 06.8 P.M.	2 06.2 A.M.	8 08.7 A.M.

Decrease per day, 3.92 m.

In order to make ordinary latitude observations it is sufficient to know the time of culmination of Polaris with an accuracy of five minutes.

CORRECTIONS TO TIMES OF ELONGATION FOR DIFFERENT LATITUDES

Latitude	10°	15°	20°	25°	30°	35°	40°	45°	50°
	m	m	m	m	m	m	m	m	m
West Elongation	+2.8	+2.4	+2.0	+1.6	+1.1	+0.6	0.0	−0.7	−1.4
East Elongation	−2 8	−2.4	−2.0	−1.6	−1.1	−0.6	0.0	+0.7	+1.4

To refer to any other longitude, *add* 0.16 m. for each 15° east of the meridian of Greenwich and *subtract* 0.16 m. for each 15° west of the meridian of Greenwich.

Table D.—Azimuth of Polaris at Elongation, Year 1935

Decl.	88° 57.′17	88° 57.′50	88° 57.′83	Decl.	88° 57.′17	88° 57.′50	88° 57.′83
Lat.	Azimuth at elongation			Lat.	Azimuth at elongation		
°	° ′	° ′	° ′	°	° ′	° ′	° ′
5	1 03.1	1 02.7	1 02 4	38	1 19.7	1 19.3	1 18.9
6	1 03.2	1 02.8	1 02.5	39	1 20.9	1 20.4	1 [illegible].0
7	1 03.3	1 03.0	1 02.6	40	1 22.0	1 21.6	1 [illegible].2
8	1 03.5	1 03.1	1 02.8	41	1 23.3	1 22.8	1 22.4
9	1 03.6	1 03.3	1 02.9	42	1 24.6	1 24.1	1 23.7
10	1 03.8	1 03.5	1 03.1	43	1 25.9	1 25.5	1 25.0
11	1 04.0	1 03.7	1 03.3	44	1 27.4	1 26.9	1 26.4
12	1 04.2	1 03.9	1 03.6	45	1 28.9	1 28 4	1 27.9
13	1 04.5	1 04.1	1 03.8	46	1 30.5	1 30.0	1 29.
14	1 04.8	1 04.4	1 04.1	47	1 32.1	1 31 6	[illegible] 31
15	1 05.1	1 04.7	1 04.4	48	1 33.9	1 33 4	1 32.9
16	1 05.4	1 05.0	1 04.7	49	1 35.8	1 35 3	1 34.8
17	1 05.7	1 05.4	1 05.0	50	1 37.8	1 37.2	1 36.7
18	1 06.1	1 05.7	1 05.4	51	1 39.9	1 39.3	1 38 8
19	1 06.5	1 06.1	1 05.7	52	1 42.1	1 41.5	1 41 0
20	1 06.9	1 06.5	1 06.2	53	1 44.4	1 43.9	1 43.3
21	1 07.3	1 06.9	1 06.6	54	1 46.9	1 46 3	1 45 8
22	1 07.8	1 07.4	1 07 0	55	1 49.6	1 49 0	1 48 4
23	1 08.3	1 07.9	1 07.5	56	1 52.4	1 51.8	1 51.2
24	1 08.8	1 08.4	1 08.1	57	1 55.4	1 54 8	1 54.2
25	1 09.3	1 09.0	1 08.6	58	1 58.6	1 58.0	1 57.3
26	1 09.9	1 09.5	1 09 2	59	2 02 0	2 01 4	2 00.7
27	1 10.5	1 10.1	1 09.8	60	2 05.7	2 05.0	2 04.4
28	1 11.2	1 10.8	1 10.4	61	2 09 6	2 08.9	2 08.3
29	1 11.8	1 11 5	1 11.1	62	2 13.9	2 13.2	2 12 4
30	1 12.6	1 12.2	1 11 8	63	2 18.4	2 17.7	2 17.0
31	1 13.3	1 12.9	1 12.5	64	2 23 4	2 22.6	2 21.8
32	1 14.1	1 13.7	1 13.3	65	2 28.7	2 27.9	2 27.1
33	1 14.9	1 14.5	1 14.1	66	2 34.5	2 33.7	2 32 9
34	1 15.8	1 15.4	1 15.0	67	2 40.9	2 40.0	2 39.2
35	1 16.7	1 16.3	1 15.9	68	2 47.8	2 46.9	2 46.0
36	1 17.6	1 17.3	1 16.8	69	2 55.4	2 54.5	2 53.5
37	1 18.7	1 18.3	1 17.8	70	3 03.8	3 02.8	3 01.8

The above table has been computed by the K. and E. Company, and has been extended to cover all latitudes of the North American Continent. To obtain the azimuth at any other declination compute

$$\text{Sine of azimuth} = \frac{\text{sine of polar distance}}{\text{cosine of latitude}}$$

$$\text{Polar distance} = 90° - \text{declination.}$$

Table of Declination of Polaris

Annual Change +0.34′

	° ′		° ′
Jan. 1	88 57.67	July 1	88 57.17
15	88 57.70	15	88 57.18
Feb. 1	88 57.70	Aug. 1	88 57.21
15	88 57.68	15	88 57.25
Mar. 1	88 57.63	Sept. 1	88 57.32
15	88 57.58	15	88 57.39
Apr. 1	88 57.50	Oct. 1	88 57.48
15	88 57.42	15	88 57.57
May 1	88 57.34	Nov. 1	88 57.68
15	88 57.28	15	88 57.77
June 1	88 57.22	Dec. 1	88 57.86
15	88 57.19	15	88 57.92

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
1° 00′	5,729.58	50.001	0.218	99.999	0.218	50.00	0.22	1° 00′
10	4,911.07	.002	.255	.998	.254	58.34	.30	10
20	4,297.18	.002	.291	.998	.291	66.67	.39	20
30	3,819.71	.003	.327	.997	.327	75.01	.49	30
40	3,437.75	.004	.364	.996	.363	83.34	.61	40
50	3,125.22	.004	.400	.996	.400	91.68	.73	50
2° 00	2,864.79	.005	.436	.995	.436	100.01	.87	2° 00
10	2,644 42	.006	.473	.994	.472	108.35	1.02	10
20	2,455.53	.007	.509	.993	.509	116.68	1.19	20
30	2,291.83	.008	.545	.992	.545	125.02	1.36	30
40	2,148.59	.009	.582	.991	.582	133.36	1.55	40
50	2,022.20	.010	.618	.990	.618	141.70	1.75	50
3° 00	1,909 86	.011	.654	.989	.654	150.04	1.96	3° 00
10	1,809.34	.013	.691	.987	.691	158.38	2.19	10
20	1,718.87	.014	.727	.986	.727	166.72	2.43	20
30	1,637.02	.016	.764	.984	.763	175.06	2.67	30
40	1,562 61	.017	.800	.983	.800	183.40	2.93	40
50	1,494.67	.019	.837	.981	.836	191.74	3.21	50
4° 00	1,432 39	.020	.873	.980	.873	200.08	3.49	4° 00
10	1,375 10	.022	.910	.978	.909	208.43	3.79	10
20	1,322.21	.024	.946	.976	.945	216.77	4.10	20
30	1,273.24	.026	.982	.974	.982	225.12	4.42	30
40	1,227.77	.028	1.019	.972	1.018	233.47	4.76	40
50	1,185.43	.030	.055	.970	.054	241.81	5.10	50
5° 00	1,145.92	.032	.092	.968	.091	250.16	5.46	5° 00
10	1,108.95	.034	.128	.966	.127	258.51	5.83	10
20	1,074.30	.036	.165	.964	.163	266.86	6.21	20
30	1,041.74	.038	.201	.961	.200	275.21	6.61	30
40	1,011.10	.041	.237	.959	.236	283.57	7.01	40
50	982.21	.043	.274	.957	.273	291.92	7.43	50
6° 00	954.93	.046	.310	.954	.310	300.28	7.86	6° 00
10	929.12	048	.347	.952	.346	308.64	8.31	10
20	904.67	.051	.383	.949	.382	316.99	8.76	20
30	881.47	.054	.420	.946	.418	325.35	9.23	30
40	859.44	.056	.456	.943	.454	333.71	9.71	40
50	838.47	.059	.493	.941	.490	342.08	10.20	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
7° 00′	818.51	50.062	1.529	99.938	1.526	350.44	10.71	7° 00′
10	799 48	.065	.566	.935	.563	358.81	11.22	10
20	781.31	.068	.602	.931	.599	367.17	11.75	20
30	763.94	.072	.639	.928	.635	375 54	12.29	30
40	747.34	.075	.676	.925	.672	383.91	12.85	40
50	731.44	.078	.712	.922	.708	392.28	13.41	50
8° 00	716 20	.081	.749	.919	.745	400.66	13 99	8° 00
10	701.58	.085	.785	.915	.781	409.03	14.58	10
20	687 55	.088	.822	.912	.817	417.41	15.18	20
30	674.07	.092	.859	.908	.854	425.79	15.80	30
40	661 11	.096	.895	.904	.890	434.17	16.43	40
50	648.63	.099	.932	.901	.926	442.55	17.07	50
9° 00	636.62	.103	.969	.897	.962	450.93	17.72	9° 00
10	625.04	.107	2.005	.893	.999	459.32	18.38	10
20	613 88	.111	.042	.889	2.035	467.71	19.06	20
30	603.11	.115	.079	.885	.071	476 10	19.75	30
40	592.72	.119	.115	.881	.108	484.49	20.45	40
50	582.67	.123	.152	.877	.144	492.88	21.16	50
10° 00	572.96	.127	.189	.873	.180	501.28	21.89	10° 00
10	563.56	.132	.225	.869	.217	509.68	22.62	10
20	554.48	.136	.262	.864	.253	518.08	23.38	20
30	545.67	.140	.299	.860	.289	526.48	24.14	30
40	537.15	.145	.335	.855	.325	534.89	24.91	40
50	528.88	.150	.372	.851	.362	543.29	25.70	50
11° 00	520.87	.154	.409	.846	.398	551.70	26.50	11° 00
10	513.10	.159	.446	.842	.434	560.11	27.31	10
20	505.55	.164	.483	.837	.471	568.53	28.14	20
30	498.22	.168	.519	.832	.507	576.95	28.97	30
40	491.11	.174	.556	.827	.543	585.36	29.82	40
50	484.19	.178	.593	.822	.579	593.79	30.68	50
12° 00	477.46	.184	.630	.817	.616	602.21	31.56	12° 00
10	470.92	.189	.667	.812	.652	610.64	32.45	10
20	464.56	.194	.704	.807	.688	619.07	33.35	20
30	458.37	.199	.740	.801	.724	627.50	34.26	30
40	452.34	.205	.777	.796	.761	635.93	35.18	40
50	446.46	.210	.814	.791	.797	644.37	36.12	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES— (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
13° 00′	440.74	50.216	2.850	99.786	2.833	652 81	37 07	13° 00′
10	435.16	.221	.888	.780	.869	661 25	38 03	10
20	429 72	.227	925	.774	.906	669.70	39.01	20
30	424.41	.232	.962	.768	.942	678.15	39 99	30
40	419.24	.238	.999	.763	.978	686.60	40.99	40
50	414.19	.244	3.036	.757	3.014	695.06	42 00	50
14° 00	409.26	.250	074	.751	.050	703.51	43.03	14° 00
10	404.44	.256	.111	.745	.087	711.97	44 07	10
20	399.74	.262	148	.739	.123	720.44	45.12	20
30	395 14	.269	.185	.733	.159	728.90	46.18	30
40	390.65	.275	.222	.727	.195	737 37	47.25	40
50	386 26	.281	.259	.721	.232	745.85	48.34	50
15° 00	381 97	.288	.296	.715	.268	754 32	49.44	15° 00
10	377.77	.294	.333	.708	.304	762.80	50.55	10
20	373.67	.301	.370	.702	.340	771.29	51.68	20
30	369.65	.307	.408	.695	.376	779.77	52.89	30
40	365.72	.314	.445	.688	.413	788.26	53.97	40
50	361.87	.321	.482	.682	.449	796.75	55.13	50
16° 00	358.10	.328	.519	.675	.485	805.25	56.31	16° 00
10	354.41	.334	.557	.668	.521	813.75	57.50	10
20	350.79	.341	.594	.661	.557	822.25	58.70	20
30	347.25	.348	.631	.654	.594	830.76	59.91	30
40	343.77	.356	.668	.648	.630	839.27	61.14	40
50	340.37	.363	.706	.641	.666	847.78	62.38	50
17° 00	337.03	.370	.743	.634	.702	856.30	63.63	17° 00
10	333.76	.377	.781	.626	.738	864.82	64.90	10
20	330.55	.385	.818	.619	.774	873.35	66.18	20
30	327.40	.392	.855	.611	.811	881 88	67.47	30
40	324.32	.400	.893	.604	.847	890.41	68.77	40
50	321.28	.408	.930	.597	.883	898.95	70.09	50
18° 00	318.31	.416	.968	.589	.919	907.49	71.42	18° 00
10	315.39	.423	4.005	.581	.955	916.03	72.76	10
20	312.52	.431	.043	.574	.991	924.58	74.12	20
30	309.71	.439	.080	.566	4.027	933.13	75.49	30
40	306.94	.447	.118	.558	.063	941.69	76.86	40
50	304.22	.455	.156	.550	.100	950.25	78.26	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
19° 00′	301.56	50.463	4 193	99.542	4.136	958.81	79.67	19° 00′
10	298.93	.472	.231	.534	.172	967.38	81.09	10
20	296.36	.480	.269	.526	.208	975.96	82 53	20
30	293.82	.488	.306	.518	.244	984.53	83.97	30
40	291.33	.497	.344	.509	.280	993.12	85.43	40
50	288.89	.505	.382	.501	.316	1,001.7	86 90	50
20° 00	286.48	.514	.419	.493	.352	1,010 3	88.39	20° 00
10	284.11	.523	.457	.484	.388	1,018.9	89.89	10
20	281 78	.531	.495	.476	.424	1,027.5	91.40	20
30	279.49	.540	.533	.467	.460	1,036.1	92 92	30
40	277.24	.549	.571	.458	.497	1,044 7	94.46	40
50	275.02	.558	.609	.450	.533	1,053.3	96.01	50
21° 00	272.84	.567	.646	.441	.569	1,061 9	97.57	21° 00′
10	270.69	.576	.684	.432	.605	1,070.6	99 16	10
20	268.57	.586	.722	.423	.641	1,079 2	100.75	20
30	266.49	.595	.760	.414	.677	1,087.8	102.35	30
40	264.44	.604	.798	.405	.713	1,096.4	103.97	40
50	262.42	.614	.836	.396	.749	1,105.1	105.60	50
22° 00	260.44	.624	.874	.387	.785	1,113.7	107.24	22° 00
10	258.48	.633	.913	.377	.821	1,122.4	108 90	10
20	256.55	.643	.951	.368	.857	1,131.0	110.57	20
30	254.65	.653	.989	.358	.893	1,139.7	112.25	30
40	252.78	.662	5.027	.349	.929	1,148.4	113.95	40
50	250.93	.672	.065	.339	.965	1,157.0	115.66	50
23° 00	249.11	.682	.103	.330	5.001	1,165.7	117.38	23° 00
10	247.32	.692	.142	.320	.037	1,174.4	119.12	10
20	245.55	.703	.180	.310	.073	1,183.1	120.87	20
30	243.81	.713	.218	.300	.109	1,191.8	122.63	30
40	242.09	.723	.257	.290	.145	1,200.5	124.41	40
50	240.40	.734	.295	.280	.181	1,209.2	126.20	50
24° 00	238.73	.744	.333	.271	.217	1,217.9	128.00	24° 00
10	237.09	.755	.372	.260	.253	1,226.6	129.82	10
20	235.46	.765	.410	.250	.289	1,235.3	131.65	20
30	233.86	.776	.449	.240	.325	1,244.0	133.50	30
40	232.28	.787	.488	.229	.361	1,252.8	135.35	40
50	230.72	.798	.526	.219	.397	1,261.5	137.23	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
25° 00′	229.18	50.809	5 565	99.209	5.432	1,270.2	139.11	25° 00′
10	227 66	.820	.603	.198	.468	1,279.0	141.01	10
20	226.17	.831	.642	.187	.504	1,287.7	142.93	20
30	224.69	.842	.681	.176	.540	1,296.5	144.85	30
40	223.23	.853	.719	.166	.576	1,305.3	146.79	40
50	221.79	.865	.758	.155	.612	1,314.0	148.75	50
26° 00	220.37	.876	.797	.144	.648	1,322.8	150.71	26° 00
10	218 96	.888	.835	.133	.684	1,331.6	152.69	10
20	217.58	.899	.874	.122	.720	1,340.4	154.69	20
30	216.21	.911	.913	.111	.756	1,349.2	156.70	30
40	214.86	.923	.952	.100	.792	1,358.0	158.72	40
50	213.52	.934	.991	.088	.827	1,366.8	160.76	50
27° 00	212 21	.946	6.030	.077	.863	1,375.6	162.81	27° 00
10	210 90	.958	.069	.066	.899	1,384.4	164.86	10
20	209 62	.970	.108	.054	.935	1,393.2	166.95	20
30	208 35	.982	.147	.042	.971	1,402.0	169.04	30
40	207 09	.995	.186	.031	6.007	1,410.9	171.15	40
50	205.85	51.007	.225	.019	.042	1,419.7	173.27	50
28° 00	204.63	.020	.264	.008	.078	1,428.6	175.41	28° 00
10	203.42	.032	.304	98.996	.114	1,437.4	177.55	10
20	202 22	.044	.343	.984	.150	1,446.3	179.72	20
30	201.04	.057	.382	.972	.186	1,455.1	181.89	30
40	199.87	.070	.422	.960	.222	1,464.0	184.08	40
50	198.71	.083	.461	.948	.257	1,472.9	186.29	50
29° 00	197.57	.096	.500	.936	.293	1,481.8	188.51	29° 00
10	196.44	.108	.540	.924	.329	1,490.7	190.74	10
20	195.33	.122	.579	.911	.365	1,499.6	192.99	20
30	194.22	.135	.619	.899	.400	1,508.5	195.25	30
40	193.13	.148	.658	.887	.436	1,517.4	197.53	40
50	192.05	.161	.698	.874	.472	1,526.3	199.82	50
30° 00	190.98	.174	.737	.862	.508	1,535.3	202.12	30° 00
10	189.93	.188	.777	.849	.543	1,544.2	204.44	10
20	188.89	.202	.817	.836	.579	1,553.1	206.77	20
30	187.86	.215	.856	.823	.615	1,562.1	209.12	30
40	186.83	.229	.896	.811	.651	1,571.0	211.48	40
50	185.82	.243	.936	.798	.686	1,580.0	213.86	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continu*

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
31° 00′	184.82	51 256	6.976	98.785	6 722	1,589 0	216.3	31° 00′
10	183.84	.271	7.016	.772	.758	1,598.0	218.7	10
20	182.86	.284	.055	758	793	1,606.9	221 1	20
30	181 89	.299	.095	745	.829	1,615 9	223.5	30
40	180 93	.313	.135	.732	.865	1,624.9	226.0	40
50	179 99	.327	.175	.719	.900	1,633.9	228 4	50
32° 00	179.05	.342	.215	.705	.936	1,643.0	230.9	32° 00
10	178.12	.356	.256	.692	.972	1,652.0	233.4	10
20	177.20	.371	.296	.678	7 007	1,661.0	235.9	20
30	176.29	.385	.336	.664	.043	1,670.0	238.4	30
40	175.40	.400	.376	.651	.079	1,679.1	241 0	40
50	174.50	.415	.417	.637	.114	1,688.1	243 5	50
33° 00	173.62	.430	.457	.624	.150	1,697.2	246 1	33° 00
10	172.75	.445	.497	.610	.185	1,706.3	248.7	10
20	171.89	.460	.538	.595	.221	1,715.3	251.3	20
30	171.03	.475	.578	.581	.257	1,724.4	253.9	30
40	170.18	.490	.619	.567	.292	1,733.5	256.5	40
50	169.35	.505	.659	.553	.328	1,742.6	259.1	50
34° 00	168.52	.521	.700	.539	.363	1,751.7	261.8	34° 00
10	167.70	.536	.740	.525	.399	1,760.8	264.5	10
20	166.88	.552	.781	.511	.434	1,770.0	267.2	20
30	166.07	.568	.822	.496	.470	1,779.1	269 9	30
40	165.28	.583	.863	.482	.505	1,788.2	272.6	40
50	164.48	.599	.904	468	.541	1,797.4	275.3	50
35° 00	163.70	.615	.944	.454	.576	1,806.6	278.1	35° 00
10	162.93	.631	.985	.439	.612	1,815.7	280.8	10
20	162.16	.647	8.026	.424	.647	1,824.9	283.6	20
30	161.40	.664	.067	.409	.683	1,834.1	286.4	30
40	160.64	.680	.108	.394	.718	1,843.3	289.2	40
50	159.90	.696	.149	.378	.754	1,852.5	292.0	50
36° 00	159.15	.713	.190	.363	.790	1,861.7	294.9	36° 00
10	158 42	.729	.232	.348	.825	1,870.9	297.7	10
20	157.69	.746	.274	.333	.861	1,880.1	300.6	20
30	156.97	.762	.315	.317	.896	1,889.4	303.5	30
40	156.26	.779	.356	.302	.931	1,898.6	306.4	40
50	155.55	.796	.397	.287	.967	1,907.9	309.3	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
37° 00′	154.85	51.813	8.438	98.271	8.002	1,917.1	312.2	37° 00′
10	154 16	.830	.480	.256	.038	1,926.4	315.2	10
20	153 47	.848	.521	.240	.073	1,935.7	318.1	20
30	152 79	.865	.563	.224	.108	1,945.0	321.1	30
40	152 11	.882	.605	.209	.144	1,954.3	324.1	40
50	151.44	.900	.646	.193	.179	1,963.6	327.1	50
38° 00	150 78	.917	.688	.177	.215	1,972.9	336 2	38° 00
10	150.12	.935	.730	.161	.250	1,982.2	333.2	10
20	149.47	.952	.772	.145	.285	1,991.5	336.3	20
30	148.82	.970	.814	.129	.321	2,000.9	339.3	30
40	148.18	.988	.855	.113	.356	2,010 2	342.4	40
50	147.54	52.006	.897	097	.391	2,019.6	345.5	50
39° 00	146 91	.024	.939	.081	.427	2,029.0	348.6	39° 00
10	146.29	.042	.982	.064	.462	2,038.4	351.8	10
20	145.67	.061	9.024	.048	.497	2,047.8	354.9	20
30	145 05	.079	.066	.031	.533	2,057.2	358.1	30
40	144.44	.098	.108	.015	.568	2,066.6	361.3	40
50	143.84	.116	.151	97.998	.603	2,076.0	364.5	50
40° 00	143.24	.135	.193	.982	.638	2,085.4	367.7	40° 00
10	142.64	.154	.235	.965	.674	2,094 9	371.0	10
20	142.06	.172	.278	.948	.709	2,104.3	374.2	20
30	141.47	.191	.320	.931	.744	2,113.8	377.5	30
40	140.89	.211	.363	.914	.779	2,123.3	380.8	40
50	140.32	.230	.405	.897	.815	2,132.7	384.1	50
41° 00′	139.74	.249	.448	.880	.850	2,142.2	387.4	41° 00
10	139 18	.268	.491	.863	.885	2,151.7	390.7	10
20	138.62	.288	.534	.845	.920	2,161.2	394.1	20
30	138.06	.307	.577	.828	.955	2,170.8	397.4	30
40	137.51	.327	.620	.811	.901	2,180.3	400.8	40
50	136.96	.346	.663	.793	9.026	2,189.9	404.2	50
42° 00	136.42	.366	.705	.776	.061	2,199.4	407.6	42° 00
10	135.88	.386	.749	.758	.096	2,209.0	411.1	10
20	135.34	.406	.792	.741	.131	2,218.6	414.5	20
30	134.81	.426	.835	.723	.166	2,228.1	418.0	30
40	134.29	.446	.878	.705	.201	2,237.7	421.4	40
50	133.76	.467	.922	.687	.236	2,247.3	425.0	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
43° 00′	133.25	52.487	9.965	97.670	9.272	2,257.0	428.5	43° 00′
10	132.73	.508	10.009	.651	.307	2,266.6	432.0	10
20	132.22	.528	.052	.633	.342	2,276.2	435.6	20
30	131.71	.549	.096	.615	.377	2,285.9	439.2	30
40	131.21	.570	.139	.597	.412	2,295.6	442.8	40
50	130.71	.590	.183	.579	.447	2,305.2	446.4	50
44° 00	130.22	.611	.227	.561	.482	2,314.9	450.0	44° 00
10	129.73	.632	.271	.542	.517	2,324.6	453.6	10
20	129.24	.654	.314	.523	.552	2,334.3	457.3	20
30	128.75	.675	.358	.505	.587	2,344.1	461.0	30
40	128.27	.696	.402	.486	.622	2,353.8	464.6	40
50	127.80	.718	.446	.467	.657	2,363.5	468.4	50
45° 00	127.32	.739	.490	.448	.692	2,373.3	472.1	45° 00
10	126.85	.761	.534	.430	.727	2,383.1	475.8	10
20	126.39	.783	.579	.411	.762	2,392.8	479.6	20
30	125.92	.805	.623	.392	.797	2,402.6	483.8	30
40	125.46	.827	.668	.373	.832	2,412.4	487.2	40
50	125.01	.849	.712	.355	.867	2,422.3	491.0	50
46° 00	124.56	.871	.757	.336	.902	2,432.1	494.8	46° 00
10	124.11	.893	.801	.316	.936	2,441.9	498.7	10
20	123.66	.916	.846	.297	.971	2,451.8	502.5	20
30	123.22	.938	.891	.278	10.006	2,461.7	506.4	30
40	122.78	.961	.936	.258	.041	2,471.5	510.3	40
50	122.34	.983	.980	.239	.076	2,481.4	514.3	50
47° 00	121.91	53.006	11.025	.220	.111	2,491.3	518.2	47° 00
10	121.48	.029	.070	.200	.146	2,501.2	522.2	10
20	121.05	.052	.115	.180	.180	2,511.2	526.1	20
30	120.62	.075	.161	.160	.215	2,521.1	530.1	30
40	120.20	.098	.206	.141	.250	2,531.1	534.2	40
50	119.78	.122	.251	.121	.285	2,541.0	538.2	50
48° 00	119.37	.145	.296	.101	.320	2,551.0	542.2	48° 00
10	118.95	.169	.342	.081	.354	2,561.0	546.3	10
20	118.54	.192	.387	.061	.389	2,571.0	550.4	20
30	118.14	.216	.433	.041	.424	2,581.0	554.5	30
40	117.73	.240	.479	.021	.459	2,591.0	558.6	40
50	117.33	.264	.524	.000	.494	2,601.1	562.8	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
49° 00′	116.93	53.238	11.570	96.980	10.528	2,611.2	566.9	49° 00′
10	116.53	.312	.616	.960	.563	2,621.2	571.1	10
20	116.14	.337	.662	.939	.598	2,631.3	575.3	20
30	115.75	.361	.708	.919	.632	2,641.4	579.5	30
40	115.36	.386	.754	.898	.667	2,651.5	583.8	40
50	114.97	.410	.800	.878	.701	2,661.6	588.0	50
50° 00	114.59	.435	.846	.857	.736	2,671.8	592.3	50° 00
10	114.21	.460	.893	.836	.771	2,681.9	596.6	10
20	113.83	.485	.939	.815	.805	2,692.1	600.9	20
30	113.46	.510	.985	.794	.840	2,702.3	605.3	30
40	113.08	.535	12.032	.773	.875	2,712.5	609.6	40
50	112.71	.560	.079	.752	.909	2,722.7	614.0	50
51° 00	112.34	.586	.125	.731	.944	2,732.9	618.4	51° 00
10	111.98	.611	.172	.710	.979	2,743.1	622.8	10
20	111.62	.639	.219	.689	11.013	2,753.4	627.2	20
30	111.25	.662	.266	.667	.048	2,763.7	631.7	30
40	110.90	.688	.313	.646	.082	2,773.9	636.2	40
50	110.54	.714	.360	.625	.117	2,784.2	640.7	50
52° 00	110.18	.741	.407	.603	.151	2,794.5	645.2	52° 00
10	109.83	.767	.454	.581	.186	2,804.9	649.7	10
20	109.48	.793	.502	.560	.220	2,815.2	654.3	20
30	109.13	.819	.549	.538	.255	2,825.6	658.8	30
40	108.79	.846	.596	.516	.289	2,835.9	663.4	40
50	108.45	.872	.644	.494	.324	2,846.3	668.0	50
53° 00	108.11	.899	.692	.473	.358	2,856.7	672.7	53° 00
10	107.77	.926	.739	.450	.393	2,867.1	677.3	10
20	107.43	.953	.787	.428	.427	2,877.5	682.0	20
30	107.09	.980	.835	.406	.461	2,888.0	686.7	30
40	106.76	54.008	.883	.384	.496	2,898.4	691.4	40
50	106.43	.035	.931	.362	.530	2,908.9	696.1	50
54° 00	106.10	.062	.979	.340	.565	2,919.4	700.9	54° 00
10	105.78	.090	13.027	.317	.599	2,929.9	705.7	10
20	105.45	.118	.076	.295	.633	2,940.4	710.5	20
30	105.13	.146	.124	.272	.668	2,951.0	715.3	30
40	104.81	.173	.173	.250	.702	2,961.5	720.1	40
50	104.49	.202	.221	.227	.736	2,972.1	725.0	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
55° 00′	104.17	54.230	13.270	96 205	11.771	2,982.7	729.9	55° 00′
10	103.86	.258	.319	.182	.805	2,993 3	734.8	10
20	103 55	.286	.368	.159	.839	3,003.9	739.7	20
30	103.24	.315	.416	.136	.873	3,014 5	744 6	30
40	102.93	.344	.466	.113	.908	3,025.2	749.6	40
50	102.62	.372	.515	.090	.942	3,035.8	754.6	50
56° 00	102.31	.401	.564	.067	.976	3,046.5	759.6	56° 00
10	102.01	.430	.613	.044	12.010	3,057.2	764 6	10
20	101.71	.460	.663	.020	.044	3,067.9	769 7	20
30	101.41	.489	.712	95.997	.079	3,078 7	774.7	30
40	101.11	.518	.762	.974	.113	3,089.4	779.8	40
50	100.81	.548	.811	.950	.147	3,100.2	784.9	50
57° 00	100.52	.577	.861	.927	.181	3,110.9	790 1	57° 00
10	100.22	.607	.911	.903	.215	3,121.7	795.2	10
20	99.93	.637	.961	.880	.249	3,132.6	800 4	20
30	99.64	.667	14.011	.856	.283	3,143.4	805.6	30
40	99.36	.697	.061	.832	.318	3,154.2	810.9	40
50	99.07	.728	.111	.808	.352	3,165.1	816.1	50
58° 00	98.78	.758	.161	.785	.386	3,176.0	821 4	58° 00
10	98 50	.788	.212	.761	.420	3,186 9	826 7	10
20	98.22	.819	.262	.736	.454	3,197.8	832.1	20
30	97.94	.850	.313	.712	.488	3,208 8	837.4	30
40	97.66	.881	.364	.688	.522	3,219.7	842.7	40
50	97.39	.912	.414	.664	.556	3,230.7	848.1	50
59° 00	97.11	.943	.465	.640	.590	3,241.7	853.5	59° 00
10	96 84	.974	.516	.615	.624	3,252.7	859.0	10
20	96.56	55.006	.567	.591	.658	3,263.7	864.4	20
30	96.30	.037	.619	.566	.692	3,274.8	869.9	30
40	96.03	.069	.670	.542	.726	3,285.8	875.4	40
50	95.76	.101	.721	.517	.760	3,296.9	880.9	50
60° 00	95.49	.133	.773	.493	.794	3,308.0	886.4	60° 00
10	95.23	.165	.824	.468	.828	3,319.1	892.0	10
20	94.96	.197	.876	.443	.861	3,330.3	897.6	20
30	94.70	.230	.928	.418	.895	3,341.4	903.2	30
40	94.44	.262	.980	.393	.929	3,352.6	908.8	40
50	94.18	.295	15.032	.368	.963	3,363.8	914.5	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
61° 00′	93.93	55.328	15.084	95.344	12.997	3,375.0	920.2	61° 00′
10	93.67	.361	.136	.318	13.030	3,386.3	925.9	10
20	93.42	.394	.189	.293	.064	3,397.5	931.6	20
30	93.16	.427	.241	.268	.098	3,408.8	937.3	30
40	92.91	.460	.294	.243	.132	3,420.1	943.1	40
50	92.66	.493	.346	.217	.166	3,431.4	948.9	50
62° 00	92.41	.527	.399	.192	.199	3,442.7	954.8	62° 00
10	92.16	.561	.452	.166	.233	3,454.1	960.6	10
20	91.92	.595	.505	.141	.267	3,465.4	966.5	20
30	91.67	.629	.558	.115	.301	3,476.8	972.4	30
40	91.43	.663	.611	.089	.334	3,488.3	978.3	40
50	91.19	.697	.665	.064	.368	3,499.7	984.3	50
63° 00	90.94	.732	.718	.038	.402	3,511.1	990.2	63° 00
10	90.71	.766	.772	.012	.435	3,522.6	996.2	10
20	90.47	.801	.825	94.986	.469	3,534.1	1,002.3	20
30	90.23	.836	.879	.960	.503	3,545.6	1,008.3	30
40	89.99	.871	.933	.934	.536	3,557.2	1,014.4	40
50	89.76	.906	.987	.908	.570	3,568.7	1,020.5	50
64° 00	89.52	.941	16.041	.882	.603	3,580.3	1,026.6	64° 00
10	89.29	.977	.095	.855	.637	3,591.9	1,032.8	10
20	89.06	56.012	.150	.829	.671	3,603.5	1,039.0	20
30	88.83	.048	.204	.802	.704	3,615.1	1,045.2	30
40	88.60	.084	.259	.776	.738	3,626.8	1,051.4	40
50	88.37	.120	.313	.749	.771	3,638.5	1,057.7	50
65° 00	88.15	.156	.368	.723	.805	3,650.2	1,063.9	65° 00
10	87.92	.192	.423	.696	.838	3,661.9	1,070.2	10
20	87.70	.229	.478	.669	.871	3,673.7	1,076.6	20
30	87.47	.266	.533	.643	.905	3,685.4	1,082.9	30
40	87.25	.302	.589	.616	.938	3,697.2	1,089.3	40
50	87.03	.339	.644	.589	.972	3,709.0	1,095.7	50
66° 00	86.81	.376	.699	.562	14.005	3,720.9	1,102.2	66° 00
10	86.59	.413	.755	.535	.039	3,732.7	1,108.6	10
20	86.38	.451	.811	.508	.072	3,744.6	1,115.1	20
30	86.16	.488	.867	.481	.105	3,756.5	1,121.7	30
40	85.94	.526	.923	.453	.139	3,768.5	1,128.2	40
50	85.73	.564	.979	.426	.172	3,780.4	1,134.8	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
67° 00′	85.52	56.602	17.035	94.399	14.205	3,792 4	1,141.4	67° 00′
10	85 30	.640	.092	.371	.239	3,804 4	1,148.0	10
20	85 09	.678	.148	.344	.272	3,816.4	1,154.7	20
30	84.88	.717	.205	.316	.305	3,828.4	1,161.3	30
40	84.67	.755	.262	.289	.338	3,840.5	1,168.1	40
50	84.46	.794	.319	.261	.372	3,852.6	1,174 8	50
68° 00	84.26	.833	.376	.234	.405	3,864 7	1,181.6	68° 00
10	84.05	.872	.433	.206	.438	3,876.8	1,188.4	10
20	83.85	.911	.490	.178	.471	3,889.0	1,195.2	20
30	83 64	.951	.548	.150	.505	3,901.2	1,202.0	30
40	83.44	.990	.605	.122	.538	3,913.4	1,208.9	40
50	83.24	57.030	.663	.095	.571	3,925.6	1,215.8	50
69° 00	83.04	.070	.721	.067	.604	3,937 9	1,222.7	69° 00
10	82.84	.110	.779	.038	.637	3,950 2	1,229.7	10
20	82.64	.150	.837	.010	.670	3,962 5	1,236.7	20
30	82.44	.191	.895	93.981	.703	3,974 8	1,243 7	30
40	82.24	.231	.954	.953	.736	3,987.2	1,250.8	40
50	82.05	.272	18.012	.924	.770	3,999.5	1,257.9	50
70° 00	81.85	.313	.071	.896	.803	4,011.9	1,265.0	70° 00
10	81.66	.354	.130	.867	.836	4,024.4	1,272.1	10
20	81.46	.395	.189	.838	.869	4,036.8	1,279.3	20
30	81.27	.436	.248	.810	.902	4,049.3	1,286.5	30
40	81 08	.478	.307	.781	.935	4,061.8	1,293.6	40
50	80.89	.520	.366	.752	.968	4,074.4	1,300.9	50
71° 00′	80.70	.561	.426	.723	15.001	4,086.9	1,308.2	71° 00
10	80 51	.604	.485	.694	.034	4,099.5	1,315.6	10
20	80.32	.646	.545	.665	.066	4,112.1	1,322.9	20
30	80.13	.688	.605	.636	.099	4,124.8	1,330.3	30
40	79.95	.731	.665	.607	.132	4,137.4	1,337.7	40
50	79.76	.774	.725	.578	.165	4,150.1	1,345.1	50
72° 00	79.58	.816	.786	.549	.198	4,162.8	1,352.6	72° 00
10	79.39	.860	.846	.519	.231	4,175 6	1,360.1	10
20	79.21	.903	.907	.490	.264	4,188.5	1,367.6	20
30	79.03	.946	.968	.460	.296	4,201.2	1,375.2	30
40	78.85	.990	19.029	.430	.329	4,214.0	1,382.8	40
50	78.67	58.034	.090	.401	.362	4,226.8	1,390.4	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
73° 00′	78.49	58.078	19.151	93.371	15.395	4,239.7	1,398.0	73° 00′
10	78.31	.122	.213	.342	.427	4,252.6	1,405.7	10
20	78.13	.166	.274	.312	.460	4,265.6	1,413.5	20
30	77.95	.211	.336	.282	.493	4,278.5	1,421.2	30
40	77.78	.255	.398	.253	.526	4,291.5	1,429.0	40
50	77.60	.300	.460	.223	.558	4,304.6	1,436.8	50
74° 00	77.43	.345	.522	.193	.591	4,317.6	1,444.6	74° 00
10	77.25	.390	.585	.163	.624	4,330.7	1,452.5	10
20	77.08	.436	.647	.132	.656	4,343.8	1,460.4	20
30	76.91	.482	.710	.102	.689	4,356.9	1,468.4	30
40	76.74	.527	.773	.072	.721	4,370.1	1,476.4	40
50	76.56	.573	.836	.041	.754	4,383.3	1,484.4	50
75° 00	76.39	.620	.899	.011	.787	4,396.5	1,492.4	75° 00
10	76.22	.666	.962	92.981	.819	4,409.8	1,500.5	10
20	76.06	.712	20.026	.950	.852	4,423.1	1,508.6	20
30	75.89	.759	.089	.920	.884	4,436.4	1,516.7	30
40	75.82	.806	.153	.889	.917	4,449.7	1,524.9	40
50	75.55	.853	.217	.859	.949	4,463.1	1,533.1	50
76° 00	75.39	.901	.281	.828	.982	4,476.5	1,541.4	76° 00
10	75.22	.948	.346	.797	16.014	4,489.9	1,549.7	10
20	75.06	.996	.410	.767	.046	4,503.4	1,558.0	20
30	74.90	59.044	.475	.736	.079	4,516.9	1,566.3	30
40	74.73	.092	.540	.705	.111	4,530.4	1,574.7	40
50	74.57	.140	.604	.674	.144	4,544.0	1,583.1	50
77° 00	74.41	.188	.669	.643	.176	4,557.6	1,591.6	77° 00
10	74.25	.237	.735	.611	.208	4,571.2	1,600.1	10
20	74.09	.286	.801	.580	.241	4,584.8	1,608.6	20
30	73.93	.335	.866	.549	.273	4,598.5	1,617.1	30
40	73.77	.384	.932	.517	.305	4,612.2	1,625.7	40
50	73.61	.434	.998	.486	.338	4,626.0	1,634.4	50
78° 00	73.46	.484	21.064	.455	.370	4,639.8	1,643.0	78° 00
10	73.30	.534	.131	.423	.402	4,653.6	1,651.7	10
20	73.14	.584	.197	.391	.434	4,667.4	1,660.5	20
30	72.99	.634	.264	.360	.467	4,681.3	1,669.2	30
40	72.83	.684	.331	.328	.499	4,695.2	1,678.1	40
50	72.68	.735	.398	.296	.531	4,709.2	1,686.9	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
79° 00′	72.53	59.786	21.465	92.265	16.563	4,723.2	1,695.8	79° 00′
10	72.37	.837	.533	.233	.595	4,737.2	1,704.7	10
20	72.22	.889	.601	.201	.627	4,751.2	1,713.7	20
30	72.07	.940	.668	.169	.660	4,765.3	1,722.7	30
40	71.92	.992	.737	.137	.692	4,779.4	1,731.7	40
50	71.77	60.044	.805	.105	.724	4,793.6	1,740.8	50
80° 00	71.62	.096	.873	.073	.756	4,807.7	1,749.9	80° 00
10	71.47	.148	.942	.040	.788	4,822.0	1,759.0	10
20	71.32	.201	22.011	.008	.820	4,836.2	1,768.2	20
30	71.17	.254	.080	91.975	.852	4,850.5	1,777.4	30
40	71.03	.307	.149	.943	.884	4,864.8	1,786.7	40
50	70.88	.360	.218	.911	.916	4,879.2	1,796.0	50
81° 00′	70.74	.414	.288	.878	.948	4,893.6	1,805.3	81° 00′
10	70.59	.468	.358	.845	.980	4,908.0	1,814.7	10
20	70.44	.522	.428	.813	17.012	4,922.5	1,824.1	20
30	70.30	.576	.498	.780	.043	4,937.0	1,833.6	30
40	70.16	.630	.568	.747	.075	4,951.5	1,843.1	40
50	70.02	.685	.639	.714	.107	4,966.1	1,852.6	50
82° 00	69.87	.740	.710	.682	.139	4,980.7	1,862.2	82° 00
10	69.73	.795	.781	.648	.171	4,995.4	1,871.8	10
20	69.59	.850	.852	.615	.203	5,010.0	1,881.5	20
30	69.45	.906	.923	.582	.234	5,024.8	1,891.2	30
40	69.31	.961	.995	.549	.266	5,039.5	1,900.9	40
50	69.17	61.017	23.067	.516	.298	5,054.3	1,910.7	50
83° 00	69.03	.074	.139	.483	.330	5,069.2	1,920.5	83° 00
10	68.89	.130	.211	.449	.362	5,084.0	1,930.4	10
20	68.75	.187	.284	.416	.393	5,099.0	1,940.3	20
30	68.62	.244	.356	.382	.425	5,113.9	1,950.3	30
40	68.48	.301	.429	.349	.457	5,128.9	1,960.2	40
50	68.34	.358	.502	.315	.488	5,143.9	1,970.3	50
84° 00	68.21	.416	.575	.282	.520	5,159.0	1,980.4	84° 00
10	68.07	.474	.649	.248	.551	5,174.1	1,990.5	10
20	67.94	.532	.723	.214	.583	5,189.3	2,000.6	20
30	67.81	.590	.797	.180	.615	5,204.4	2,010.8	30
40	67.67	.649	.871	.146	.646	5,219.7	2,021.1	40
50	67.54	.708	.945	.113	.678	5,234.9	2,031.4	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
85° 00′	67 41	61 767	24 020	91.079	17.709	5,250.3	2,041 7	85° 00′
10	67 27	.826	.095	.045	.741	5,265.6	2,052.1	10
20	67.14	.886	.170	.010	.772	5,281.0	2,062.5	20
30	67 01	.946	.245	90.976	.804	5,296.4	2,073.0	30
40	66 88	62 006	.321	.942	.835	5,311.9	2,083.5	40
50	66.75	.066	.397	.908	.867	5,327.4	2,094.1	50
86° 00	66 62	.127	.472	.874	.898	5,343.0	2,104.7	86° 00
10	66 49	.188	.549	.839	.929	5,358 6	2,115.3	10
20	66 36	.249	625	804	.961	5,374.2	2,126 0	20
30	66 24	.310	.702	.770	.992	5,389.9	2,136.7	30
40	66 11	372	.779	.735	18.023	5,405.6	2,147.5	40
50	65 98	.434	.856	.701	.055	5,421 4	2,158.4	50
87° 00	65 86	.496	.934	.666	.086	5,437.2	2,169.2	87° 00
10	65 73	.559	25 011	.631	.117	5,453.1	2,180.2	10
20	65 61	.621	089	596	.149	5,469 0	2,191.1	20
30	65 48	.684	.167	.561	.180	5,484 9	2,202.2	30
40	65 36	.748	.246	.527	.211	5,500.9	2,213.2	40
50	65.23	.811	.324	.492	.242	5,517.0	2,224.3	50
88° 00	65.11	.875	.403	.457	.273	5,533.1	2,235 5	88° 00
10	64.98	.939	.482	.422	.305	5,549.2	2,246.7	10
20	64.86	63.003	.562	.386	.336	5,565.4	2,258.0	20
30	64.74	.068	641	.351	.367	5,581.6	2,269.3	30
40	64 62	.133	.721	.316	.398	5,597.8	2,280.6	40
50	64.50	.198	.882	.280	.429	5,614.2	2,292.0	50
89° 00	64.38	.263	.882	.245	.460	5,630.5	2,303.5	89° 00
10	64.26	.329	.963	.210	.491	5,646.9	2,315.0	10
20	64.14	.395	26.044	.174	.522	5,663.4	2,326.6	20
30	64.02	.461	.124	.138	.553	5,679 9	2,338.2	30
40	63.90	.528	.206	.103	.584	5,696.4	2,349.8	40
50	63.78	.595	.288	.067	.615	5,713 0	2,361.5	50
90° 00	63.66	.662	.370	.032	.646	5,729.7	2,373.3	90° 00
10	63.54	.731	.452	89.996	.677	5,746.3	2,385.1	10
20	63.43	.797	.535	.960	.708	5,763.1	2,397.0	20
30	63.31	.865	.617	.924	.739	5,779.9	2,408.9	30
40	63.19	.934	.701	.888	.770	5,796.7	2,420.9	40
50	63.08	64.002	.784	.852	.801	5,813.6	2,432.9	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
91° 00′	62.96	64.071	26.867	89.816	18.831	5,830.5	2,444.9	91° 00′
10	62.85	.140	.951	.780	.862	5,847.5	2,457.1	10
20	62.73	.210	27.036	.743	.893	5,864.6	2,469.3	20
30	62.62	.280	.120	.707	.924	5,881.7	2,481.5	30
40	62.50	.350	.205	.671	.954	5,898.8	2,493.8	40
50	62.39	.420	.290	.635	.985	5,916.0	2,506.1	50
92° 00	62.28	.491	.375	.598	19.016	5,933.2	2,518.5	92° 00
10	62.16	.562	.461	.561	.047	5,950.5	2,531.0	10
20	62.05	.633	.546	.525	.077	5,967.9	2,543.5	20
30	61.94	.705	.632	.488	.108	5,985.3	2,556.0	30
40	61.83	.777	.719	.452	.139	6,002.7	2,568.6	40
50	61.72	.849	.806	.415	.169	6,020.2	2,581.3	50
93° 00	61.61	.922	.892	.378	.200	6,037.8	2,594.0	93° 00
10	61.50	.995	.980	.341	.231	6,055.4	2,606.8	10
20	61.39	65.068	28.068	.304	.261	6,073.1	2,619.7	20
30	61.28	.141	.156	.267	.292	6,090.8	2,632.6	30
40	61.17	.215	.244	.230	.322	6,108.6	2,645.5	40
50	61.06	.290	.333	.193	.353	6,126.4	2,658.5	50
94° 00	60.95	.364	.421	.156	.383	6,144.3	2,671.6	94° 00
10	60.84	.439	.511	.119	.414	6,162.6	2,684.7	10
20	60.74	.514	.600	.082	.444	6,180.2	2,697.9	20
30	60.63	.590	.690	.044	.474	6,198.3	2,711.2	30
40	60.52	.666	.780	.007	.505	6,216.4	2,724.5	40
50	60.42	.742	.870	88.970	.535	6,234.6	2,737.9	50
95° 00	60.31	.818	.961	.932	.565	6,252.8	2,751.3	95° 00
10	60.21	.895	29.052	.895	.596	6,271.1	2,764.8	10
20	60.10	.972	.143	.857	.626	6,289.4	2,778.3	20
30	60.00	66.050	.235	.819	.656	6,307.9	2,792.0	30
40	59.89	.128	.327	.782	.687	6,326.3	2,805.6	40
50	59.79	.206	.420	.744	.717	6,344.8	2,819.4	50
96° 00	59.68	.285	.512	.706	.747	6,363.4	2,833.2	96° 00
10	59.58	.364	.605	.668	.777	6,382.1	2,847.0	10
20	59.48	.443	.699	.630	.808	6,400.8	2,861.0	20
30	59.37	.523	.792	.592	.838	6,419.5	2,875.0	30
40	59.27	.603	.886	.554	.868	6,438.4	2,889.0	40
50	59.17	.683	.981	.516	.898	6,457.3	2,903.1	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
97° 00′	59.07	66.764	30.075	88.478	19.928	6,476.2	2,917.3	97° 00′
10	58.97	.845	.170	.440	.958	6,495.2	2,931.6	10
20	58.86	.927	.266	.402	.988	6,514.3	2,945 9	20
30	58.76	67.009	.361	.363	20.018	6,533.4	2,960 3	30
40	58.66	.091	.458	.325	.048	6,552.6	2,974 7	40
50	58.56	.173	.554	.287	.079	6,571.9	2,989.2	50
98° 00	58.46	.256	.651	.248	.109	6,591.2	3,003.8	98° 00
10	58.36	.340	.748	.210	.138	6,610 6	3,018 4	10
20	58.27	.424	.846	.171	.168	6,630.1	3,033.1	20
30	58.17	.508	.944	.132	.198	6,649.6	3,047 9	30
40	58.07	.592	31.042	.094	.228	6,669 2	3,062 8	40
50	57.97	.677	.140	.055	.258	6,688.8	3,077 7	50
99° 00	57.87	.762	.239	.016	.288	6,708.6	3,092 7	99° 00
10	57.78	.848	.339	87.977	318	6,728 4	3,107.7	10
20	57.68	.934	.438	.938	.348	6,748 2	3,122 9	20
30	57 58	68.021	.538	.899	.377	6,768.1	3,138.1	30
40	57.49	.108	.639	.860	.407	6,788 1	3,153.3	40
50	57.39	.195	.740	.821	.437	6,808.2	3,168.7	50
100° 00	57.30	.282	.841	.782	.467	6,828.3	3,184 1	100° 00
10	57.20	.371	.943	.743	.496	6,848.5	3,199 6	10
20	57 11	.459	32.045	.704	.526	6,868.8	3,215.1	20
30	57.01	.548	.147	.664	.556	6,889 2	3,230.8	30
40	56.92	.637	.250	.625	.585	6,909.6	3,246 5	40
50	56.82	.727	.353	.586	.615	6,930.1	3,262.3	50
101° 00	56.73	.817	.456	.546	.645	6,950.6	3,278.1	101° 00
10	56.64	.908	.561	.506	.674	6,971.3	3,294 1	10
20	56.54	.999	.665	.467	.704	6,992.0	3,310.1	20
30	56 45	69.090	.770	.427	.733	7,012.7	3,326 1	30
40	56.36	.182	.875	.388	.763	7,033.6	3,342.3	40
50	56.26	.274	.981	348	.792	7,054.5	3,358.5	50
102° 00	56.17	.367	33.086	.308	.822	7,075.5	3,374.9	102° 00
10	56.08	.460	.193	.268	.851	7,096.6	3,391.2	10
20	55.99	.554	.300	.228	.881	7,117.8	3,407.7	20
30	55.90	.648	.407	.188	.910	7,139.0	3,424.3	30
40	55.81	.742	.515	.148	.940	7,160.3	3,440.9	40
50	55.72	.837	.623	.108	.969	7,181.7	3,457.6	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
103° 00′	55.63	69 933	33.732	87 068	21 998	7,203 2	3,474 4	103° 00′
10	55.54	70.029	.841	.028	21.028	7,224.7	3,491 3	10
20	55.45	.125	.950	86.988	.057	7,246 3	3,508 2	20
30	55.36	.222	34.060	.947	.086	7,268 0	3,525 2	30
40	55.27	.319	.171	.907	.115	7,289 8	3,542.4	40
50	55.18	.417	.281	.867	.145	7,311 7	3,559 6	50
104° 00	55.09	.515	.392	.826	.174	7,333.6	3,576 8	104° 00
10	55.00	.613	.504	.786	.203	7,355.6	3,594.2	10
20	54.92	.712	.617	.745	.232	7,377.8	3,611.7	20
30	54.83	.812	.729	.704	.261	7,399.9	3,629.2	30
40	54.74	.912	.842	.664	.291	7,422.2	3,646.8	40
50	54.65	71.013	.956	.623	.320	7,444.6	3,664 5	50
105° 00	54.57	.114	35.069	.582	.349	7,467.0	3,682.3	105° 00
10	54.48	.215	.184	.541	.378	7,489.6	3,700 2	10
20	54.39	.317	.299	.500	.407	7,512.2	3,718.2	20
30	54.31	.420	.414	.459	.436	7,534.9	3,736 2	30
40	54.22	.523	.531	.419	.465	7,557.7	3,754.4	40
50	54.14	.626	.647	.378	.494	7,580 5	3,772.6	50
106° 00	54.05	.730	.763	.337	.523	7,603 5	3,791 0	106° 00
10	53.97	.835	.881	.295	.552	7,626.6	3,809.4	10
20	53.88	.940	.999	.254	.581	7,649.7	3,827.9	20
30	53.80	72.046	36.117	.213	.610	7,672.9	3,846 5	30
40	53.71	.152	.236	.172	.638	7,696.3	3,865 2	40
50	53.63	.258	.356	.130	.667	7,719.7	3,884.0	50
107° 00	53.55	.365	.475	.089	.696	7,743.2	3,902.9	107° 00
10	53.46	.473	.596	.047	.725	7,766.8	3,921.9	10
20	53.38	.579	.717	.006	.754	7,790.5	3,940.9	20
30	53.30	.690	.838	85.964	.782	7,814.3	3,960.1	30
40	53.22	.799	.960	.923	.811	7,838.1	3,979.4	40
50	53.13	.909	37.083	.881	.840	7,862.1	3,998.7	50
108° 00	53.05	73.019	.205	.839	.869	7,886.2	4,018.2	108° 00
10	52.97	.130	.329	.797	.897	7,910.4	4,037.8	10
20	52.89	.242	.453	.756	.926	7,934.6	4,057.4	20
30	52.81	.354	.577	.714	.954	7,959.0	4,077.2	30
40	52.73	.466	.703	.672	.983	7,983.5	4,097.1	40
50	52.64	.580	.829	.630	22.012	8,008.0	4,117.0	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
109° 00′	52.56	73.693	37.955	85.588	22.040	8,032 7	4,137.1	109° 00′
10	52.48	.808	38.082	.546	.069	8,057.4	4,157 3	10
20	52.40	.923	.209	.503	097	8,082.3	4,177.5	20
30	52.32	74.038	.337	.461	.126	8,107.3	4,197 9	30
40	52.24	.154	.466	.419	.154	8,132.3	4,218.4	40
50	52.17	.271	.595	.377	.183	8,157.5	4,239.0	50
110° 00	52.09	.388	.724	.334	.211	8,182.8	4,259.7	110° 00
10	52.01	.506	.855	.292	.240	8,208.2	4,280.5	10
20	51.93	.624	.986	.249	.268	8,233.7	4,301.4	20
30	51.85	.743	39.116	.207	.296	8,259 3	4,322.4	30
40	51.77	.863	.249	.164	.325	8,285.0	4,343 6	40
50	51.70	.983	.382	.122	.353	8,310.8	4,364.8	50
111° 00	51.62	75.104	.514	.079	.381	8,336.7	4,386.1	111° 00
10	51.54	.226	.649	.036	.409	8,362.7	4,407.6	10
20	51 46	.348	.783	84 993	.438	8,388.9	4,429 2	20
30	51.39	.471	.918	.951	.466	8,415.1	4,450 9	30
40	51.31	.594	40.054	.908	.494	8,441.5	4,472 7	40
50	51.23	.719	.190	.864	.522	8,468.0	4,494.6	50
112° 00	51.16	.843	.327	.822	.550	8,494.6	4,516.6	112° 00
10	51.08	.969	.465	.779	.578	8,521.3	4,538.8	10
20	51.01	76.095	.604	.736	.606	8,548.1	4,561.1	20
30	50.93	.222	.742	.692	.634	8,575.0	4,583 4	30
40	50.85	.349	.882	.649	.663	8,602.1	4,606.0	40
50	50.78	.477	41.022	.606	.691	8,629.3	4,628 6	50
113° 00	50.70	.606	.162	.563	.719	8,656.6	4,651.3	113° 00
10	50.63	.735	.304	.520	.747	8,684.0	4,674 2	10
20	50.56	.865	.447	.476	.775	8,711.5	4,697.2	20
30	50.48	.994	.589	.433	.802	8,739.2	4,720 3	30
40	50.41	77.128	.733	.389	.830	8,767.0	4,743.6	40
50	50.33	.260	.877	.346	.858	8,794.9	4,766.9	50
114° 00	50.26	.393	42.021	.302	.886	8,822.9	4,790.4	114° 00
10	50.19	.526	.167	.258	.914	8,851.0	4,814.1	10
20	50.11	.661	.313	.215	.942	8,879.3	4,837.8	20
30	50.04	.796	.460	.171	.969	8,907.7	4,861.7	30
40	49.97	.932	.608	.127	.997	8,936.3	4,885.7	40
50	49.89	78.068	.757	.083	23.025	8,965.0	4,909.9	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
115° 00′	49.82	78.206	42.905	81.040	23.053	8,993.8	4,934.1	115° 00′
10	49.75	.344	43.056	83.996	.080	9,022.7	4,958.6	10
20	49.68	.482	.206	.951	.108	9,051.7	4,983 1	20
30	49 61	.622	.357	.907	.136	9,080 9	5,007.8	30
40	49.54	.762	.510	.863	.163	9,110 3	5,032.6	40
50	49.46	.903	.663	.819	.191	9,139 8	5,057.6	50
116° 00	49.39	79.045	.816	.775	.219	9,169.4	5,082.7	116° 00
10	49.32	.188	.971	.731	.246	9,199 1	5,107.9	10
20	49.25	.331	44.126	.686	.274	9,229 0	5,133.3	20
30	49 18	.475	.281	.642	.301	9,259.0	5,158.8	30
40	49.11	.621	.438	.598	.329	9,289 2	5,184.5	40
50	49.04	.766	.596	.553	.356	9,319.5	5,210 3	50
117° 00	48.97	.913	.753	.509	.384	9,349.9	5,236.2	117° 00
10	48 90	80.060	.913	.464	.411	9,380.5	5,262.3	10
20	48.83	.209	45.073	.419	.438	9,411.3	5,288 6	20
30	48.76	.358	.233	.375	.466	9,442.2	5,315 0	30
40	48.69	.508	.395	.330	.493	9,473.2	5,341 5	40
50	48.62	.659	.558	.285	.520	9,504.4	5,368.2	50
118° 00	48.56	.810	.720	.241	.548	9,535.7	5,395.1	118° 00
10	48 49	.963	.885	.196	.575	9,567.2	5,422.1	10
20	48.42	81.116	46.050	.151	.602	9,598.9	5,449.2	20
30	48.35	.271	.215	.106	.629	9,630.7	5,476.5	30
40	48.28	.426	.382	.061	.656	9,662.6	5,504.0	40
50	48.22	.582	.550	.016	.684	9,694.7	5,531.7	50
119° 00	48.15	.739	.718	82.971	.711	9,727.0	5,559.4	119° 00
10	48.08	.895	.888	.926	.738	9,759.4	5,587.4	10
20	48.01	82.055	47.058	.880	.765	9,792.0	5,615 5	20
30	47.95	.215	.228	.835	.792	9,824.8	5,643 8	30
40	47.88	.375	.401	.790	.819	9,857 7	5,672.3	40
50	47.81	.537	.574	.745	.846	9,890.8	5,700.9	50
120° 00	47.75	.699	.746	.699	.873	9,924.0	5,729.7	120° 00
10	47.68	.863	.922	.654	.900	9,957.5	5,758.6	10
20	47.61	83.027	48.098	.608	.927	9,991.0	5,787.7	20
30	47.55	.192	.274	.563	.954	10,025	5,817 0	30
40	47.48	.359	.452	.517	.981	10,059	5,846.5	40
50	47.42	.530	.630	.472	24.008	10,093	5,876.1	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
121° 00′	47.35	83.694	48 809	82 426	24 035	10,127	5,906 0	121° 00′
10	47.29	.864	.990	.380	.061	10,161	5,936 0	10
20	47.22	84.034	49.172	.334	.088	10,196	5,966 2	20
30	47.16	.205	.353	.288	.115	10,231	5,996.4	30
40	47 09	.377	538	.243	.142	10,266	6,027 1	40
50	47.03	.551	.722	.197	.169	10,301	6,057 8	50
122° 00	46.96	.725	.907	.151	.195	10,336	6,088 6	122° 00
10	46.90	.900	50.094	.105	.222	10,372	6,119 8	10
20	46 84	85.077	.282	.059	.248	10,408	6,151 1	20
30	46 77	.255	.469	.012	.275	10,444	6,182.5	30
40	46 71	.433	.660	81.966	.302	10,480	6,214.3	40
50	46 64	.612	.851	.920	.328	10,516	6,246 2	50
123° 00	46.58	.793	51.042	.874	.355	10,553	6,278.1	123° 00
10	46 52	.975	.236	.828	.381	10,589	6,310 6	10
20	46.46	86.158	.430	.781	.408	10,626	6,343 0	20
30	46 39	.342	.624	.735	.434	10,663	6,375 6	30
40	46.33	.527	.821	.688	.461	10,701	6,408 6	40
50	46.27	.714	52.018	.642	.487	10,738	6,441 6	50
124° 00	46.21	.901	.216	.595	.513	10,776	6,474 7	124° 00
10	46.14	87.090	.416	.549	.540	10,814	6,508 3	10
20	46.08	.280	.617	.502	.566	10,852	6,542 0	20
30	46.02	.471	.818	.455	.593	10,890	6,575 8	30
40	45.96	.663	53.022	.409	.619	10,929	6,610.1	40
50	.90	.857	.227	.362	.645	10,967	6,644 5	50
125° 00	.84	88.051	.431	.315	.672	11,006	6,678 9	125° 00
10	.78	.247	.639	.268	.698	11,046	6,713 8	10
20	.71	.444	.847	.221	.724	11,085	6,748 9	20
30	.65	.643	54.055	.174	.750	11,125	6,783.9	30
40	.59	.842	.266	.127	.776	11,165	6,819 4	40
50	.53	89.043	.478	.080	.802	11,205	6,855.1	50
126° 00	.47	.246	.690	.033	.829	11,245	6,890.9	126° 00
10	.41	.449	.905	80.936	.855	11,286	6,927.2	10
20	.35	.654	55.121	.939	.881	11,326	6,963 6	20
30	.29	.860	.336	.891	.907	11,367	7,000.0	30
40	.23	90.067	.556	.844	.933	11,409	7,037.1	40
50	.17	.276	.775	.797	.959	11,450	7,074.1	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
127° 00′	45.11	90 486	55.995	80.750	24 985	11,492	7,111.3	127° 00′
10	.06	.698	56 218	.702	25.011	11,534	7,149 1	10
20	.00	.911	.442	.654	.036	11,576	7,186.9	20
30	44.94	91.125	.665	.607	.062	11,618	7,224 8	30
40	.88	.341	.893	.559	.088	11,661	7,263 4	40
50	.82	.558	57.121	.512	.114	11,704	7,301 9	50
128° 00	76	.776	.348	.464	.140	11,747	7,340 6	128° 00
10	.70	.997	.580	.416	.165	11,791	7,379 9	10
20	.65	92.218	.812	.368	.191	11,835	7,419.2	20
30	.59	.441	58.044	.321	.216	11,879	7,458.7	30
40	.53	.665	.281	.273	.242	11,923	7,498 8	40
50	.47	.891	.517	.225	.267	11,967	7,538 9	50
129° 00	.42	93.119	.754	.177	.293	12,012	7,579 2	129° 00
10	.36	.348	.995	.129	.319	12,057	7,620 2	10
20	.30	.578	59.236	.081	344	12,103	7,661.2	20
30	.24	.810	.476	.033	.370	12,148	7,702 1	30
40	.19	94.044	.722	79.985	.396	12,194	7,744.0	40
50	.13	.280	.968	.937	.422	12,241	7,785 8	50
130° 00	.07	.516	60.214	.889	.447	12,287	7,827 7	130° 00
10	.02	.755	.464	.840	.473	12,334	7,870 4	10
20	43.96	.995	.715	.792	.498	12,381	7,913 2	20
30	.90	95.237	.965	.743	.524	12,428	7,955.9	30
40	.85	.480	61.221	.695	.549	12,476	8,000 0	40
50	.79	.726	.476	.647	.574	12,524	8,043.1	50
131° 00	.74	.973	.732	.598	.600	12,572	8,086.9	131° 00
10	.68	96.221	.992	.550	.625	12,621	8,131 3	10
20	.63	.472	62.253	.501	.650	12,670	8,175.9	20
30	57	.724	.514	.452	.675	12,719	8,220.6	30
40	.52	.978	.780	.404	.701	12,769	8,266.1	40
50	.46	97.234	63.046	.355	.726	12,819	8,311.5	50
132° 00	.41	.491	.312	.307	.751	12,869	8,357.1	132° 00
10	.35	.751	.583	.258	.776	12,919	8,403.6	10
20	.30	98.012	.854	.209	.801	12,970	8,450.0	20
30	.24	.275	64.126	.160	.826	13,021	8,496.7	30
40	.19	.540	.403	.111	.851	13,073	8,544.2	40
50	.13	.807	.680	.062	.877	13,125	8,591.6	50

TABLE 1.—FUNCTIONS OF CIRCULAR CURVES—(*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 feet						Tangents and externals for a 1° curve		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
133° 00′	43 08	99 076	64 957	79.013	25 903	13,177	8,639.3	133° 00′
10	.02	.347	65 240	78.964	.927	13,230	8,687 8	10
20	42.97	.620	.523	.915	.951	13,283	8,736.4	20
30	.92	.895	.806	.865	.976	13,336	8,785.1	30
40	.86	100.172	66.095	.816	26.001	13,390	8,834.7	40
50	81	.322	.384	.767	.026	13,426	8,884.4	50
134° 00	.76	.732	.673	.718	.051	13,498	8,934.2	134° 00
10	.70	101.015	.968	.669	.076	13,553	8,985.0	10
20	.65	.300	67.263	.619	.101	13,608	9,035.8	20
30	.60	.587	.558	.570	.125	13,663	9,086.6	30
40	.55	.877	.860	.520	.150	13,719	9,138.5	40
50	.49	102.169	68.162	.471	.175	13,776	9,190.5	50
135° 00	.44	.462	.463	.421	.200	13,832	9,242.5	135° 00
10	39	759	771	.372	.224	13,890	9,295.6	10
20	.34	103 057	69 080	.322	.249	13,947	9,349.0	20
30	.28	.358	.388	.272	.274	14,005	9,402.1	30
40	.23	.661	.703	.223	.298	14,063	9,456.4	40
50	18	.966	70.018	.173	.323	14,122	9,510.8	50
136° 00	.13	104.274	.334	.123	.347	14,181	9,565.4	136° 00
10	.08	.584	.656	.073	.372	14,241	9,621.0	10
20	.03	.896	.978	.023	.396	14,301	9,676.6	20
30	41 97	105.211	71.300	77.973	.421	14,361	9,732.5	30
40	.92	.528	.630	.923	.445	14,422	9,789 5	40
50	.87	.848	.959	.873	.470	14,483	9,846.4	50
137° 00	.82	106.171	72.289	.823	.494	14,545	9,903.6	137° 00
10	.77	.496	.626	.773	.519	14,608	9,961 9	10
20	.72	.823	.964	.723	.543	14,670	10,020	20
30	.67	107.153	73.301	.673	.567	14,734	10,079	30
40	.62	.486	.646	.623	.591	14,797	10,139	40
50	.57	.822	.991	.572	.615	14,844	10,198	50
138° 00	.52	108.160	74.336	.522	.640	14,926	10,258	138° 00
10	.47	.501	.690	.472	.664	14,992	10,320	10
20	.42	.844	75.043	.421	.688	15,057	10,381	20
30	.37	109.191	.396	.371	.712	15,123	10,442	30
40	.32	.541	.758	.320	.736	15,190	10,505	40
50	.27	.893	76.120	.270	.760	15,257	10,568	50

TABLE 1A.—CORRECTIONS FOR TANGENT DISTANCES IN TABLE 1

Applying only to the "Chord" Definition.

Add after dividing (T for 1° curve) by D.

Δ	D =												Δ
	2°	3°	4°	5°	6°	7°	8°	9°	10°	15°	20°	25°	
10°	.01	.02	.02	.03	.04	.04	.05	.06	.06	.09	.13	.16	10°
12	.01	.02	.03	.04	.04	.05	.06	.07	.08	.1[illegible]	.15	.19	12
14	.01	.02	.03	.04	.05	.06	.07	.08	.09	.13	.17	.22	14
16	.02	.03	.04	.05	.06	.07	.08	.09	.10	.15	.20	.25	16
18	.02	.03	.04	.06	.07	.08	.09	.10	.11	.17	.23	.28	18
20	.02	.03	.05	.06	.07	.09	.10	.11	.13	.19	.26	32	20
22	.02	.04	.05	.07	.08	.10	.11	.13	.14	.21	.29	.35	22
24	.02	.04	.06	.07	.09	.11	.12	.14	.15	.23	.32	.38	24
26	.03	.04	.06	.08	.10	12	.13	.15	.17	.25	.35	.42	26
28	.03	.05	.07	.09	.11	.12	.14	.16	.18	.27	.37	.45	28
30	.03	.05	.07	.09	.11	.13	.15	.17	.19	.29	.40	.49	30
32	.03	.06	.08	.10	.12	.14	.16	.19	.21	31	.43	.53	32
34	.03	.06	.08	.11	.13	.15	.18	.20	.22	33	.46	.56	34
36	.04	.06	.09	.11	.14	.16	.19	.21	.23	.35	48	59	36
38	.04	.07	.09	.12	.15	.17	.20	.22	.25	38	.51	.63	38
40	.04	.07	.10	.13	.15	.18	.21	.24	.26	.40	.53	.67	40
42	04	.07	.10	.13	.16	.19	.22	.25	.28	.42	.56	.70	42
44	.04	08	.11	.14	.17	.20	.23	26	.29	.44	.59	.74	44
46	.05	.08	.12	.15	.18	.21	.24	.27	.31	.46	.62	.77	46
48	.05	.09	.12	.16	.19	.22	.26	.29	.32	.49	65	.81	48
50	.05	.09	.13	.16	.20	.23	27	.30	.34	.51	.68	.85	50
52	.05	.09	.13	.17	.21	.24	.28	.32	.35	.53	.71	.89	52
54	.06	.10	.14	.18	.22	.25	.29	.33	.37	.56	.74	.93	54
56	.06	.10	.15	.19	.23	.27	.30	.34	.38	.58	.77	.97	56
58	.06	.11	.15	.19	.24	.28	.32	.36	.40	.61	.80	1.01	58
60	.06	.11	.16	.20	.24	.29	33	.37	.42	.63	.84	1.05	60
62	.07	.12	.16	.21	.25	.30	.34	.39	.43	.65	.88	1.10	62
64	.07	.12	.17	.22	.27	.31	.36	.40	.45	68	.91	1.14	64
66	.07	13	.18	.23	.28	.32	.37	.42	.47	.71	.95	1.19	66
68	.07	.13	.18	.24	.29	.34	.39	.44	.49	.74	.98	1.23	68
70	.08	.14	.19	.24	.30	.35	.40	.45	.50	.76	1.02	1.28	70
72	08	.14	.20	.25	.31	.36	.42	.47	.52	.79	1.06	1.33	72
74	.08	.15	.21	.26	.32	.38	.43	.49	.54	.82	1.10	1.38	74
76	.09	.15	.21	.27	.33	.39	.45	.51	.56	.85	1.14	1.43	76
78	.09	.16	.22	.28	.34	.40	.46	.52	.58	.88	1.18	1.48	78
80	.09	.16	.23	.29	.36	.42	.48	.54	.60	.91	1.22	1.53	80
82	.09	.17	.24	.30	.37	.43	.50	.56	.63	.95	1 26	1.59	82
84	.10	.17	.25	.31	.38	.45	.52	.58	.65	.98	1.31	1.65	84
86	.10	.18	.25	.33	.40	.47	.53	.60	.67	1.02	1.36	1.71	86
88	.11	.19	.26	.34	.41	.48	.55	.62	.70	1.05	1.40	1.77	88
90	.11	.19	.27	.35	.42	.50	.57	.65	.72	1.09	1.45	1.83	90
92	.11	.20	.28	.36	.44	.52	.59	.67	.75	1.13	1.49	1.90	92
94	.12	.21	.29	.37	.46	.53	.61	.69	.77	1.17	1.53	1.96	94
96	.12	.22	.30	.39	.47	.55	.64	.72	.80	1.21	1.61	2.03	96
98	.13	.22	.31	.40	.49	.57	.66	.74	.83	1.26	1.68	2.11	98
100	.13	.23	.33	.42	.51	.59	.68	.77	.86	1.30	1.74	2.18	100

Table 2.—Chord Lengths of Circular Arcs

Degree of curve	Chords								Degree of curve
	For arcs of								
	100′	95′	90′	85′	80′	75′	70′	60′	
1°	100′	95′	90′	85′	80′	75′	70′	60′	1°
2	100	95	90	85	80	75	70	60	2
3	99.99	94.99	89.99	85	80	75	70	60	3
4	99.98	94.98	89.98	84.99	79.99	74.99	70	60	4
5	99.97	94.97	89.98	84.98	79.98	74.99	69.99	59.99	5
6	99.95	94.96	89.97	84.97	79.98	74.98	69.98	59.99	6
7	99.94	94.95	89.96	84.96	79.96	74.97	69.98	59.99	7
8	99.92	94.93	89.94	84.95	79.96	74.97	69.97	59.98	8
9	99.90	94.91	89.93	84.94	79.95	74.96	69.96	59.98	9
10	99.87	94.89	89.91	84.92	79.94	74.95	69.96	59.97	10
	For arcs of								
	100′	60′	55′	50′	45′	40′	35′	30′	
11	99.85′	59.97′	54.97′	49.98′	44.99′	39.99′	34.99′	30′	11
12	99.82	59.96	54.97	49.98	44.98	39.99	34.99	30	12
13	99.79	59.95	54.96	49.97	44.98	39.99	34.99	29.99	13
14	99.75	59.95	54.96	49.97	44.98	39.98	34.99	29.99	14
15	99.71	59.94	54.95	49.96	44.97	39.98	34.99	29.99	15
16	99.68	59.93	54.95	49.96	44.97	39.98	34.99	29.99	16
17	99.63	59.92	54.94	49.95	44.97	39.98	34.98	29.99	17
18	99.59	59.91	54.93	49.95	44.96	39.97	34.98	29.99	18
19	99.54	59.90	54.92	49.94	44.96	39.97	34.98	29.99	19
20	99.49	59.89	54.92	49.94	44.95	39.97	34.98	29.99	20
21	99.44	59.88	54.91	49.93	44.95	39.96	34.98	29.98	21
22	99.39	59.87	54.90	49.92	44.94	39.96	34.97	29.98	22
23	99.33	59.85	54.89	49.92	44.94	39.96	34.97	29.98	23
24	99.27	59.84	54.88	49.91	44.93	39.95	34.97	29.98	24
25	99.21	59.83	54.87	49.90	44.93	39.95	34.97	29.98	25
	For arcs of								
	60′	50′	45′	40′	35′	30′	25′	20′	
26	59.81′	49.89′	44.92′	39.94′	34.96′	29.98′	24.99′	19.99′	26
27	59.80	49.88	44.92	39.94	34.96	29.98	24.99	19.99	27
28	59.79	49.88	44.91	39.94	34.96	29.97	24.98	19.99	28
29	59.77	49.87	44.90	39.93	34.95	29.97	24.98	19.99	29
30	59.75	49.86	44.90	39.93	34.95	29.97	24.98	19.99	30

TABLE 2.—CHORD LENGTHS OF CIRCULAR ARCS—(*Continued*)

Degree of curve	Chords								Degree of curve
	For arcs of								
	60′	50′	45′	40′	35′	30′	25′	20′	
31°	59.74′	49.85′	44.89′	39.92′	34.95′	29.97′	24.98′	19.99′	31°
32	59.72	49.84	44.88	39.92	34.94	29.96	24.98	19.99	32
33	59.70	49.83	44.87	39.91	34.94	29.96	24.98	19.99	33
34	59.68	49.82	44.87	39.91	34.94	29.96	24.98	19.99	34
35	59.66	49.81	44.86	39.90	34.93	29.96	24.98	19.99	35
36	59.65	49.79	44.85	39.89	34.93	29.96	24.97	19.99	36
37	59.63	49.78	44.84	39.89	34.93	29.95	24 97	19 99	37
38	59.60	49.77	44.83	39.88	34.92	29.95	24.97	19.99	38
39	59.58	49.76	44.82	39.88	34.92	29.95	24.97	19.98	39
40	59.56	49.75	44.82	39.87	34.91	29.95	24.97	19.98	40
41	59.54	49.73	44.81	39.86	34.91	29.94	24.97	19.98	41
42	59.52	49.72	44.80	39.86	34.90	29.94	24.97	19.98	42
43	59.49	49.71	44.79	39.85	34.90	29.94	24.96	19.98	43
44	59.47	49.69	44.78	39.84	34.89	29.93	24.96	19.98	44
45	59.45	49.68	44.77	39.84	34.89	29.93	24.96	19.98	45
46	59.42	49.66	44.76	39.83	34.88	29.93	24.96	19.98	46
47	59.40	49.65	44.74	39.82	34.88	29.92	24.96	19.98	47
48	59.37	49.64	44.73	39.81	34.87	29.92	24.95	19.98	48
49	59.34	49.62	44.72	39.81	34.87	29.92	24.95	19.98	49
50	59.32	49.60	44.71	39.80	34.86	29 91	24.95	19.97	50
	For arcs of								
	25′	22′	20′	18′	16′	14′	12′	10′	
51	24.95	21.96′	19.97′	17.98′	15.99′	13.99′	12′	10′	51
52	24.95	21.96	19.97	17.98	15.99	13.99	11.99	10	52
53	24.94	21.96	19.97	17.98	15.99	13.99	11.99	10	53
54	24.94	21.96	19.97	17.98	15.99	13.99	11.99	10	54
55	24.94	21.96	19.97	17.98	15.98	13.99	11.99	10	55
56	24.94	21.96	19.97	17.98	15.98	13.99	11.99	10	56
57	24.94	21.96	19.97	17.98	15.98	13.99	11.99	10	57
58	24.94	21.95	19.97	17.98	15.98	13.99	11.99	10	58
59	24.94	21.95	19.96	17.97	15.98	13.99	11.99	10	59
60	24.93	21.95	19.96	17.97	15.98	13.99	11.99	10	60
62	24.92	21.95	19.96	17.97	15.98	13.99	11.99	10	62
64	24.92	21.94	19.96	17.97	15.98	13.99	11.99	10	64
66	24.91	21.94	19.96	17.97	15.98	13.98	11.99	10	66
68	24.91	21.94	19.95	17.97	15.98	13.98	11.99	10	68
70	24.90	21.93	19.95	17.96	15.97	13.98	11.99	9.99	70

TABLE 2.—CHORD LENGTHS OF CIRCULAR ARCS—(*Continued*)

Degree of curve	Chords: For arcs of 25′	22′	20′	18′	16′	14′	12′	10′	Degree of curve
72°	24.90′	21.93′	19.95′	17.96′	15.97′	13.98′	11.99	9.99′	72°
74	24.89	21.93	19.94	17.96	15.97	13.98	11.99	9.99	74
76	24.89	21.92	19.94	17.96	15.97	13.98	11.99	9.99	76
78	24.88	21.92	19.94	17.96	15.97	13.98	11.99	9.99	78
80	24.87	21.91	19.94	17.95	15.97	13.98	11.99	9.99	80
82	24.87	21.91	19.93	17.95	15.97	13.98	11.98	9.99	82
84	24.86	21.90	19.93	17.95	15.96	13.98	11.98	9.99	84
86	24.85	21.90	19.93	17.95	15.96	13.97	11.98	9.99	86
88	24.85	21.90	19.92	17.94	15.96	13.97	11.98	9.99	88
90	24.84	21.89	19.92	17.94	15.96	13.97	11.98	9.99	90
92	24.83	21.89	19.91	17.94	15.96	13.97	11.98	9.99	92
94	24.83	21.88	19.91	17.93	15.95	13.97	11.98	9.99	94
96	24.82	21.88	19.91	17.93	15.95	13.97	11.98	9.99	96
98	24.81	21.87	19.90	17.93	15.95	13.97	11.98	9.99	98
100	24.80	21.87	19.90	17.93	15.95	13.97	11.98	9.99	100
102	24.79	21.86	19.89	17.92	15.95	13.96	11.98	9.99	102
104	24.79	21.85	19.89	17.92	15.94	13.96	11.98	9.99	104
106	24.78	21.85	19.89	17.92	15.94	13.96	11.98	9.99	106
108	24.77	21.84	19.88	17.91	15.94	13.96	11.97	9.99	108
110	24.76	21.84	19.88	17.91	15.94	13.96	11.97	9.98	110
112	24.75	21.83	19.87	17.91	15.93	13.96	11.97	9.98	112
114	24.74	21.83	19.87	17.90	15.93	13.95	11.97	9.98	114
116	24.73	21.82	19.86	17.90	15.93	13.95	11.97	9.98	116
118	24.72	21.81	19.86	17.90	15.93	13.95	11.97	9.98	118
120	24.72	21.81	19.85	17.89	15.93	13.95	11.97	9.98	120
122	24.71	21.80	19.85	17.89	15.92	13.95	11.97	9.98	122
124	24.70	21.79	19.84	17.89	15.92	13.95	11.97	9.98	124
126	24.69	21.79	19.84	17.88	15.92	13.94	11.97	9.98	126
128	24.68	21.78	19.83	17.88	15.91	13.94	11.96	9.98	128
130	24.67	21.77	19.83	17.88	15.91	13.94	11.96	9.98	130
132	24.66	21.77	19.82	17.87	15.91	13.94	11.96	9.98	132
134	24.65	21.76	19.82	17.87	15.91	13.94	11.96	9.98	134
136	24.63	21.75	19.81	17.86	15.90	13.94	11.96	9.98	136
138	24.62	21.74	19.81	17.86	15.90	13.93	11.96	9.98	138
140	24.61	21.74	19.80	17.86	15.90	13.93	11.96	9.98	140
142	24.60	21.73	19.80	17.85	15.90	13.93	11.96	9.97	142
144	24.59	21.72	19.79	17.85	15.89	13.93	11.95	9.97	144
146	24.58	21.71	19.78	17.84	15.89	13.93	11.95	9.97	146
148	24.57	21.71	19.78	17.84	15.89	13.92	11.95	9.97	148
150	24.56	21.70	19.77	17.83	15.88	13.92	11.95	9.97	150

TABLE 3.—RADII

Deg. D	Radius R (ft.) Arc definition	Radius R (ft.) Chord definition	Deg. D	Radius R (ft.) Arc definition	Radius R (ft.) Chord definition	Deg. D	Radius R (ft.) Arc definition	Radius R (ft.) Chord definition
0° 0′	∞	∞	1° 0′	**5,729.58**	5,729.65	2° 0′	**2,864.79**	2,864.93
1	**343,774.7**	343,744.7	1	**5,635.65**	5,635.72	1	**2,841.11**	2,841.26
2	**171,887.3**	171,887.3	2	**5,544.75**	5,544.83	2	**2,817.83**	2,817.97
3	**114,591.6**	114,591.6	3	**5,456.74**	5,456.82	3	**2,794.92**	2,795.06
4	**85,943.7**	85,943.7	4	**5,371.48**	5,371.56	4	**2,772.38**	2,772.53
5	**68,754.9**	68,754.9	5	**5,288.84**	5,288.92	5	**2,750.20**	2,750.35
6	**57,295.8**	57,295.8	6	**5,208.71**	5,208.79	6	**2,728.37**	2,728.52
7	**49,110.7**	49,110.7	7	**5,130.97**	5,131.05	7	**2,706.89**	2,707.04
8	**42,971.8**	42,971.8	8	**5,055.51**	5,055.59	8	**2,685.74**	2,685.89
9	**38,197.2**	38,197.2	9	**4,982.24**	4,982.33	9	**2,664.92**	2,655.08
10	**34,377.5**	34,377.5	10	**4,911.07**	4,911.15	10	**2,644.42**	2,644.58
11	**31,252.2**	31,252.3	11	**4,841.90**	4,841.98	11	**2,624.23**	2,624.39
12	**28,647.8**	28,647.9	12	**4,774.65**	4,774.74	12	**2,604.35**	2,604.51
13	**26,444.2**	26,444.2	13	**4,709.24**	4,709.33	13	**2,584.52**	2,584.93
14	**24,555.3**	24,555.4	14	**4,645.60**	4,645.69	14	**2,565.48**	2,565.65
15	**22,918.3**	22,918.3	15	**4,583.66**	4,583.75	15	**2,546.48**	2,546.64
16	**21,485.9**	21,485.9	16	**4,523.35**	4,523.44	16	**2,527.76**	2,527.92
17	**20,222.0**	20.222.1	17	**4,464.61**	4,464.70	17	**2,509.30**	2,509.47
18	**19,098.6**	19,098.6	18	**4,407.37**	4,407.46	18	**2,491.12**	2,491.29
19	**18,093.4**	18,093.4	19	**4,351.58**	4,351.67	19	**2,473.20**	2,473.37
20	**17,188.8**	17,188.8	20	**4,297.18**	4,297.28	20	**2,455.53**	2,455.70
21	**16,370.2**	16,370.3	21	**4,244,13**	4,244.23	21	**2,438.12**	2,438.29
22	**15,626.1**	15,626.2	22	**4,192.37**	4,192.47	22	**2,420.95**	2,421.12
23	**14,946.7**	14,946.8	23	**4,141.86**	4,141.96	23	**2,404.02**	2,404.19
24	**14,324.0**	14,324.0	24	**4,092.56**	4,092.66	24	**2,387.32**	2,387.50
25	**13,751.0**	13,751.0	25	**4,044.41**	4,044.51	25	**2,370.86**	2,371.04
26	**13,222.1**	13,222.1	26	**3,997.38**	3,997.48	26	**2,354.62**	2,354.80
27	**12,732.4**	12,732.4	27	**3,951.43**	3,951.54	27	**2,338.60**	2,338.78
28	**12,277.7**	12,277.7	28	**3,906.53**	3,906.64	28	**2,322.80**	2,322.98
29	**11,854.3**	11,854.3	29	**3,862.64**	3,862.64	29	**2,307.21**	2,307.39
30	**11,459.2**	11,459.2	30	**3,819.71**	3,819.83	30	**2,291.83**	2,292.01
31	**11,089.5**	11,089.5	31	**3,777.74**	3,777.85	31	**2,276.65**	2,276.84
32	**10,743.0**	10,743.0	32	**3,736.68**	3,736.79	32	**2,261.68**	2,261.86
33	**10,417.4**	10,417.5	33	**3,696.50**	3,696.61	33	**2,246.89**	2,247.08
34	**10,111.0**	10.111.1	34	**3,657.18**	3,657.29	34	**2,232.30**	2,232.49
35	**9,822.13**	9,822.18	35	**3,618.68**	3,618.80	35	**2,217.90**	2,218.09
36	**9,549.30**	9,549.34	36	**3,580.99**	3,581.10	36	**2,203.68**	2,203.87
37	**9,291.21**	9,291.25	37	**3,544.07**	3,544.19	37	**2,189.65**	2,189.84
38	**9,046.70**	9,046.75	38	**3,507.91**	3,508.02	38	**2,175.79**	2,175.98
39	**8,814.74**	8,814.78	39	**3,472.47**	3,472.59	39	**2,162.10**	2,162.30
40	**8,594.37**	8,594.42	40	**3,437.75**	3,437.87	40	**2,148.59**	2,148.79
41	**8,384.75**	8,384.80	41	**3,403.71**	3,403.83	41	**2,135.25**	2,135.44
42	**8,185.11**	8,185.16	42	**3,370.34**	3,370.46	42	**2,122.07**	2,122.26
43	**7,994.76**	7,994.81	43	**3,337.62**	3,337.74	43	**2,109.05**	2,109.24
44	**7,813.06**	7,813.11	44	**3,305.53**	3,305.65	44	**2,096.19**	2,096.39
45	**7,639.44**	7,639.49	45	**3,274.05**	3,274.17	45	**2,083.48**	2,083.68
46	**7,473.36**	7,473.42	46	**3,243.16**	3,243.29	46	**2,070.93**	2,071.13
47	**7,314.35**	7,314.41	47	**3,212.85**	3,212.98	47	**2,058.53**	2,058.73
48	**7,161.97**	7,162.03	48	**3,183.10**	3,183.23	48	**2,046.28**	2,046.48
49	**7,015.81**	7,015.87	49	**3,153.90**	3,154.03	49	**2,034.17**	2,034.37
50	**6,875.50**	6,875.55	50	**3,125.22**	3,125.36	50	**2,022.20**	2,022.41
51	**6,740.68**	6,740.74	51	**3,097.07**	3,097.20	51	**2,010.38**	2,010.59
52	**6,611.05**	6,611.12	52	**3,069.42**	3,069.55	52	**1,998.69**	1,998.90
53	**6,486.31**	6,486.38	53	**3,042.25**	3,042.39	53	**1,987.14**	1,987.35
54	**6,366.20**	6,366.26	54	**3,015.57**	3,015.71	54	**1,975.71**	1,975.93
55	**6,250.45**	6,250.51	55	**2,989.35**	2,989.48	55	**1,964.43**	1,964.64
56	**6,138.83**	6,138.90	56	**2,963.58**	2,963.72	56	**1,953.27**	1,953.48
57	**6,031.13**	6,031.20	57	**2,938.25**	2,938.39	57	**1,942.23**	1,942.44
58	**5,927.15**	5,927.22	58	**2,913.34**	2,913.49	58	**1,931.32**	1,931.53
59	**5,826.69**	5,826.76	59	**2,888.86**	2.889.01	59	**1,920.53**	1,920.75
60	**5,729.58**	5,729.65	60	**2,864.79**	2,864.93	60	**1,909.86**	1,910.08

TABLE 3.—RADII—(*Continued*)

Deg. D	Radius R (ft.) Arc definition	Radius R (ft.) Chord definition	Deg. D	Radius R (ft.) Arc definition	Radius R (ft.) Chord definition	Deg. D	Radius R (ft.) Arc definition	Radius R (ft.) Chord definition
3° 0′	**1,909.86**	1,910.08	4° 0′	**1,432.29**	1,432.69	5° 0′	**1,145.92**	1,146.28
1	**1,899.31**	1,899.53	1	**1,426.45**	1,426.74	1	**1,142.11**	1,142.47
2	**1,888.86**	1,889 09	2	**1,420.56**	1,420.85	2	**1,138.31**	1,138.69
3	**1,878.55**	1,878.77	3	**1,414.71**	1,415.01	3	**1,134.57**	1,134.94
4	**1,868.33**	1,868.56	4	**1,408.91**	1,409.21	4	**1,130.84**	1,131.21
5	**1,858.24**	1,858.47	5	**1,403.16**	1,403.46	5	**1,127.13**	1,127.50
6	**1,848.25**	1,848.48	6	**1,397.46**	1,397.76	6	**1,123.45**	1,123.82
7	**1,838.37**	1,838.59	7	**1,391.80**	1,392.10	7	**1,119.79**	1,120.16
8	**1,828.59**	1,828.82	8	**1,386.19**	1,386.49	8	**1,116.15**	1,116.52
9	**1,818.91**	1,819.14	9	**1,380.62**	1,380.92	9	**1,112.54**	1,112.91
10	**1,809.34**	1,809.57	10	**1,375.10**	1,375.40	10	**1,108.95**	1,109.33
11	**1,799.87**	1,800.10	11	**1,369.62**	1,369.92	11	**1,105.39**	1,105.76
12	**1,790.49**	1,790.73	12	**1,364.19**	1,364.49	12	**1,101.84**	1,102.22
13	**1,781.22**	1,781.45	13	**1,358.79**	1,359.10	13	**1,098.32**	1,098.70
14	**1,772.03**	1,772.27	14	**1,353.44**	1,353.75	14	**1,094.82**	1,095.20
15	**1,762.95**	1,763.18	15	**1,348.14**	1,348.45	15	**1,091.35**	1,091.73
16	**1,753.95**	1,754.19	16	**1,342.87**	1,343.18	16	**1,087.89**	1,088.28
17	**1,745.05**	1,745.29	17	**1,337.65**	1,337.96	17	**1,084.46**	1,084.85
18	**1,736.24**	1,736.48	18	**1,332.46**	1,332.77	18	**1,081.05**	1,081.44
19	**1,727.51**	1,727.75	19	**1,327.32**	1,327.63	19	**1,077.66**	1,078.05
20	**1,718.87**	1,719.12	20	**1,322.21**	1,322.53	20	**1,074.30**	1,074.68
21	**1,710.32**	1,710.57	21	**1,317.14**	1,317.46	21	**1,070.95**	1,071.34
22	**1,701.86**	1,702.10	22	**1,312.12**	1,312.43	22	**1,067.62**	1,068.01
23	**1,693.47**	1,693.72	23	**1,307.13**	1,307.45	23	**1,064.32**	1,064.71
24	**1,685.17**	1,685.42	24	**1,302.08**	1,302.50	24	**1,061.03**	1,061.43
25	**1,676.95**	1,677.20	25	**1,297.26**	1,297.58	25	**1,057.77**	1,058.16
26	**1,668.81**	2,669.06	26	**1,292.39**	1,292.71	26	**1,054.52**	1,054.92
27	**1,660.75**	1,661.00	27	**1,287.55**	1,287.87	27	**1,051.30**	1,051.70
28	**1,652.76**	1,653.02	28	**1,282.74**	1,283.07	28	**1,048.09**	1,048.49
29	**1,644.85**	1,645.11	29	**1,277.97**	1,278.30	29	**1,044.91**	1,045.31
30	**1,637.02**	1,637.28	30	**1,273.24**	1,273.57	30	**1,041.74**	1,042.14
31	**1,629.36**	1,629.52	31	**1,268.45**	1,268.87	31	**1,038.59**	1,039.00
32	**1,621.58**	1,621.84	32	**1,263.88**	1,264.21	32	**1,035.47**	1,035.87
33	**1,613.94**	1,614.22	33	**1,259.25**	1,259.58	33	**1,032.36**	1,032.76
34	**1,606.42**	1,606.68	34	**1,254.65**	1,254.98	34	**1,029.27**	1,029.67
35	**1,598.95**	1,599.21	35	**1,250.09**	1,250.42	35	**1,026.19**	1,026.60
36	**1,591.55**	1,591.81	36	**1,245.56**	1,245.89	36	**1,023.14**	1,023.55
37	**1,584.22**	1,584.48	37	**1,241.06**	1,241.40	37	**1,020.10**	1,020.51
38	**1,576.95**	1,577.21	38	**1,236.60**	1,236.94	38	**1,017.09**	1,017.49
39	**1,569.75**	1,570.01	39	**1,232.17**	1,232.51	39	**1,014.08**	1,014.50
40	**1,562.61**	1,562.88	40	**1,227.77**	1,228.11	40	**1,011.10**	1,011.51
41	**1,555.54**	1.555.81	41	**1,223.40**	1,223.74	41	**1,008.14**	1,008.55
42	**1,548.53**	1,548.80	42	**1,219.06**	1,219.40	42	**1,005.19**	1,005.60
43	**1,541.59**	1,541.86	43	**1,214.75**	1,215.09	43	**1,002.26**	1,002.67
44	**1,534.71**	1,534.98	44	**1,210.47**	1,210.82	44	**999.35**	999.76
45	**1,527.89**	1,528.16	45	**1,206.23**	1,206.57	45	**996.45**	996.87
46	**1,521.13**	1,521.40	46	**1,202.01**	1,202.36	46	**993.57**	993.99
47	**1,514.43**	1,514.70	47	**1,197.82**	1,198.17	47	**990.71**	991.13
48	**1,507.78**	1,508.06	48	**1,193.66**	1,194.01	48	**987.86**	988.28
49	**1,501.20**	1,501.48	49	**1,189.53**	1,189.88	49	**985.03**	985.45
50	**1,494.67**	1,494.95	50	**1,185.43**	1,185.78	50	**982.21**	982.64
51	**1,488.20**	1,488.48	51	**1,181.36**	1,181.71	51	**979.42**	979.84
52	**1,481.79**	1,482.07	52	**1,177.31**	1,177.66	52	**976.63**	977.06
53	**1,475.43**	1,475.71	53	**1,173.29**	1,173.65	53	**973.87**	974.29
54	**1,469.12**	1,469.41	54	**1,169.30**	1,169.66	54	**971.12**	971.54
55	**1,462.87**	1,463.16	55	**1,165.34**	1,165.70	55	**968.38**	968.81
56	**1,456.67**	1,456.96	56	**1,161.40**	1,161.76	56	**965.66**	966.09
57	**1,450.53**	1,450.81	57	**1,157.49**	1,157.85	57	**962.95**	963.39
58	**1,444.43**	1,444.72	58	**1,153.61**	1,153.97	58	**960.26**	960.70
59	**1,438.39**	1,438.68	59	**1,149.75**	1,150.11	59	**957.59**	958.02
60	**1,432.39**	1,432.69	60	**1,145.92**	1,146.28	60	**954.93**	955.37

TABLE 3.—RADII—(*Continued*)

Deg. D	Radius R (ft.) Arc definition	Radius R (ft.) Chord definition	Deg. D	Radius R (ft.) Arc definition	Radius R (ft.) Chord definition	Deg. D	Radius R (ft.) Arc definition	Radius R (ft.) Chord definition
6° 0′	**954.93**	955.37	7° 0′	**818.51**	819.02	8° 0′	**716.20**	716.78
1	**952.28**	952.72	1	**816.57**	817.08	1	**714.71**	715.29
2	**949.65**	950.09	2	**814.63**	815.14	2	**713.23**	713.81
3	**947.04**	947.48	3	**812.70**	813.22	3	**711.75**	712.33
4	**944.44**	944.88	4	**810.79**	811.30	4	**710.28**	710.87
5	**941.85**	942.29	5	**808.88**	809.40	5	**708.81**	709.40
6	**939.28**	939.72	6	**806.98**	806.50	6	**707.35**	707.94
7	**936.72**	937.16	7	**805.09**	805.61	7	**705.90**	706.49
8	**934.17**	934.62	8	**803.21**	803.73	8	**704.46**	705.05
9	**931.64**	932.09	9	**801.34**	801.86	9	**703.02**	703.61
10	**929.12**	929.57	10	**799.48**	800.00	10	**701.58**	702.18
11	**926.62**	927.07	11	**797.62**	798.14	11	**700.16**	700.75
12	**924.13**	924.58	12	**795.78**	796.30	12	**698.73**	699.33
13	**921.65**	922.10	13	**793.94**	794.46	13	**697.31**	697.91
14	**919.18**	919.64	14	**792.11**	792.63	14	**695.90**	696.50
15	**916.73**	917.19	15	**790.29**	790.81	15	**694.49**	695.09
16	**914.29**	914.75	16	**788.47**	789.00	16	**693.09**	693.70
17	**911.87**	912.33	17	**786.67**	787.20	17	**691.70**	692.30
18	**909.46**	909.92	18	**784.87**	785.40	18	**690.31**	690.91
19	**907.06**	907.52	19	**783.09**	783.62	19	**688.92**	689.53
20	**904.67**	905.13	20	**781.31**	781.84	20	**687.55**	688.16
21	**902.30**	902.76	21	**779.53**	780.07	21	**686.18**	686.78
22	**899.93**	900.40	22	**777.77**	778.31	22	**684.81**	685.42
23	**897.58**	898.05	23	**776.02**	776.55	23	**683.45**	684.06
24	**895.25**	895.71	24	**774.27**	774.81	24	**682.09**	682.70
25	**892.92**	893.39	25	**772.53**	773.07	25	**680.74**	681.35
26	**890.61**	891.08	26	**770.80**	771.34	26	**679.40**	680.01
27	**888.31**	888.78	27	**769.07**	769.61	27	**678.06**	678.67
28	**886.02**	886.49	28	**767.35**	767.90	28	**676.72**	677.34
29	**883.74**	884.21	29	**765.65**	766.19	29	**675.39**	676.01
30	**881.47**	881.95	30	**763.94**	764.49	30	**674.07**	674.69
31	**879.22**	879.69	31	**762.25**	762.80	31	**672.75**	673.37
32	**876.98**	877.45	32	**760.56**	761.11	32	**671.43**	672.06
33	**874.75**	875.22	33	**758.88**	759.43	33	**670.13**	670.75
34	**872.52**	873.00	34	**757.21**	757.76	34	**668.82**	669.45
35	**870.32**	870.79	35	**755.55**	756.10	35	**667.52**	668.15
36	**868.12**	868.60	36	**753.89**	754.44	36	**666.23**	666.86
37	**865.93**	866.41	37	**752.24**	752.80	37	**664.94**	665.57
38	**863.76**	864.24	38	**750.60**	751.16	38	**663.66**	664.29
39	**861.59**	862.07	39	**748.96**	749.52	39	**662.38**	663.01
40	**859.44**	859.92	40	**747.34**	747.89	40	**661.11**	661.74
41	**857.29**	857.78	41	**745.72**	746.27	41	**659.84**	660.47
42	**855.16**	855.65	42	**744.10**	744.66	42	**658.57**	659.21
43	**853.04**	853.53	43	**742.49**	743.06	43	**657.31**	657.95
44	**850.93**	851.42	44	**740.09**	741.46	44	**656.06**	656.69
45	**848.83**	849.32	45	**739.30**	739.87	45	**654.81**	655.45
46	**846.74**	847.23	46	**737.71**	738.28	46	**653.56**	654.20
47	**844.66**	845.15	47	**736.13**	736.70	47	**652.32**	652.96
48	**842.59**	843.08	48	**734.56**	735.13	48	**651.09**	651.73
49	**840.52**	841.02	49	**733.00**	733.56	49	**649.86**	650.50
50	**838.47**	838.97	50	**731.44**	732.01	50	**648.53**	649.27
51	**836.43**	836.93	51	**729.88**	730.45	51	**647.41**	648.05
52	**834.40**	834.90	52	**728.34**	728.91	52	**646.19**	646.84
53	**832.38**	832.88	53	**726.80**	727.37	53	**644.98**	645.63
54	**830.37**	830.88	54	**725.26**	725.84	54	**643.77**	644.42
55	**828.37**	828.88	55	**723.74**	724.31	55	**642.57**	643.22
56	**826.38**	826.89	56	**722.23**	722.79	56	**641.37**	642.02
57	**824.40**	824.91	57	**720.70**	721.28	57	**640.18**	640.83
58	**822.43**	822.93	58	**719.19**	719.77	58	**638.99**	639.64
59	**820.46**	820.97	59	**717.69**	718.27	59	**637.80**	638.45
60	**818.51**	819.02	60	**716.20**	716.78	60	**636.62**	637.27

TABLE 3.—RADII—(*Continued*)

Deg. D	Radius R (ft.) Arc definition	Chord definition	Deg. D	Radius R (ft.) Arc definition	Chord definition	Deg. D	Radius R (ft.) Arc definition	Chord definition
9° 0′	**636.62**	637.27	10° 0′	**572.96**	573.69	11° 0′	**520.87**	521.67
1	**635.44**	636.10	1	**572.00**	572.73	1	**520.01**	520.88
2	**634.27**	634.93	2	**571.05**	571.78	2	**519.30**	520.10
3	**633.10**	633.76	3	**570.11**	570.84	3	**518.52**	519.32
4	**631.94**	632.60	4	**569.16**	569.90	4	**517.73**	518.54
5	**630.78**	631.44	5	**568.22**	568.96	5	**516.95**	517.76
6	**629.62**	630.29	6	**567.29**	568.02	6	**516.18**	516.99
7	**628.47**	629.14	7	**566.35**	567.09	7	**515.40**	516.21
8	**627.33**	627.99	8	**565.42**	566.16	8	**514.63**	515.44
9	**626.18**	626.85	9	**564.49**	565.23	9	**513.86**	514.68
10	**625.04**	625.71	10	**563.56**	564.31	10	**513.10**	513.91
11	**623.91**	624.58	11	**562.64**	563.38	11	**512.33**	513.15
12	**622.78**	623.45	12	**561.72**	562.47	12	**511.57**	512.38
13	**621.65**	622.32	13	**560.81**	561.55	13	**510.81**	511.63
14	**620.53**	621.20	14	**559.89**	560.64	14	**510.05**	510.87
15	**619.41**	620.09	15	**558.98**	559.73	15	**509.30**	510.11
16	**618.30**	618.97	16	**558.08**	558.82	16	**508.54**	509.36
17	**617.19**	617.87	17	**557.17**	557.92	17	**507.79**	508.61
18	**616.08**	616.76	18	**556.27**	557.02	18	**507.04**	507.86
19	**614.98**	615.66	19	**555.37**	556.12	19	**506.30**	507.12
20	**613.88**	614.56	20	**554.48**	555.23	20	**505.55**	506.38
21	**612.79**	613.47	21	**553.58**	554.34	21	**504.81**	505.64
22	**611.70**	612.38	22	**552.69**	553.45	22	**504.07**	504.90
23	**610.61**	611.30	23	**551,81**	552.56	23	**503.33**	504.16
24	**609.53**	610.21	24	**550.92**	551.68	24	**502.59**	503.42
25	**608.45**	609.14	25	**550.04**	550.80	25	**501.86**	502.69
26	**607.38**	608.06	26	**549.16**	549.92	26	**501.13**	501.96
27	**606.30**	606.99	27	**548.28**	549.05	27	**500.40**	501.23
28	**605.24**	605.93	28	**547.41**	548.17	28	**499.67**	500.51
29	**604.17**	604.86	29	**546.54**	547.30	29	**498.95**	499.78
30	**603.11**	603.80	30	**545.67**	546.44	30	**498.22**	499.06
31	**602.06**	602.75	31	**544.81**	545.57	31	**497.50**	498.34
32	**601.00**	601.70	32	**543.95**	544.71	32	**496.78**	497.62
33	**599.96**	600.65	33	**543.09**	543.86	33	**496.07**	496.91
34	**598.91**	599.61	34	**542.23**	543.00	34	**495.35**	496.19
35	**597.87**	598.57	35	**541.38**	542.15	35	**494.64**	495.48
36	**596.83**	597.53	36	**540.53**	541.30	36	**493.93**	494.77
37	**595.80**	596.50	37	**539.68**	540.45	37	**493.22**	494.07
38	**594.77**	595.47	38	**538.83**	539.61	38	**492.51**	493.36
39	**593.74**	594.44	39	**537.99**	538.76	39	**491.81**	492.66
40	**592.72**	593.42	40	**537.15**	537.92	40	**491.11**	491.96
41	**591.69**	592.40	41	**536.31**	537.09	41	**490.41**	491.26
42	**590.68**	591.38	42	**535.47**	536.25	42	**489.71**	490.56
43	**589.68**	590.37	43	**534.64**	535.42	43	**489.01**	489.86
44	**588.67**	589.36	44	**533.81**	534.59	44	**488.32**	489.17
45	**587.66**	588.36	45	**532.99**	533.77	45	**487.62**	488.48
46	**586.65**	587.36	46	**532.16**	532.94	46	**486.93**	487.79
47	**585.65**	586.36	47	**531.34**	532.12	47	**486.24**	487.10
48	**584.66**	585.36	48	**530.52**	531.30	48	**485.56**	486.42
49	**583.66**	584.37	49	**529.70**	530.49	49	**484.87**	485.73
50	**582.67**	583.38	50	**528.88**	529.67	50	**484.19**	485.05
51	**581.68**	582.40	51	**528.07**	528.86	51	**483.51**	484.37
52	**580.70**	581.42	52	**527.26**	528.05	52	**482.83**	483.69
53	**579.72**	580.44	53	**526.45**	527.25	53	**482.15**	483.02
54	**578.75**	579.46	54	**525.65**	526.44	54	**481.48**	482.34
55	**577.77**	578.49	55	**524.85**	525.64	55	**480.80**	481.67
56	**576.80**	577.53	56	**524.05**	524.84	56	**480.13**	481.00
57	**575.84**	576.56	57	**523.25**	524.05	57	**479.46**	480.33
58	**574.87**	575.60	58	**522.45**	523.25	58	**478.79**	479.67
59	**573.91**	574.64	59	**521.66**	522.46	59	**478.13**	479.00
60	**572.96**	573.69	60	**520.87**	521.67	60	**477.46**	478.34

TABLE 3.—RADII—(*Continued*)

Deg. D	Radius R (ft.)		Deg. D	Radius R (ft.)		Deg. D	Radius R (ft.)	
	Arc definition	Chord definition		Arc definition	Chord definition		Arc definition	Chord definition
12° 0′	**477.46**	478.34	14° 0′	**409.26**	410.28	16° 0′	**358.10**	359.26
2	**476.14**	477.02	2	**408.28**	409.31	2	**357.35**	358.52
4	**474.83**	475.71	4	**407.32**	408.34	4	**356.61**	357.78
6	**473.52**	474.40	6	**406.35**	407.38	6	**355.81**	357.05
8	**472.22**	473.10	8	**405.39**	406.42	8	**355.14**	356.32
10	**470.92**	471.81	10	**404.44**	405.47	10	**354.41**	355.59
12	**469.64**	470.53	12	**403.49**	404.53	12	**353.68**	354.86
14	**468.36**	469.25	14	**402.55**	403.58	14	**352.95**	354.13
16	**467.09**	467.98	16	**401.61**	402.65	16	**352.23**	353.41
18	**465.82**	466.72	18	**400.66**	401.71	18	**351.51**	352.70
20	**464.56**	465.46	20	**399.74**	400.78	20	**350.79**	351.98
22	**463.31**	464.21	22	**398.81**	399.86	22	**350.08**	351.02
24	**462.06**	462.96	24	**397.89**	398.94	24	**349.36**	350.56
26	**460.82**	461.73	26	**396.97**	398.02	26	**348.66**	349.85
28	**459.59**	460.50	28	**396.05**	397.11	28	**347.95**	349.15
30	**458.37**	459.28	30	**395.14**	396.20	30	**347.25**	348.45
32	**457.15**	458.06	32	**394.24**	395.30	32	**346.55**	347.75
34	**455.93**	456.85	34	**393.33**	394.40	34	**345.85**	347.06
36	**454.73**	455.65	36	**392.44**	393.50	36	**345.16**	346.37
38	**453.53**	454.45	38	**391.54**	392.61	38	**344.46**	345.68
40	**452.34**	453.26	40	**390.65**	391.72	40	**343.77**	344.99
42	**451.15**	452.07	42	**389.77**	390.84	42	**343.09**	344.31
44	**449.97**	450.89	44	**388.89**	389.96	44	**342.41**	343.62
46	**448.79**	449.72	46	**388.01**	389.08	46	**341.72**	342.95
48	**447.62**	448.56	48	**387.13**	388.21	48	**341.05**	342.27
50	**446.46**	447.40	50	**386.26**	387.34	50	**340.37**	341.60
52	**445.30**	446.24	52	**385.40**	386.48	52	**339.70**	340.93
54	**444.15**	445.09	54	**384.54**	385.62	54	**339.03**	340.26
56	**443.01**	443.95	56	**383.68**	384.77	56	**338.36**	339.60
58	**441.87**	442.81	58	**382.82**	383.91	58	**337.70**	338.93
13° 0′	**440.70**	441.68	15° 0′	**381.97**	383.06	17° 0′	**337.03**	338.27
2	**439.61**	440.56	2	**381.12**	382.22	2	**336.38**	337.62
4	**438.49**	439.44	4	**380.28**	381.33	4	**335.72**	336.96
6	**437.37**	438.33	6	**379.44**	380.54	6	**335.06**	336.31
8	**436.25**	437.22	8	**378.61**	379.71	8	**334.41**	335.66
10	**435.16**	436.12	10	**377.77**	378.88	10	**333.76**	335.01
12	**434.06**	435.02	12	**376.95**	378.05	12	**333.12**	334.37
14	**432.97**	433.93	14	**376.12**	377.23	14	**332.47**	333.73
16	**431.88**	432.84	16	**375.30**	376.41	16	**331.83**	333.09
18	**430.79**	431.76	18	**374.48**	375.60	18	**331.19**	332.45
20	**429.72**	430.69	20	**373.67**	374.79	20	**330.55**	331.82
22	**428.65**	429.62	22	**372.86**	373.98	22	**329.92**	331.18
24	**427.58**	428.56	24	**372.05**	373.17	24	**329.29**	330.55
26	**426.52**	427.50	26	**371.25**	372.37	26	**328.66**	329.93
28	**425.46**	426.44	28	**370.45**	371.57	28	**328.03**	329.30
30	**424.41**	425.40	30	**369.65**	370.78	30	**327.40**	328.68
32	**423.37**	424.35	32	**368.86**	369.99	32	**326.78**	328.06
34	**422.33**	423.32	34	**368.07**	369.20	34	**326.16**	327.44
36	**421.29**	422.28	36	**367.28**	368.42	36	**325.54**	326.83
38	**420.26**	421.26	38	**366.50**	367.64	38	**324.93**	326.22
40	**419.24**	420.23	40	**365.72**	366.86	40	**324.32**	325.60
42	**418.22**	419.22	42	**364.94**	366.09	42	**323.70**	325.00
44	**417.20**	418.20	44	**364.17**	365.31	44	**323.09**	324.39
46	**416.19**	417.19	46	**363.40**	364.55	46	**322.49**	323.79
48	**415.19**	416.19	48	**362.63**	363.78	48	**321.89**	323.18
50	**414.19**	415.19	50	**361.87**	363.02	50	**321.28**	322.59
52	**413.20**	414.20	52	**361.11**	362.26	52	**320.69**	321.99
54	**412.20**	413.21	54	**360.35**	361.51	54	**320.09**	321.39
56	**411.21**	412.23	56	**359.59**	360.76	56	**319.49**	320.81
58	**410.23**	411.25	58	**358.85**	360.01	58	**318.90**	320.21
60	**409.26**	410 28	60	**358.10**	359.26	60	**318.31**	319.62

TABLE 3.—RADII—(*Continued*)

Deg. D	Radius R (ft.) Arc definition	Radius R (ft.) Chord definition
18° 0′	**318.31**	319.62
2	**317.72**	319.03
4	**317.14**	318.45
6	**316.55**	317.87
8	**315.97**	317.29
10	**315.39**	316.71
12	**314.81**	316.14
14	**314.24**	315.56
16	**313.66**	314.99
18	**313.09**	314.42
20	**312.52**	313.86
22	**311.96**	313.30
24	**311.39**	313.73
26	**310.83**	312.17
28	**310.27**	311.61
30	**309.71**	311.06
32	**309.15**	310.51
34	**308.59**	309.95
36	**308.04**	309.39
38	**307.49**	308.85
40	**306.94**	308.30
42	**306.39**	307.56
44	**305.85**	307.22
46	**305.31**	306.68
48	**304.76**	306.14
50	**304.22**	305.60
52	**303.69**	305.06
54	**303.15**	304.53
56	**302.62**	304.00
58	**302.09**	303.47
19° 0′	**301.56**	302.94
2	**301.03**	302.42
4	**300.50**	301.89
6	**299.98**	301.37
8	**299.38**	300.85
10	**298.93**	300.33
12	**298.42**	299.82
14	**297.90**	299.30
16	**297.38**	298.78
18	**296.87**	298.27
20	**296.36**	297.77
22	**295.85**	297.26
24	**295.34**	296.75
26	**294.83**	296.25
28	**294.33**	295.75
30	**293.82**	295.25
32	**293.22**	294.75
34	**292.82**	294.25
36	**292.33**	293.75
38	**291.83**	293.27
40	**291.33**	202.77
42	**290.84**	292.28
44	**290.35**	291.79
46	**289.86**	291.31
48	**289.37**	290.82
50	**288.89**	290.33
52	**288.40**	289.85
54	**287.92**	289.37
56	**287.43**	288.89
58	**286.96**	288.41
60	**286.48**	287.94

Deg. D	Radius R (ft.) Arc definition	Radius R (ft.) Chord definition
20° 0′	**286.48**	287.94
10	**284.11**	285.58
20	**281.78**	283.27
30	**279.49**	280.99
40	**277.24**	278.75
50	**275.02**	276.54
21° 0′	**272.84**	274.37
10	**270.69**	272.23
20	**268.57**	270.13
30	**266.49**	268.06
40	**264.44**	266.02
50	**262.42**	264.02
22° 0′	**260.44**	262.04
10	**258.48**	260.10
20	**256.55**	258.18
30	**254.65**	256.29
40	**252.78**	254.43
50	**250.93**	252.60
23° 0′	**249.11**	250.79
10	**247.32**	249.01
20	**245.55**	247.26
30	**243.81**	245.53
40	**242.09**	243.82
50	**240.40**	242.14
24° 0′	**238.73**	240.49
10	**237.09**	238.85
20	**235.46**	237.24
30	**333.86**	235.65
40	**232.28**	234.08
50	**230.72**	232.54
25° 0′	**229.18**	231.01
10	**227.66**	229.51
20	**226.17**	228.02
30	**224.69**	226.55
40	**223.23**	225.11
50	**221.79**	223.68
26° 00′	**220.37**	222.27
10	**218.96**	220.88
20	**217.58**	219.51
30	**216.21**	218.15
40	**214.86**	216.81
50	**213.52**	215.49
27° 0′	**212.21**	214.18
10	**210.90**	212.89
20	**209.62**	211.62
30	**208.35**	210.36
40	**207.09**	209.12
50	**205.85**	207.89
28° 0′	**204.63**	206.68
10	**203.42**	205.48
20	**202.22**	204.30
30	**201.04**	203.13
40	**199.87**	201.97
50	**198.71**	200.83
29° 0′	**197.57**	199.70
10	**196.44**	198.58
20	**195.33**	197.48
30	**194.22**	196.38
40	**193.13**	195.31
50	**192.05**	194.24

Deg. D	Radius R (ft.) Arc definition	Radius R (ft.) Chord definition
30° 0′	**190.98**	193.19
10	**189.93**	192.14
20	**188.89**	191.11
30	**187.86**	190.09
40	**186.83**	189.08
50	**185.82**	188.09
31° 0′	**184.82**	187.10
10	**183.84**	186.12
20	**182.86**	185.16
30	**181.89**	184.20
40	**180.93**	183.26
50	**179.99**	182.32
32° 0′	**179.05**	181.40
10	**178.12**	180.48
20	**177.20**	179.58
30	**176.29**	178.68
40	**175.40**	177.79
50	**174.50**	176.92
33° 0′	**173.62**	176.05
10	**172.75**	175.19
20	**171.89**	174.34
30	**171.03**	173.49
40	**170.18**	172.66
50	**169.35**	171.83
34° 0′	**168.52**	171.02
10	**167.70**	170.21
20	**166.88**	169.40
30	**166.07**	168.61
40	**165.28**	167.82
50	**164.48**	167.05
35° 0′	**163.70**	166.28
10	**162.93**	165.51
20	**162.16**	164.76
30	**161.40**	164.01
40	**160.64**	163.27
50	**159.90**	162.53
36° 0′	**159.15**	161.80
10	**158.42**	161.08
20	**157.69**	160.37
30	**156.97**	159.66
40	**156.26**	158.96
50	**155.55**	158.27
37° 0′	**154.85**	157.58
10	**154.16**	156.90
20	**153.47**	156.22
30	**152.79**	155.55
40	**152.11**	154.89
50	**151.44**	154.23
38° 0′	**150.78**	153.58
10	**150.12**	152.93
20	**149.47**	152.29
30	**148.82**	151.66
40	**148.18**	151.03
50	**147.54**	150.41
39° 0′	**146.91**	149.79
10	**146.29**	149.17
20	**145.67**	148.57
30	**145.05**	147.97
40	**144.44**	147.37
50	**143.84**	146.78

TABLE 3.—RADII—(*Continued*)

Deg. *D*	Radius *R* (ft.) Arc definition	Radius *R* (ft.) Chord definition	Deg. *D*	Radius *R* (ft.) Arc definition	Radius *R* (ft.) Chord definition	Deg. *D*	Radius *R* (ft.) Arc definition	Radius *R* (ft.) Chord definition
40° 0′	143.24	146.19	50° 0′	114.59	118.31	60° 0′	95.49	100.00
10	142.64	145.61	10	114.21	117.94	10	95.23	99.75
20	142.06	145.03	20	113.83	117.58	20	94.96	99.50
30	141.47	144.46	30	113.46	117.21	30	94.70	99.25
40	140.89	143.89	40	113.08	116.85	40	94.44	99.00
50	140.32	143.33	50	112.71	116.50	50	94.18	98.76
41° 0′	139.74	142.77	51° 0′	112.34	116.14	61° 0′	93.93	98.52
10	139.18	142.22	10	111.98	115.79	10	93.67	98.27
20	138.62	141.67	20	111.62	115.44	20	93.42	98.03
30	138.06	141.13	30	111.25	115.09	30	93.16	97.79
40	137.51	140.59	40	110.90	114.74	40	92.91	97.55
50	136.96	140.05	50	110.54	114.40	50	92.66	97.32
42° 0′	136.42	139.52	52° 0′	110.18	114.06	62° 0′	92.41	97.08
10	135.88	138.99	10	109.83	113.72	10	92.16	96.85
20	135.34	138.47	20	109.48	113.38	20	91.92	96.61
30	134.81	137.95	30	109.13	113.05	30	91.67	96.38
40	134.29	137.44	40	108.79	112.72	40	91.43	96.15
50	133.76	136.93	50	108.45	112.39	50	91.19	95.92
43° 0′	133.25	136.43	53° 0′	108.11	112.06	63° 0′	90.94	95.69
10	132.73	135.92	10	107.77	111.73	10	90.71	95.47
20	132.22	135.43	20	107.43	111.41	20	90.47	95.24
30	131.71	134.93	30	107.09	111.09	30	90.23	95.02
40	131.21	134.44	40	106.76	110.77	40	89.99	94.80
50	130.71	133.96	50	106.43	110.45	50	89.76	94.57
44° 0′	130.22	133.47	54° 0′	106.10	110.13	64° 0′	89.52	94.35
10	129.73	132.99	10	105.78	109.82	10	89.29	94.13
20	129.24	132.52	20	105.45	109.51	20	89.06	93.92
30	128.75	132.05	30	105.13	109.20	30	88.83	93.70
40	128.27	131.58	40	104.81	108.89	40	88.60	93.49
50	127.80	131.12	50	104.49	108.59	50	88.37	93.27
45° 0′	127.32	130.66	55° 0′	104.17	108.28	65° 0′	88.15	93.06
10	126.85	130.20	10	103.86	107.98	10	87.92	92.85
20	126.39	129.75	20	103.55	107.68	20	87.70	92.64
30	125.92	129.30	30	103.24	107.38	30	87.47	92.43
40	125.46	128.85	40	102.93	107.09	40	87.25	92.22
50	125.01	128.41	50	102.62	106.79	50	87.03	92.01
46° 0′	124.56	127.97	56° 0′	102.31	106.50	66° 0′	86.81	91.81
10	124.11	127.53	10	102.01	106.21	10	86.59	91.60
20	123.66	127.09	20	101.71	105.92	20	86.38	91.39
30	123.22	126.66	30	101.41	105.63	30	86.16	91.19
40	122.78	126.24	40	101.11	105.35	40	85.94	90.99
50	122.34	125.81	50	100.81	105.07	50	85.73	90.79
47° 0′	121.91	125.39	57° 0′	100.52	104.89	67° 0′	85.52	90.59
10	121.48	124.97	10	100.22	104.51	10	85.30	90.39
20	121.05	124.56	20	99.93	104.23	20	85.09	90.19
30	120.62	124.15	30	99.64	103.95	30	84.88	90.00
40	120.20	123.74	40	99.36	103.68	40	84.67	89.80
50	119.78	123.33	50	99.07	103.40	50	84.46	89.61
48° 0′	119.37	122.93	58° 0′	98.78	103.13	68° 0′	84.26	89.42
10	118.95	122.53	10	98.50	102.86	10	84.05	89.22
20	118.54	122.13	20	98.22	102.60	20	83.85	89.03
30	118.14	121.74	30	97.94	102.33	30	83.64	88.84
40	117.73	121.35	40	97.66	102.06	40	83.44	88.65
50	117.33	120.96	50	97.39	101.80	50	83.24	88.46
49° 0′	116.93	120.57	59° 0′	97.11	101.59	69° 0′	83.04	88.27
10	116.53	120.19	10	96.84	101.28	10	82.84	88.09
20	116.14	119.81	20	96.56	101.02	20	82.64	87.90
30	115.75	119.43	30	96.30	100.76	30	82.44	87.72
40	115.36	119.05	40	96.03	100.51	40	82.24	87.54
50	114.97	118.68	50	95.76	100.25	50	82.05	87.35

TABLE 3.—RADII—(*Continued*)

Deg. D	Radius R (ft.)		Deg. D	Radius R (ft.)		Deg. D	Radius R (ft.)	
	Arc definition	Chord definition		Arc definition	Chord definition		Arc definition	Chord definition
70° 0′	**81.85**	87.17	79° 0′	**72.53**	78.61	88° 0′	**65.11**	71.98
10	**81.66**	86.99	10	**72.37**	78.48	10	**64.98**	71.87
20	**81.46**	86.81	20	**72.22**	78.33	20	**64.86**	71.76
30	**81.27**	86.63	30	**72.07**	78.19	30	**64.74**	71.65
40	**81.08**	86.46	40	**71.92**	78.06	40	**64.62**	71.55
50	**80.89**	86.28	50	**71.77**	77.92	50	**64.50**	71.44
71° 0′	**80.70**	86.10	80° 0′	**71.62**	77.78	89° 0′	**64.38**	71.33
10	**80.51**	85.93	10	**71.47**	77.65	10	**64.26**	71.23
20	**80.32**	85.75	20	**71.32**	77.52	20	**64.14**	71.13
30	**80.13**	85.58	30	**71.17**	77.38	30	**64.02**	71.02
40	**79.95**	85.41	40	**71.03**	77.25	40	**63.90**	70.92
50	**79.76**	85.24	50	**70.88**	77.12	50	**63.78**	70.81
72° 0′	**79.58**	85.07	81° 0′	**70.74**	76.99	90° 0′	**63.66**	70.71
10	**79.39**	84.89	10	**70.59**	76.86	10	**63.54**	70.61
20	**79.21**	84.73	20	**70.44**	76.73	20	**63.43**	70.51
30	**79.03**	84.56	30	**70.30**	76.60	30	**63.31**	70.40
40	**78.85**	84.39	40	**70.16**	76.47	40	**63.19**	70.30
50	**78.67**	84.25	50	**70.02**	76.34	50	**63.08**	70.20
73° 0′	**78.49**	84.07	82° 0′	**69.87**	76.21	91° 0′	**62.96**	70.10
10	**78.31**	83.89	10	**69.73**	76.09	10	**62.85**	70.00
20	**78.13**	83.73	20	**69.59**	75.96	20	**62.73**	69.90
30	**77.95**	83.57	30	**69.45**	75.83	30	**62.62**	69.80
40	**77.78**	83.40	40	**69.31**	75.71	40	**62.50**	69.70
50	**77.60**	83.24	50	**69.17**	75.58	50	**62.39**	69.61
74° 0′	**77.43**	83.08	83° 0′	**69.03**	75.45	92° 0′	**62.28**	69.51
10	**77.25**	82.92	10	**68.89**	75.33	10	**62.16**	69.41
20	**77.08**	82.76	20	**68.75**	75.21	20	**62.05**	69.31
30	**76.91**	82.60	30	**68.62**	75.09	30	**61.94**	69.22
40	**76.74**	82.45	40	**68.48**	74.97	40	**61.83**	69.12
50	**76.56**	82.29	50	**68.34**	74.84	50	**61.72**	69.03
75° 0′	**76.39**	82.13	84° 0′	**68.21**	74.72	93° 0′	**61.61**	68.93
10	**76.22**	81.98	10	**68.07**	74.60	10	**61.50**	68.83
20	**76.06**	81.82	20	**67.94**	74.48	20	**61.39**	68.74
30	**75.89**	81.67	30	**67.81**	74.36	30	**61.28**	68.65
40	**75.72**	81.52	40	**67.67**	74.25	40	**61.17**	68.55
50	**75.55**	81.37	50	**67.54**	74.11	50	**61.06**	68.45
76° 0′	**75.39**	81.22	85° 0′	**67.41**	74.00	94° 0′	**60.95**	68.36
10	**75.22**	81.06	10	**67.27**	73.89	10	**60.84**	68.27
20	**75.06**	80.91	20	**67.14**	73.78	20	**60.74**	68.18
30	**74.90**	80.77	30	**67.01**	73.66	30	**60.63**	68.09
40	**74.73**	80.61	40	**66.88**	73.54	40	**60.52**	68.00
50	**74.57**	80.47	50	**66.75**	73.43	50	**60.42**	67.91
77° 0′	**74.41**	80.32	86° 0′	**66.62**	73.32	95° 0′	**60.31**	67.81
10	**74.25**	80.17	10	**66.49**	73.20	10	**60.21**	67.72
20	**74.09**	80.03	20	**66.36**	73.09	20	**60.10**	67.64
30	**73.93**	79.88	30	**66.24**	72.97	30	**60.00**	67.56
40	**73.77**	79.74	40	**66.11**	72.86	40	**59.89**	67.46
50	**73.61**	79.59	50	**65.98**	72.75	50	**59.79**	67.37
78° 0′	**72.53**	79.45	87° 0′	**65.86**	72.64	96° 0′	**59.68**	67.28
10	**73.30**	79.31	10	**65.73**	72.53	10	**59.58**	67.19
20	**73.14**	79.17	20	**65.61**	72.42	20	**59.48**	67.11
30	**72.99**	79.03	30	**65.48**	72.31	30	**59.37**	67.02
40	**72.83**	78.89	40	**65.36**	72.20	40	**59.27**	66.93
50	**72.68**	78.75	50	**65.23**	72.09	50	**59.17**	66.85

TABLE 4.—DEFLECTIONS AND CHORDS FOR EVEN RADIUS CURVES

Radius (ft.)	Deflection per foot of arc (min.)	Deflections for arcs of			Chords for arcs of			Radius (ft.)
		10 ft.	**15 ft.**	**25 ft.**	**10 ft.**	**15 ft.**	**25 ft.**	
100	17.190	2° 51.89	4° 17.83′	7° 09.72′	10.00	14.99	24.93	100
125	13.751	2° 17.51	3° 26.26′	5° 43.77′	10.00	14.99	24.96	125
150	11.460	1° 54.59′	2° 51.89′	4° 46.48′	10.00	14.99	24.97	150
175	9.822	1° 38.22′	2° 27.33′	4° 05.55′	10.00	15.00	24.98	175
200	8.594	1° 25.94′	2° 08.92′	3° 34.86′	10.00	15.00	24.98	200
225	7.639	1° 16.39′	1° 54.59′	3° 10.99′	10.00	15.00	24.99	225
		15 ft.	**25 ft.**	**50 ft.**	**15 ft.**	**25 ft.**	**50 ft.**	
250	6.875	1° 43.13′	2° 51.89′	5° 43.77′	15.00	24.99	49.92	250
275	6.250	1° 33.76′	2° 36.26′	5° 12.52′	15.00	24.99	49.93	275
300	5.730	1° 25.94′	2° 23.24′	4° 46.48′	15.00	25.00	49.94	300
325	5.289	1° 19.33′	2° 12.22′	4° 24.44′	15.00	25.00	49.95	325
350	4.911	1° 13.67′	2° 02.78′	4° 05.55′	15.00	25.00	49.96	350
375	4.584	1° 08.75′	1° 54.59′	3° 49.18′	15.00	25.00	49.96	375
400	4.297	1° 04.46′	1° 47.43′	3° 34.86′	15.00	25.00	49.97	400
425	4.044	1° 00.67′	1° 41.11′	3° 22.22′	15.00	25.00	49.97	425
450	3.820	0° 57.30′	1° 35.49′	3° 10.99′	15.00	25.00	49.97	450
475	3.619	0° 54.28′	1° 30.47′	3° 00.93′	15.00	25.00	49.98	475
500	3.438	0° 51.57′	1° 25.94′	2° 51.89′	15.00	25.00	49.98	500
525	3.274	0° 49.11′	1° 21.85′	2° 43.70′	15.00	25.00	49.98	525
		25 ft.	**50 ft.**	**100 ft.**	**25 ft.**	**50 ft.**	**100 ft.**	
550	3.125	1° 18.13′	2° 36.26′	5° 12.52′	25.00	49.98	99.86	550
600	2.865	1° 11.62′	2° 23.24′	4° 46.48′	25.00	49.99	99.88	600
650	2.644	1° 06.11′	2° 12.22′	4° 24.44′	25.00	49.99	99.90	650
700	2.456	1° 01.39′	2° 02.78′	4° 05.55′	25.00	49.99	99.91	700
750	2.292	0° 57.30′	1° 54.59′	3° 49.18′	25.00	50.00	99.93	750
800	2.149	0° 53.71′	1° 47.43′	3° 34.86′	25.00	50.00	99.93	800
850	2.022	0° 50.56′	1° 41.11′	3° 22.22′	25.00	50.00	99.94	850
900	1.911	0° 47.75′	1° 35.49′	3° 10.99′	25.00	50.00	99.95	900
950	1.809	0° 45.23′	1° 30.47′	3° 00.93′	25.00	50.00	99.95	950
1,000	1.719	0° 42.97	1° 25.94′	2° 51.89′	25.00	50.00	99.96	1,000
1,500	1.146	0° 28.65	0° 57.30′	1° 54.59′	25.00	50.00	99.97	1,500
2,000	0.859	0° 21.48′	0° 42.97′	1° 25.94′	25.00	50.00	99.98	2,000
2,500	0.688	0° 17.19′	0° 34.38′	1° 08.75′	25.00	50.00	99.99	2,500
3,000	0.573	0° 14.32′	0° 28.65′	0° 57.30′	25.00	50.00	100.00	3,000
3,500	0.491	0° 12.28′	0° 24.56′	0° 49.11′	25.00	50.00	100.00	3,500
4,000	0.430	0° 10.74′	0° 21.48′	0° 42.97′	25.00	50.00	100.00	4,000
4,500	0.382	0° 09.55′	0° 19.10′	0° 38.20′	25.00	50.00	100.00	4,500
5,000	0.344	0° 08.60′	0° 17.19′	0° 34.38′	25.00	50.00	100.00	5,000
5,500	0.313	0° 07.81′	0° 15.62′	0° 31.25′	25.00	50.00	100.00	5,500
6,000	0.286	0° 07.16′	0° 14.32′	0° 28.65′	25.00	50.00	100.00	6,000
7,000	0.246	0° 06.14′	0° 12.28′	0° 24.56′	25.00	50.00	100.00	7,000
8,000	0.215	0° 05.37′	0° 10.74′	0° 21.49′	25.00	50.00	100.00	8,000
9,000	0.191	0° 04.77′	0° 09.55′	0° 19.10′	25.00	50.00	100.00	9,000
10,000	0.172	0° 04.30′	0° 08.60′	0° 17.19′	25.00	50.00	100.00	10,000

Table 5.—Arc Lengths for Radius = 1

Deg.	Length	Deg.	Length	Min.	Length	Sec.	Length
1	0.0174533	61	1.0646508	1	0.0002909	1	0.0000048
2	0.0349066	62	1.0821041	2	0.0005818	2	0.0000097
3	0.0523599	63	1.0995574	3	0.0008727	3	0.0000145
4	0.0698132	64	1.1170107	4	0.0011636	4	0.0000194
5	0.0872665	65	1.1344640	5	0.0014544	5	0.0000242
6	0.1047198	66	1.1519173	6	0.0017453	6	0.0000291
7	0.1221730	67	1.1693706	7	0.0020362	7	0.0000339
8	0.1396263	68	1.1868239	8	0.0023271	8	0.0000388
9	0.1570796	69	1.2042772	9	0.0026180	9	0.0000436
10	0.1745329	70	1.2217305	10	0.0029089	10	0.0000485
11	0.1919862	71	1.2391838	11	0.0031998	11	0.0000533
12	0.2094395	72	1 2566371	12	0.0034907	12	0.0000582
13	0.2268928	73	1.2740904	13	0.0037815	13	0.0000630
14	0.2443461	74	1.2915436	14	0.0040724	14	0.0000679
15	0.2617994	75	1.3089969	15	0.0043633	15	0.0000727
16	0 2792527	76	1.3264502	16	0.0046542	16	0.0000776
17	0.2967060	77	1.3439035	17	0.0049451	17	0.0000824
18	0.3141593	78	1.3613568	18	0.0052360	18	0.0000873
19	0.3316126	79	1.3788101	19	0.0055269	19	0.0000921
20	0.3490659	80	1 3962634	20	0.0058178	20	0.0000970
21	0.3665191	81	1 4137167	21	0.0061087	21	0.0001018
22	0.3839724	82	1.4311700	22	0.0063995	22	0.0001067
23	0.4014257	83	1.4486233	23	0 0066904	23	0.0001115
24	0.4188790	84	1.4660766	24	0.0069813	24	0.0001164
25	0.4363323	85	1.4835299	25	0.0072722	25	0.0001212
26	0 4537856	86	1.5009832	26	0.0075631	26	0.0001261
27	0.4712389	87	1.5184364	27	0.0078540	27	0.0001309
28	0.4886922	88	1.5358897	28	0.0081449	28	0.0001357
29	0 5061455	89	1.5533430	29	0.0084358	29	0.0001406
30	0.5235988	90	1.5707963	30	0.0087266	30	0.0001454
31	0.5410521	91	1.5882496	31	0.0090175	31	0.0001503
32	0.5585054	92	1.6057029	32	0.0093084	32	0.0001551
33	0.5759587	93	1.6231562	33	0.0095993	33	0.0001600
34	0.5934119	94	1.6406095	34	0.0098902	34	0.0001648
35	0.6108652	95	1.6580628	35	0.0101811	35	0.0001697
36	0.6283185	96	1.6755161	36	0.0104720	36	0.0001745
37	0.6457718	97	1.6929694	37	0.0107629	37	0.0001794
38	0.6632251	98	1.7104227	38	0.0110538	38	0.0001842
39	0.6806784	99	1.7278760	39	0.0113446	39	0.0001891
40	0.6981317	100	1.7453293	40	0.0116355	40	0.0001939
41	0.7155850	101	1.7627825	41	0.0119264	41	0.0001988
42	0.7330383	102	1.7802358	42	0.0122173	42	0.0002036
43	0.7504916	103	1.7976891	43	0.0125082	43	0.0002085
44	0.7679449	104	1.8151424	44	0.0127991	44	0.0002133
45	0.7853982	105	1.8325957	45	0.0130900	45	0.0002182
46	0.8028515	106	1.8500490	46	0.0133809	46	0.0002230
47	0.8203047	107	1.8675023	47	0.0136717	47	0.0002279
48	0.8377580	108	1.8849556	48	0.0139626	48	0.0002327
49	0.8552113	109	1.9024089	49	0.0142535	49	0.0002376
50	0.8726646	110	1.9198622	50	0.0145444	50	0.0002424
51	0.8901179	111	1.9373155	51	0.0148353	51	0.0002473
52	0.9075712	112	1.9547688	52	0.0151262	52	0.0002521
53	0.9250245	113	1.9722221	53	0.0154171	53	0.0002570
54	0.9424778	114	1.9896753	54	0.0157080	54	0.0002618
55	0.9599311	115	2.0071286	55	0.0159989	55	0.0002666
56	0.9773844	116	2.0245819	56	0.0162897	56	0.0002715
57	0.9948377	117	2.0420352	57	0.0165806	57	0.0002763
58	1.0122910	118	2.0594885	58	0.0168715	58	0.0002812
59	1.0297443	119	2.0769418	59	0.0171624	59	0.0002860
60	1.0471976	120	2.0943951	60	0.0174533	60	0.0002909

TABLE 6.—INNER AND OUTER PARALLEL CIRCULAR CURVES

Values of a and j for radial distance = 10 ft.

See page 127

Δ	a	j	Δ	a	j	Δ	a	j	Δ	a	j
20° 00′	3.49	10.15	26° 00′	4.54	10.26	32° 00′	5.59	10.40	38° 00′	5.63	10.58
10	.52	.16	10	.57	.27	10	.61	.41	10	.66	.58
20	.55	.16	20	.60	.27	20	.64	.41	20	.69	.59
30	.58	.17	30	.63	.27	30	.67	.42	30	.72	.59
40	.61	.18	40	.65	.28	40	.70	.42	40	.75	.60
50	.64	.18	50	.68	.28	50	.73	.42	50	.78	.60
21° 00	.67	.19	27° 00	.71	.28	33° 00	.76	.43	39° 00	.81	.61
10	.69	.19	10	.74	.29	10	.79	.43	10	.84	.61
20	.72	.20	20	.77	.29	20	.82	.44	20	.86	.62
30	.75	.20	30	.80	.30	30	.85	.44	30	.89	.62
40	.78	.21	40	.83	.30	40	.88	.45	40	.92	.63
50	.81	.22	50	.86	.30	50	.91	.45	50	.95	.64
22° 00	.84	.22	28° 00	.89	.31	34° 00	.93	.46	40° 00	.98	.64
10	.87	.22	10	.92	.31	10	.96	.46	10	7.01	.65
20	.90	.23	20	.95	.31	20	.99	.47	20	.04	.65
30	.93	.24	30	.97	.32	30	6.02	.47	30	.07	.66
40	.96	.24	40	5.00	.32	40	.05	.48	40	.10	.66
50	.99	.25	50	.03	.33	50	.08	.48	50	.13	.67
23° 00	4.01	.26	29° 00	.06	.33	35° 00	.11	.49	41° 00	.16	.63
10	.04	.26	10	.09	.33	10	.14	.49	10	.18	.69
20	.07	.27	20	.12	.34	20	.17	.49	20	.21	.69
30	.10	.28	30	.15	.34	30	.20	.50	30	.24	.69
40	.13	.28	40	.18	.34	40	.23	.50	40	.27	.70
50	.16	.29	50	.21	.35	50	.25	.51	50	.30	.71
24° 00	.19	.22	30° 00	.24	.35	36° 00	.28	.52	42° 00	.33	.71
10	.22	.23	10	.27	.36	10	.31	.52	10	.36	.72
20	.25	.23	20	.29	.36	20	.34	.52	20	.39	.72
30	.28	.23	30	.32	.37	30	.37	.53	30	.42	.73
40	.31	.24	40	.35	.37	40	.40	.53	40	.45	.74
50	.33	.24	50	.38	.37	50	.43	.54	50	.48	.74
25° 00	.36	.24	31° 00	.41	.38	37° 00	.46	.54	43° 00	.50	.75
10	.39	.25	10	.44	.38	10	.49	.55	10	.53	.75
20	.42	.25	20	.47	.39	20	.52	.56	20	.56	.76
30	.45	.25	30	.50	.39	30	.54	.56	30	.59	.77
40	.48	.26	40	.53	.39	40	.57	.57	40	.62	.77
50	.51	.26	50	.56	.40	50	.60	.57	50	.65	.78

TABLE 6.—INNER AND OUTER PARALLEL CIRCULAR CURVES—(*Continued*).

Values of a and j for radial distance = 10 ft.

See page 127

Δ	a	j	Δ	a	j	Δ	a	j	Δ	a	j
44° 00′	7.68	10.79	50° 00′	8.73	11.03	56° 00′	9.77	11.33	62° 00′	10.82	11.67
10	.71	.79	10	.76	.04	10	.80	.33	10	.85	.68
20	.74	.80	20	.78	.05	20	.83	.34	20	.88	.69
30	.77	.80	30	.81	.06	30	.86	.35	30	.91	.70
40	.80	.81	40	.84	.06	40	.89	.36	40	.94	.71
50	.82	.82	50	.87	.07	50	.92	.37	50	.97	.72
45° 00	.85	.82	51° 00	.90	.08	57° 00	.95	.38	63° 00	11.00	.73
10	.88	.83	10	.93	.09	10	.98	.39	10	.02	.74
20	.91	.84	20	.96	.09	20	10.01	.40	20	.05	.75
30	.94	.84	30	.99	.11	30	.04	.41	30	.08	76
40	.97	.85	40	9.02	.11	40	.06	.42	40	.11	.77
50	8.00	.86	50	.05	.12	50	.09	.42	50	.14	.78
46° 00	.03	.86	52° 00	.08	.13	58° 00	.12	.43	64° 00	.17	.79
10	.06	.87	10	.10	.13	10	.15	.44	10	.20	.80
20	.09	.88	20	.13	.14	20	.18	.45	20	.23	.81
30	.12	.88	30	.16	.15	30	.21	.46	30	.26	.82
40	.14	.89	40	.19	.16	40	.24	.47	40	.29	.83
50	.17	.90	50	.22	.17	50	.27	.48	50	.32	.85
47° 00	.20	.90	53° 00	.25	.17	59° 00	.30	.49	65° 00	.34	.86
10	.23	.91	10	.28	.18	10	.33	.50	10	.37	.87
20	.26	.92	20	.31	.19	20	.36	.51	20	.40	.88
30	.29	.93	30	.34	.20	30	.38	.52	30	.43	.89
40	.32	.93	40	.37	.21	40	.41	.53	40	.46	.90
50	.35	.94	50	.40	.21	50	.44	.54	50	.49	.91
48° 00	.38	.95	54° 00	.42	.22	60° 00	.47	.55	66° 00	.52	.92
10	.41	.95	10	.45	.23	10	.50	.56	10	.55	.93
20	.44	.96	20	.48	.24	20	.53	.57	20	.58	.95
30	.46	.97	30	.51	.25	30	.56	.58	30	.61	.96
40	.49	.97	40	.54	.26	40	.59	.59	40	.64	.97
50	.52	.98	50	.57	.27	50	.62	.60	50	.66	.98
49° 00	.55	.99	55° 00	.60	.27	61° 00	.65	.61	67° 00	.69	.99
10	.58	11.00	10	.63	.28	10	.68	.62	10	.72	12.00
20	.61	.00	20	.66	.29	20	.70	.63	20	.75	.02
30	.64	.01	30	.69	.30	30	.73	.64	30	.78	.03
40	.67	.02	40	.72	.31	40	.76	.65	40	.81	.04
50	.70	.03	50	.74	.32	50	.79	.66	50	.84	.05

TABLE 6.—INNER AND OUTER PARALLEL CIRCULAR CURVES—(*Continued*).

Values of a and j for radial distance = 10 ft

See page 127

Δ	a	j	Δ	a	j	Δ	a	j	Δ	a	j
68° 00′	11.87	12.06	74° 00′	12.92	12.52	80° 00′	13.96	13.05	86° 00′	15.01	13.67
10	.90	.07	10	.94	.54	10	.99	.07	10	.04	.69
20	.93	.09	20	.97	.55	20	14.02	.09	20	.07	.71
30	.96	.10	30	13.00	.56	30	.05	.10	30	.10	.73
40	.98	.11	40	.03	.58	40	.08	.12	40	.13	.75
50	12.01	.12	50	.06	.59	50	.11	.13	50	.16	.77
69° 00	.04	.13	75° 00	.09	.60	81° 00	.14	.15	87° 00	.18	.79
10	.07	.15	10	.12	.62	10	.17	.17	10	.21	.80
20	.10	.16	20	.15	.63	20	.20	.18	20	.24	.82
30	.13	.17	30	.18	.65	30	.22	.20	30	.27	.84
40	.16	.18	40	.21	.66	40	.25	.22	40	.30	.86
50	.19	.20	50	.24	.68	50	.28	.23	50	.33	.88
70° 00	.22	.21	76° 00	.26	.69	82° 00	.31	.25	88° 00	.36	.90
10	.25	.22	10	.29	.70	10	.34	.27	10	.39	.92
20	.28	.23	20	.32	.72	20	.37	.28	20	.42	.94
30	.30	.25	30	.35	.73	30	.40	.30	30	.45	.96
40	.33	.26	40	.38	.75	40	.43	.32	40	.48	.98
50	.36	.27	50	.41	.76	50	.46	.33	50	.50	14.00
71° 00	.39	.28	77° 00	.44	.78	83° 00	.49	.35	89° 00	.53	.02
10	.42	.30	10	.47	.79	10	.52	.37	10	.56	.04
20	.45	.31	20	.50	.81	20	.54	.39	20	.59	.06
30	.48	.32	30	.53	.82	30	.57	.40	30	.62	.08
40	.51	.33	40	.56	.84	40	.60	.42	40	.65	.10
50	.54	.35	50	.58	.85	50	.63	.44	50	.68	.12
72° 00	.57	.36	78° 00	.61	.87	84° 00	.66	.46	90° 00	.71	.14
10	.60	.37	10	.64	.88	10	.69	.47	10	.74	.16
20	.62	.39	20	.67	.90	20	.72	.49	20	.77	.18
30	.65	.40	30	.70	.91	30	.75	.51	30	.80	.20
40	.68	.41	40	.73	.93	40	.78	.53	40	.82	.23
50	.71	.43	50	.76	.94	50	.81	.55	50	.85	.25
73° 00	.74	.44	79° 00	.79	.96	85° 00	.84	.56	91° 00	.88	.27
10	.77	.45	10	.82	.98	10	.86	.58	10	.91	.29
20	.80	.47	20	.85	.99	20	.89	.60	20	.94	.31
30	.83	.48	30	.88	13.01	30	.92	.62	30	.97	.33
40	.86	.49	40	.90	.02	40	.95	.64	40	16.00	.35
50	.89	.51	50	.93	.04	50	.98	.65	50	.03	.37

TABLE 6.—INNER AND OUTER PARALLEL CIRCULAR CURVES—(*Continued*).

Values of a and j for radial distance = 10 ft.

See page 127

Δ	a	j	Δ	a	j	Δ	a	j	Δ	a	j
92° 00′	16.06	14.40	98° 00′	17.10	15.24	104° 00′	18.15	16.24	110° 00′	19.20	17.43
10	.09	.42	10	.13	.27	10	.18	.27	10	.23	.47
20	.12	.44	20	.16	.29	20	.21	.30	20	.26	.50
30	.14	.46	30	.19	.32	30	.24	.33	30	.29	.54
40	.17	.48	40	.22	.35	40	.27	.36	40	.31	.58
50	.20	.51	50	.25	.37	50	.30	.40	50	.34	.62
93° 00	.23	.53	99° 00	.28	.40	105° 00	.33	.43	111° 00	.37	.66
10	.26	.55	10	.31	.42	10	.36	.46	10	.40	.69
20	.29	.57	20	.34	.45	20	.38	.49	20	.43	.73
30	.32	.59	30	.37	.48	30	.41	.52	30	.46	.77
40	.35	.62	40	.40	.50	40	.44	.55	40	.49	.81
50	.38	.64	50	.42	.53	50	.47	.58	50	.52	.84
94° 00	.41	.66	100° 00	.45	.56	106° 00	.50	.62	112° 00	.55	.88
10	.44	.69	10	.48	.58	10	.53	.65	10	.58	.92
20	.46	.71	20	.51	.61	20	.56	.68	20	.61	.96
30	.49	.73	30	.54	.64	30	.59	.71	30	.63	18.00
40	.52	.76	40	.57	.67	40	.62	.75	40	.66	.04
50	.55	.78	50	.60	.69	50	.65	.78	50	.69	.08
95° 00	.58	.80	101° 00	.63	.72	107° 00	.68	.81	113° 00	.72	.12
10	.61	.83	10	.66	.75	10	.70	.84	10	.75	.16
20	.64	.85	20	.69	.78	20	.73	.88	20	.78	.20
30	.67	.87	30	.72	.81	30	.76	.91	30	.81	.24
40	.70	.90	40	.74	.83	40	.79	.95	40	.84	.28
50	.73	.92	50	.77	.86	50	.82	.98	50	.87	.32
96° 00	.76	.94	102° 00	.80	.89	108° 00	.85	17.01	114° 00	.90	.36
10	.78	.97	10	.83	.92	10	.88	.05	10	.93	.40
20	.81	.99	20	.86	.95	20	.91	.08	20	.95	.44
30	.84	15.02	30	.89	.98	30	.94	.12	30	.98	.49
40	.87	.04	40	.92	16.00	40	.97	.15	40	20.01	.53
50	.90	.07	50	.95	.03	50	19.00	.19	50	.04	.57
97° 00	.93	.09	103° 00	.98	.06	109° 00	.02	.22	115° 00	.07	.61
10	.96	.12	10	18.01	.09	10	.05	.26	10	.10	.65
20	.99	.14	20	.04	.12	20	.08	.29	20	.13	.70
30	17.02	.17	30	.07	.15	30	.11	.33	30	.16	.74
40	.05	.19	40	.09	.18	40	.14	.36	40	.19	.78
50	.08	.22	50	.12	.21	50	.17	.40	50	.22	.83

TABLE 6.—INNER AND OUTER PARALLEL CIRCULAR CURVES—(*Continued*).

Values of a and j for radial distance = 10 ft.

See page 127

Δ	a	j	Δ	a	j	Δ	a	j	Δ	a	j
116° 00′	20.25	18.87	122° 00′	21.29	20.63	128° 00′	22 34	22.81	134° 00′	23.39	25.59
10	.27	.91	10	.32	.68	10	.37	.88	10	.42	.68
20	.30	.96	20	.35	.74	20	.40	.95	20	.45	.77
30	.33	19.00	30	.38	.79	30	.43	23.02	30	.47	.86
40	.36	.05	40	.41	.85	40	.46	.09	40	.50	.95
50	.39	.09	50	.44	.90	50	.49	.16	50	.53	26.04
117° 00	.42	.14	123° 00	.47	.96	129° 00	.51	.23	135° 00	.56	.13
10	.45	.18	10	.50	21.01	10	.54	.30	10	.59	.22
20	.48	.23	20	.53	.07	20	.57	.37	20	62	.32
30	.51	.28	30	.55	.13	30	.60	.44	30	.65	.41
40	.54	.32	40	.58	.18	40	.63	.51	40	.68	.50
50	.57	.37	50	.61	.24	50	.66	.59	50	.71	.60
118° 00	.59	.42	124° 00	.64	.30	130° 00	.69	.66	136° 00	.74	.69
10	.62	.46	10	.67	.36	10	.72	.74	10	.77	.79
20	.65	.51	20	.70	.42	20	.75	.81	20	.79	.89
30	.68	.56	30	.73	.48	30	.78	.89	30	.82	.98
40	.71	.61	40	.76	.54	40	.81	.96	40	.85	.09
50	.74	.65	50	.79	.60	50	.83	24.04	50	.88	27.18
119° 00	.77	.70	125° 00	.82	.66	131° 00	.86	.11	137° 00	.91	.28
10	.80	.75	10	.85	.72	10	.89	.19	10	.94	.39
20	.83	.80	20	.87	.78	20	.92	.27	20	.97	.49
30	.86	.85	30	.90	.84	30	.95	.35	30	24.00	.59
40	.89	.90	40	.93	.90	40	.98	.43	40	.03	.69
50	.91	.95	50	.96	.96	50	23.01	.51	50	.06	.80
120° 00	.94	20.00	126° 00	.99	22.03	132° 00	.04	.59	138° 00	.09	.90
10	.97	.05	10	22.02	.09	10	.07	.67	10	.11	28.01
20	21.00	.10	20	.05	.15	20	.10	.75	20	.14	.12
30	.03	.15	30	.08	.22	30	.13	.83	30	.17	.23
40	.06	.20	40	.11	.28	40	.15	.91	40	.20	.33
50	.09	.26	50	.14	.35	50	.18	.99	50	.23	.44
121° 00	.12	.31	127° 00	.17	.41	133° 00	.21	25.08	139° 00	.26	.55
10	.15	.36	10	.19	.48	10	.24	.16	10	.29	.67
20	.18	.41	20	.22	.54	20	.27	.25	20	.32	.78
30	.21	.47	30	.25	.61	30	.30	.33	30	.35	.89
40	.23	.52	40	.28	.68	40	.33	.42	40	.38	29.10
50	.26	.57	50	.31	.74	50	.36	.51	50	.41	.12

TABLE 7.—VALUES OF x AND y FOR THE UNIT SPIRAL OF LENGTH 100 FT.

θ	x	y	θ	x	y	θ	x	y
0° 00′	100.000	0.000						
1° 00′	99.997	0.582	7° 00′	99.851	4.068	13° 00′	99.486	7.535
10	.996	0.679	10	.843	4.165	10	.473	7.631
20	.994	0.776	20	.836	4.261	20	.459	7.727
30	.993	0.873	30	.828	4.358	30	.446	7.823
40	.991	0.970	40	.820	4.455	40	.432	7.918
50	.990	1.067	50	.813	4.551	50	.419	8.014
2° 00	.988	1.164	8° 00	.805	4.648	14° 00	.405	8.110
10	.986	1.261	10	.797	4.745	10	.390	8 206
20	.983	1.358	20	.788	4.841	20	.376	8.301
30	.981	1.454	30	.780	4.938	30	.361	8.397
40	.978	1.551	40	.771	5.034	40	.346	8.493
50	.976	1.648	50	.763	5.131	50	.332	8.588
3° 00	.973	1.745	9° 00	.754	5.227	15° 00	.317	8.684
10	.969	1.842	10	.744	5.323	10	.301	8.780
20	.966	1.939	20	.735	5.420	20	.286	8.875
30	.962	2.036	30	.725	5.516	30	.270	8.971
40	.958	2.132	40	.715	5.612	40	.254	9.066
50	.955	2.229	50	.706	5.709	50	.239	9.162
4° 00	.951	2.326	10° 00	.696	5.805	16° 00	.223	9.257
10	.947	2.423	10	.685	5.901	10	.206	9.352
20	.942	2.520	20	.675	5.998	20	.190	9.447
30	.938	2.617	30	.664	6.094	30	.173	9.543
40	.933	2.713	40	.653	6.190	40	.156	9.638
50	.929	2.810	50	.643	6.287	50	.140	9.733
5° 00	.924	2.907	11° 00	.632	6.383	17° 00	.123	9.828
10	.918	3.004	10	.620	6.479	10	.106	9.923
20	.913	3.101	20	.609	6.575	20	.088	10.018
30	.907	3.198	30	.597	6.671	30	.071	10.113
40	.901	3.294	40	.585	6.767	40	.053	10.208
50	.896	3.391	50	.574	6.863	50	.036	10.303
6° 00	.890	3.488	12° 00	.562	6.959	18° 00	.018	10.398
10	.884	3.585	10	.549	7.055	10	.000	10.493
20	.877	3.681	20	.537	7.151	20	98.981	10.588
30	.871	3.778	30	.524	7.247	30	.962	10.683
40	.864	3.875	40	.511	7.343	40	.943	10.777
50	.858	3.971	50	.499	7.439	50	.925	10.872

TABLE 7.—VALUES OF x AND y FOR THE UNIT SPIRAL OF LENGTH 100 FT.—(*Continued*)

θ	x	y	θ	x	y	θ	x	y
19° 00′	98.906	10.967	25° 00′	98.113	14.348	31° 00′	97.112	17.661
10	.886	11.062	10	.087	14.441	10	.081	17.753
20	.867	11.156	20	.062	14.534	20	.050	17.843
30	.847	11.251	30	.037	14.627	30	.019	17.934
40	.827	11.346	40	.011	14.720	40	96.988	18.025
50	.807	11.440	50	97.986	14.812	50	.956	18.115
20° 00	.788	11.535	26° 00	.960	14.905	32° 00	.925	18.206
10	.768	11.629	10	.934	14.998	10	.893	18.297
20	.747	11.724	20	.907	15.090	20	.862	18.388
30	.727	11.818	30	.881	15.183	30	.830	18.478
40	.707	11.912	40	.855	15.276	40	.798	18.568
50	.686	12.007	50	.828	15.368	50	.765	18.659
21° 00	.665	12.101	27° 00	.802	15.461	33° 00	.733	18.749
10	.644	12.195	10	.775	15.553	10	.700	18.840
20	.622	12.289	20	.748	15.645	20	.668	18.929
30	.601	12.383	30	.721	15.737	30	.635	19.019
40	.579	12.477	40	.693	15.829	40	.602	19.109
50	.558	12.571	50	.666	15.921	50	.569	19.198
22° 00	.536	12.665	28° 00	.638	16.014	34° 00	.536	19.288
10	.514	12.759	10	.610	16.106	10	.502	19.378
20	.491	12.852	20	.582	16.198	20	.469	19.467
30	.469	12.946	30	.554	16.290	30	.435	19.557
40	.446	13.040	40	.526	16.382	40	.401	19.647
50	.424	13.134	50	.497	16.473	50	.366	19.736
23° 00	.401	13.228	29° 00	.469	16.565	35° 00	.332	19.826
10	.378	13.321	10	.440	16.657	10	.298	19.916
20	.354	13.415	20	.410	16.748	20	.263	20.005
30	.331	13.508	30	.381	16.840	30	.229	20.094
40	.307	13.602	40	.352	16.931	40	.194	20.183
50	.284	13.695	50	.322	17.023	50	.159	20.272
24° 00	.260	13.789	30° 00	.293	17.114	36° 00	.124	20.361
10	.235	13.882	10	.263	17.206	10	.088	20.450
20	.211	13.975	20	.233	17.297	20	.053	20.538
30	.186	14.068	30	.203	17.388	30	.017	20.627
40	.161	14.161	40	.173	17.479	40	95.981	20.716
50	.137	14.255	50	.142	17.570	50	.946	20.805

TABLE 7.—VALUES OF x AND y FOR THE UNIT SPIRAL OF LENGTH 100 FT.—(*Continued*)

θ	x	y	θ	x	y	θ	x	y
37° 00′	95.910	20.893	43° 00′	94.513	24.028	49° 00′	92.930	27.052
10	.873	20 981	10	.471	24.113	10	.884	27.134
20	.837	21.070	20	.430	24.199	20	.837	27.217
30	.800	21.158	30	.388	24.284	30	.790	27.299
40	.763	21 246	40	.346	24.370	40	.743	27.381
50	.727	21.335	50	.303	24.455	50	.695	27.462
38° 00	.690	21.423	44° 00	.261	24.540	50° 00	.648	27.544
10	.653	21.511	10	.218	24.625	10	.599	27.626
20	.615	21.598	20	.176	24.710	20	.551	27.708
30	.578	21.686	30	.133	24.795	30	.504	27.789
40	.540	21.774	40	.090	24.880	40	.457	27.870
50	.503	21.861	50	.048	24.965	50	.409	27.952
39° 00	.465	21.949	45° 00	.005	25.049	51° 00	.362	28.033
10	.427	22 037	10	93.962	25.133	10	.313	28.115
20	.389	22 124	20	.918	25.217	20	.265	28.195
30	.351	22.212	30	.875	25.302	30	.215	28.276
40	.312	22.298	40	.831	25.386	40	.167	28.357
50	.274	22.385	50	.788	25.470	50	.118	28.437
40° 00	.235	22.473	46° 00	.744	25.554	52° 00	.070	28.518
10	.196	22.560	10	.700	25.639	10	.021	28.599
20	.156	22.648	20	.655	25.722	20	91.973	28.679
30	.117	22.735	30	.611	25.806	30	.924	28.760
40	.077	22.822	40	.566	25.890	40	.875	28.841
50	.038	22.909	50	.522	25.973	50	.826	28.921
41° 00	94.999	22.995	47° 00	.477	26.057	53° 00	.776	29.001
10	.959	23.081	10	.432	26.140	10	.726	29.081
20	.919	23.168	20	.387	26.223	20	.677	29.161
30	.879	23.254	30	.342	26.306	30	.627	29.241
40	.839	23.341	40	.297	26.390	40	.577	29.320
50	.799	23.427	50	.252	26.473	50	.526	29.400
42° 00	.759	23.513	48° 00	.206	26.556	54° 00	.476	29.479
10	.718	23.599	10	.160	26.639	10	.425	29.559
20	.677	23.685	20	.114	26.722	20	.375	29.637
30	.636	23.771	30	.068	26.805	30	.324	29.716
40	.595	23.857	40	.022	26.887	40	.273	29.795
50	.554	23.942	50	92.976	26.970	50	.221	29.873

TABLE 8.—VALUES OF k AND p FOR THE UNIT SPIRAL OF LENGTH 100 FT.

θ_s	k	p	θ_s	k	p	θ_s	k	p
0° 00′	50.000	0.000						
1° 00′	49.999	0 146	7° 00′	49.975	1.018	13° 00′	49.914	1.887
10	.999	.170	10	.974	1.042	10	.912	1.911
20	.999	.194	20	.972	1.066	20	.910	1.935
30	.999	.218	30	.971	1.090	30	.908	1.960
40	.999	.243	40	.970	1.115	40	.905	1.984
50	.998	.267	50	.968	1.139	50	.903	2.008
2° 00	.998	.291	8° 00	.967	1.163	14° 00	.901	2.032
10	.998	.315	10	.966	1.187	10	.899	2.056
20	.997	.339	20	.964	1 211	20	.896	2.080
30	.997	.363	30	.963	1.235	30	.893	2.104
40	.996	.387	40	.962	1.260	40	.891	2.128
50	.996	.411	50	.960	1.284	50	.889	2.152
3° 00	.995	.435	9° 00	.959	1.308	15° 00	.886	2.176
10	.995	.459	10	.957	1.332	10	.883	2.200
20	.994	.484	20	.956	1.356	20	.881	2.224
30	.994	.508	30	.954	1.380	30	.878	2.248
40	.993	.532	40	.952	1.404	40	.875	2.272
50	.993	.557	50	.951	1.429	50	.873	2.296
4° 00	.992	.581	10° 00	.949	1.453	16° 00	.870	2.320
10	.991	.605	10	.947	1.477	10	.867	2.344
20	.990	.630	20	.946	1.501	20	.865	2.368
30	.990	.654	30	.944	1.525	30	.862	2.393
40	.989	.678	40	.942	1.550	40	.859	2.417
50	.988	.703	50	.941	1.574	50	.857	2.441
5° 00	.987	.727	11° 00	.939	1.598	17° 00	.854	2.465
10	.986	.751	10	.937	1.622	10	.851	2.489
20	.985	.775	20	.935	1.646	20	.848	2.512
30	.985	.799	30	.933	1.670	30	.845	2.536
40	.984	.824	40	.931	1.695	40	.842	2.560
50	.983	.848	50	.929	1.719	50	.839	2.584
6° 00	.982	.872	12° 00	.927	1.743	18° 00	.836	2.608
10	.981	.896	10	.925	1.767	10	.833	2.632
20	.980	.921	20	.923	1.791	20	.830	2.656
30	.978	.945	30	.921	1.815	30	.827	2.680
40	.977	.969	40	.918	1.839	40	.823	2.704
50	.976	.994	50	.916	1.863	50	.820	2.728

TABLE 8.—VALUES OF k AND p FOR THE UNIT SPIRAL OF LENGTH 100 FT.—(*Continued*)

θ_s	k	p	θ_s	k	p	θ_s	k	p
19° 00′	49.817	2.752	25° 00′	49.684	3.611	31° 00′	49.516	4.462
10	.814	2.776	10	.680	3.635	10	.511	4.485
20	.811	2.800	20	.675	3.659	20	.505	4.509
30	.808	2.824	30	.671	3.683	30	.500	4.532
40	.804	2.848	40	.667	3.706	40	.495	4.556
50	.801	2.872	50	.662	3.730	50	.490	4.579
20° 00	.798	2.896	26° 00	.658	3.753	32° 00	.484	4.602
10	.794	2.920	10	.654	3.777	10	.479	4.626
20	.791	2.944	20	.650	3.801	20	.474	4.649
30	.787	2.968	30	.645	3.825	30	.469	4.673
40	.784	2.992	40	.641	3.849	40	.463	4.696
50	.780	3.016	50	.637	3.872	50	.458	4.720
21° 00	.777	3.040	27° 00	.632	3.896	33° 00	.452	4.743
10	.773	3.064	10	.627	3.920	10	.446	4.766
20	.770	3.087	20	.623	3.943	20	.441	4.790
30	.766	3.111	30	.618	3.967	30	.435	4.813
40	.762	3.135	40	.614	3.990	40	.430	4.836
50	.759	3.159	50	.609	4.014	50	.424	4.860
22° 00	.756	3.183	28° 00	.605	4.037	34° 00	.419	4.883
10	.752	3.207	10	.600	4.061	10	.413	4.906
20	.748	3.230	20	.595	4.084	20	.407	4.930
30	.745	3.254	30	.590	4.108	30	.401	4.953
40	.741	3.278	40	.585	4.132	40	.396	4.976
50	.737	3.302	50	.581	4.155	50	.390	5.000
23° 00	.733	3.326	29° 00	.576	4.179	35° 00	.384	5.023
10	.729	3.350	10	.571	4.203	10	.379	5.046
20	.725	3.373	20	.566	4.226	20	.373	5.070
30	.721	3.397	30	.561	4.250	30	.367	5.093
40	.717	3.421	40	.556	4.274	40	.361	5.116
50	.713	3.445	50	.551	4.297	50	.355	5.140
24° 00	.709	3.469	30° 00	.546	4.321	36° 00	.349	5.163
10	.705	3.493	10	.541	4.344	10	.343	5.186
20	.701	3.516	20	.536	4.368	20	.337	5.209
30	.697	3.540	30	.531	4.391	30	.331	5.232
40	.693	3.564	40	.526	4.415	40	.325	5.255
50	.688	3.587	50	.521	4.438	50	.319	5.278

TABLE 8.—VALUES OF k AND p FOR THE UNIT SPIRAL OF LENGTH 100 FT.—(*Continued*)

θ_s	k	p	θ_s	k	p	θ_s	k	p
37° 00′	49.313	5.301	43° 00′	49.075	6.130	49° 00′	48.805	6.943
10	.307	5.324	10	.068	6.153	10	.797	6.965
20	.301	5.348	20	.061	6.175	20	.789	6.987
30	.295	5.371	30	.054	6.198	30	.781	7.010
40	.289	5.394	40	.047	6 221	40	.773	7.032
50	.282	5.418	50	.040	6.244	50	.765	7.055
38° 00	.276	5.441	44° 00	.033	6.267	50° 00	.757	7.077
10	.269	5.464	10	.025	6.290	10	.749	7.099
20	.263	5.487	20	.018	6.312	20	.741	7.122
30	.256	5.510	30	.011	6.335	30	.733	7.144
40	.250	5.533	40	.004	6.358	40	.725	7 166
50	.244	5.566	50	48.997	6.380	50	.716	7.189
39° 00	.238	5.579	45° 00	.990	6.403	51° 00	.708	7.211
10	.231	5.602	10	.982	6.425	10	.700	7.233
20	.225	5.625	20	.975	6 448	20	.691	7.256
30	.218	5.648	30	.967	6.470	30	.683	7.278
40	.212	5.671	40	.959	6.493	40	.675	7.300
50	.205	5.695	50	.952	6.515	50	.666	7.322
40° 00	.199	5.718	46° 00	.944	6.538	52° 00	.658	7.344
10	.192	5.741	10	.936	6.561	10	.650	7.366
20	.185	5.763	20	.929	6.583	20	.641	7.389
30	.178	5.786	30	.921	6.606	30	.633	7.411
40	.172	5.809	40	.914	6.629	40	.624	7.433
50	.165	5.832	50	.906	6.651	50	.616	7.455
41° 00	.159	5.855	47° 00	.899	6.674	53° 00	.607	7.477
10	.152	5.878	10	.891	6.696	10	.598	7.499
20	.145	5.901	20	.884	6.719	20	.590	7.521
30	.138	5.924	30	.876	6.741	30	.581	7.543
40	.131	5.947	40	.868	6.763	40	.573	7.565
50	.124	5.970	50	.860	6.786	50	.564	7.587
42° 00	.118	5.993	48° 00	.852	6.808	54° 00	.556	7.609
10	.111	6.016	10	.844	6.830	10	.547	7.631
20	.104	6.038	20	.837	6.853	20	.538	7.653
30	.097	6.061	30	.829	6.875	30	.529	7.675
40	.090	6.084	40	.821	6.898	40	.520	7.697
50	.082	6.107	50	.813	6.920	50	.512	7.719

TABLE 9.—DEFLECTIONS FROM THE T.S. TO THE 10-POINT SPIRAL

θ_s	1	2	3	4	5	6	7	8	9	10	θ_s
1° 00′	0° 00.2′	0° 00.8′	0° 01.8′	0° 03.2′	0° 05.0′	0° 07.2′	0° 09.8′	0° 12.8′	0° 16.2′	0° 20	1° 00′
2 00	00.4	01.6	03.6	06.4	10.0	14.4	19.6	25.6	32.4	40	2 00
3 00	00.6	02.4	05.4	09.6	15.0	21.6	29.4	38.4	48.6	1 00	3 00
4 00	00.8	03.2	07.2	12.8	20.0	28.8	39.2	51.2	1 04.8	20	4 00
5	01.0	04.0	09.0	16.0	25.0	36.0	49.0	1 04.0	21.0	40	5
30	01.1	04.4	09.9	17.6	27.5	39.6	53.9	10.4	29.1	50	30
6	01.2	04.8	10.8	19.2	30.0	43.2	58.8	16.8	37.2	2 00	6
30	01.3	05.2	11.7	20.8	32.5	46.8	1 03.7	23.2	45.3	10	30
7	01.4	05.6	12.6	22.4	35.0	50.4	08.6	29.6	53.4	20	7
30	01.5	06.0	13.5	24.0	37.5	54.0	13.5	36.0	2 01.5	30	30
8	01.6	06.4	14.4	25.6	40.0	57.6	18.4	42.4	09.6	40	8
30	01.7	06.8	15.3	27.2	42.5	1 01.2	23.3	48.8	17.7	50	30
9	01.8	07.2	16.2	28.8	45.0	04.8	28.2	55.2	25.8	3 00	9
30	01.9	07.6	17.1	30.4	47.5	08.4	33.1	2 01.6	33.9	10	30
10	02.0	08.0	18.0	32.0	50.0	12.0	38.0	08.0	42.0	20	10
30	02.1	08.4	18.9	33.6	52.5	15.6	42.9	14.4	50.1	29.9	30
11	02.2	08.8	19.8	35.2	55.0	19.2	47.8	20.8	58.2	39.9	11
30	02.3	09.2	20.7	36.8	57.5	22.8	52.7	27.2	3 06.3	49.9	30
12	02.4	09.6	21.6	38.4	1 00.0	26.4	57.6	33.6	14.4	59.9	12
30	02.5	10.0	22.5	40.0	02.5	30.0	2 02.5	40.0	22.5	4 09.9	30
13	02.6	10.4	23.4	41.6	05.0	33.6	07.4	46.4	30.5	19.9	13
30	02.7	10.8	24.3	43.2	07.5	37.2	12.3	52.8	38.6	29.9	30
14	02.8	11.2	25.2	44.8	10.0	40.8	17.2	59.2	46.7	39.9	14
30	02.9	11.6	26.1	46.4	12.5	44.4	22.1	3 05.6	54.8	49.9	30
15	03.0	12.0	27.0	48.0	15.0	48.0	27.0	12.0	4 02.9	59.8	15
30	03.1	12.4	27.9	49.6	17.5	51.6	31.9	18.4	11.0	5 09.8	30
16	03.2	12.8	28.8	51.2	20.0	55.2	36.8	24.8	19.1	19.8	16
30	03.3	13.2	29.7	52.8	22.5	58.8	41.7	31.1	27.2	29.8	30
17	03.4	13.6	30.6	54.4	25.0	2 02.4	46.6	37.5	35.3	39.8	17
30	03.5	14.0	31.5	56.0	27.5	06.0	51.5	43.9	43.4	49.7	30

TABLE 9.—DEFLECTIONS FROM THE T.S. TO THE 10-POINT SPIRAL—(*Continued*)

θ_s	1	2	3	4	5	6	7	8	9	10	θ_s
18° 00′	0° 03.6′	0° 14.4′	0° 32.4′	0° 57.6′	1° 30.0′	2° 09.6′	2° 56.4′	3° 50.3′	4° 51.5′	5° 59.7′	18° 00′
30	03.7	14.8	33.3	59.2	32.5	13.2	3 01.3	56.7	59.5	6 09.7	30
19	03.8	15.2	34.2	1 00.8	35.0	16.8	06.2	4 03.1	5 07.6	19.7	19
30	03.9	15.6	35.1	02.4	37.5	20.4	11.1	09.5	15.7	29.6	30
20	04.0	16.0	36.0	04.0	40.0	24.0	16.0	15.9	23.8	39.6	20
30	04.1	16.4	36.9	05.6	42.5	27.6	20.9	22.3	31.9	49.6	30
21	04.2	16.8	37.8	07.2	45.0	31.2	25.8	28.7	40.0	59.5	21
30	04.3	17.2	38.7	08.8	47.5	34.8	30.8	35.1	48.1	7 09.5	30
22	04.4	17.6	39.6	10.4	50.0	38.4	35.5	41.5	56.1	19.5	22
30	04.5	18.0	40.5	12.0	52.5	42.0	40.5	47.9	6 04.2	29.5	30
23	04.6	18.4	41.4	13.6	55.0	45.6	45.3	54.2	12.3	39.4	23
30	04.7	18.8	42.3	15.2	57.5	49.2	50.2	5 00.7	20.4	49.4	30
24	04.8	19.2	43.2	16.8	2 00.0	52.8	55.1	07.0	28.4	59.3	24
30	04.9	19.6	44.1	18.4	02.5	56.4	4 00.0	13.4	36.5	8 09.3	30
25	05.0	20.0	45.0	20.0	05.0	3 00.0	04.9	19.8	44.6	19.2	25
30	05.1	20.4	45.9	21.6	07.5	03.6	09.8	26.2	52.7	29.2	30
26	05.2	20.8	46.8	23.2	10.0	07.2	14.7	32.6	7 00.7	39.1	26
30	05.3	21.2	47.7	24.8	12.5	10.8	19.6	39.0	08.8	49.1	30
27	05.4	21.6	48.6	26.4	15.0	14.4	24.5	45.4	16.9	59.0	27
30	05.5	22.0	49.5	28.0	17.5	18.0	29.4	51.8	25.0	9 09.0	30
28	05.6	22.4	50.4	29.6	20.0	21.5	34.3	58.1	33.0	18.9	28
30	05.7	22.8	51.3	31.2	22.5	25.2	39.2	6 04.5	41.1	28.9	30
29	05.8	23.2	52.2	32.8	25.0	28.8	44.1	10.9	49.2	38.8	29
30	05.9	23.6	53.1	34.4	27.5	32.4	49.0	17.3	57.2	48.8	30
30	06.0	24.0	54.0	36.0	30.0	35.9	53.9	23.7	8 05.3	58.7	30
30	06.1	24.4	54.9	37.6	32.5	39.6	58.8	30.1	13.4	10 08.6	30
31	06.2	24.8	55.8	39.2	35.0	43.1	5 03.6	36.4	21.4	18.5	31
30	06.3	25.2	56.7	40.8	37.5	46.7	08.5	42.8	29.5	28.4	30
32	06.4	25.6	57.6	42.4	40.0	50.3	13.4	49.2	37.5	38.4	32
30	06.5	26.0	58.5	44.0	42.5	53.9	18.3	55.5	45.6	48.3	30

TABLE 9.—DEFLECTIONS FROM THE T.S. TO THE 10-POINT SPIRAL—(*Continued*)

θ_s	1	2	3	4	5	6	7	8	9	10	θ_s
33° 00′	0° 06.6′	0° 26.4′	0° 59.4′	1° 45.6′	2° 45.0′	3° 57.5′	5° 23.2′	7° 01.9′	8° 53.7′	10° 58.2′	33° 00′
30	06.7	26.8	1 00.3	47.2	47.5	4 01.1	28.1	08.3	9 01 7	11 08.1	30
34	06.8	27.2	01.2	48.8	50.0	04.7	33.0	14.7	09.8	18.1	34
30	06.9	27.6	02.1	50.4	52.5	08 3	37.9	21.1	17.8	28.0	30
35	07.0	28.0	03.0	52.0	55.0	11.9	42.8	27.5	25.9	37.9	35
30	07.1	28.4	03.9	53.6	57.5	15 5	47.6	33.8	33.9	47.8	30
36	07.2	28.8	04.8	55.2	3 00.0	19.1	52.5	40.2	42.0	57.7	36
30	07.3	29.2	05.7	56 8	02.5	22.7	57.4	46.5	50.0	12 07.6	30
37	07.4	29.6	06.6	58.4	05.0	26.3	6 02.3	52.9	58.1	17.5	37
30	07.5	30.0	07.5	2 00.0	07.5	29.9	07.2	59.3	10 06.1	27.4	35
38	07.6	30.4	08.4	01.6	10.0	33.5	12.1	8 05.7	14.2	37.3	38
30	07.7	30.8	09 3	03.2	12 5	37.1	17.0	12 0	22.2	47.2	30
39	07.8	31.2	10.2	04.8	15.0	40.7	21.9	18.4	30.2	57.1	39
30	07.9	31.6	11.1	06.4	17.5	44.3	26.7	24.8	38.2	13 06.9	30
40	08.0	32.0	12.0	08.0	20.0	47 9	31 6	31.2	46.3	16.8	40
30	08.1	32 4	12.9	09.6	22.4	51 4	36 5	37.5	54.3	26.7	30
41	08.2	32.8	13 8	11.2	24.9	55 0	41.4	43 9	11 02.4	36 6	41
30	08.3	33.2	14 7	12.8	27.4	58.6	46.3	50.2	10.4	46.4	30
42	08.4	33.6	15.6	14 4	29.9	5 02 2	51.2	56 0	18 4	56.3	42
30	08.5	34.0	16.5	16.0	32.4	05.8	56.1	9 03 0	26.4	14 06.2	30
43	08.6	34 4	17.4	17.6	34.9	09.4	7 01.0	09.4	34 5	16.1	43
30	08.7	34 8	18.3	19.2	37.4	13.0	05.8	15.7	42.5	26.0	30
44	08.8	35.2	19.2	20.8	39.9	16.6	10 7	22.1	50.6	35.8	44
30	08.9	35.6	20.1	22.4	42.4	20.2	15 6	28.4	58 6	45.6	30
45	09.0	36.0	21.0	24 0	44.9	23.8	20.5	34.8	12 06.6	55.5	45
30	09.1	36.4	21.9	25.6	47.4	27.4	25.4	41.1	14.6	15 05.3	30
46	09.2	36.8	22.8	27 2	49.9	31.0	30.2	47.5	22 6	15.1	46
30	09.3	37.2	23.7	28.8	52.4	34.6	35.1	53.8	30.6	24.9	30
47	09.4	37.6	24.6	30.4	54.9	38 2	40.0	10 00.2	38.6	34.7	47
30	09.4	38.0	25.5	32.0	57.4	41.7	44 8	06.5	46.6	44.4	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL

Spiral angle θ_s	Distance ratio $= l/l_s =$										Spiral angle θ_s
	0.00	0.01	0.02	0.03	0 04	0.05	0 06	0.07	0.08	0.09	
5°	0° 00.0′	0° 00.0′	0° 00.1′	0° 00.1′	0° 00 2′	0° 00.2′	0° 00 4′	0° 00 5′	0° 00.6′	0° 00.8′	5°
6	00.0	00.0	00.1	00.1	00.2	00.3	00 4	00 6	00 8	01 0	6
7	00.0	00.0	00.1	00 1	00 2	00 3	00 5	00.7	00.9	01.1	7
8	00.0	00.0	00.1	00.1	00 3	00 4	00.6	00.8	01 0	01.3	8
9	00.0	00.0	00.1	00.2	00.3	00.4	00.6	00.9	01.2	01 5	9
10	00.0	00.0	00.1	00 2	00 3	00 5	00.7	01 0	01.3	01.6	10
11	00.0	00.0	00.1	00.2	00.4	00.5	00.8	01.1	01 4	01.8	11
12	00 0	00.0	00.1	00.2	00.4	00.6	00.9	01 2	01.5	01.9	12
13	00.0	00.0	00.1	00 2	00.4	00 6	00.9	01.3	01 7	02 1	13
14	00.0	00.0	00.1	00.3	00.4	00.7	01.0	01.4	01.8	02 3	14
15	00.0	00.0	00.1	00.3	00.5	00.7	01.1	01.5	01.9	02 4	15
16	00.0	00.0	00.1	00.3	00.5	00 8	01.2	01 6	02.0	02 6	16
17	00.0	00.0	00.1	00 3	00.5	00.8	01 2	01.7	02.2	02 8	17
18	00.0	00.0	00.1	00 3	00.6	00 9	01.3	01.8	02.3	02.9	18
19	00.0	00.0	00.2	00.3	00.6	00.9	01.4	01.9	02.4	03.1	19
20	00.0	00.0	00.2	00.4	00 6	01 0	01.4	02.0	02.6	03.2	20
21	00 0	00.0	00.2	00.4	00 7	01.0	01.5	02.1	02 7	03 4	21
22	00.0	00.0	00.2	00.4	00.7	01.1	01.6	02.2	02 8	03 6	22
23	00.0	00.0	00.2	00.4	00.7	01.1	01 7	02.3	02 9	03 7	23
24	00.0	00.0	00.2	00.4	00.8	01.2	01.7	02.4	03.1	03.9	24
25	00.0	00.0	00.2	00 5	00 8	01.2	01.8	02.5	03 2	04 0	25
26	00.0	00.0	00.2	00 5	00.8	01.3	01.9	02.5	03.3	04 2	26
27	00.0	00.1	00.2	00 5	00 9	01.3	01 9	02 6	03.5	04.4	27
28	00.0	00.1	00 2	00 5	00.9	01.4	02.0	02 7	03.6	04.5	28
29	00.0	00.1	00.2	00.5	00.9	01.4	02.1	02.8	03.7	04.7	29
30	00.0	00.1	00.2	00.5	01.0	01.5	02.2	02.9	03.8	04.9	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio $= l/l_s =$ 0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	Spiral angle θ_s
5°	0° 01.0′	0° 01.2′	0° 01.4′	0° 01.7′	0° 02.0′	0° 02.3′	0° 02.6′	0° 02.9′	0° 03.2′	0° 03.6′	5°
6	01.2	01.5	01.7	02.0	02.4	02.7	03.1	03.5	03.9	04.3	6
7	01.4	01.7	02.0	02.4	02.7	03.2	03.6	04.1	04.5	05.1	7
8	01.6	01.9	02.3	02.7	03.1	03.6	04.1	04.6	05.2	05.8	8
9	01.8	02.2	02.6	03.0	03.5	04.1	04.6	05.2	05.8	06.5	9
10	02.0	02.4	02.9	03.4	03.9	04.5	05.1	05.8	06.5	07.2	10
11	02.2	02.7	03.2	03.7	04.3	05.0	05.6	06.4	07.1	07.9	11
12	02.4	02.9	03.5	04.1	04.7	05.4	06.1	06.9	07.8	08.7	12
13	02.6	03.1	03.7	04.4	05.1	05.9	06.7	07.5	08.4	09.4	13
14	02.8	03.4	04.0	04.7	05.5	06.3	07.2	08.1	09.1	10.1	14
15	03.0	03.6	04.3	05.1	05.9	06.8	07.7	08.7	09.7	10.8	15
16	03.2	03.9	04.6	05.4	06.3	07.2	08.2	09.3	10.4	11.6	16
17	03.4	04.1	04.9	05.8	06.7	07.7	08.7	09.8	11.0	12.3	17
18	03.6	04.4	05.2	06.1	07.1	08.1	09.2	10.4	11.7	13.0	18
19	03.8	04.6	05.5	06.4	07.5	08.6	09.7	11.0	12.3	13.7	19
20	04.0	04.8	05.8	06.8	07.8	09.0	10.2	11.6	13.0	14.4	20
21	04.2	05.1	06.1	07.1	08.2	09.5	10.8	12.1	13.6	15.2	21
22	04.4	05.3	06.3	07.4	08.6	09.9	11.3	12.7	14.3	15.9	22
23	04.6	05.6	06.6	07.8	09.0	10.4	11.8	13.3	14.9	16.6	23
24	04.8	05.8	06.9	08.1	09.4	10.8	12.3	13.9	15.6	17.3	24
25	05.0	06.1	07.2	08.5	09.8	11.3	12.8	14.5	16.2	18.1	25
26	05.2	06.3	07.5	08.8	10.2	11.7	13.3	15.0	16.9	18.8	26
27	05.4	06.5	07.8	09.1	10.6	12.2	13.8	15.6	17.5	19.5	27
28	05.6	06.8	08.1	09.5	11.0	12.6	14.3	16.2	18.1	20.2	28
29	05.8	07.0	08.4	09.8	11.4	13.1	14.9	16.8	18.8	20.9	29
30	06.0	07.3	08.6	10.1	11.8	13.5	15.4	17.3	19.4	21.7	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s =										Spiral angle θ_s
	0.20	0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	
5°	0° 04.0′	0° 04.4′	0° 04.8′	0° 05.3′	0° 05.8′	0° 06.3′	0° 06.8′	0° 07.3′	0° 07.8′	0° 08.4′	5°
6	04.8	05.3	05.8	06.4	06.9	07.5	08.1	08.8	09.4	10.1	6
7	05.6	06.2	06.8	07.4	08.1	08.8	09.5	10.2	11.0	11.8	7
8	06.4	07.1	07.7	08.5	09.2	10.0	10.8	11.7	12.5	13.5	8
9	07.2	07.9	08.7	09.5	10.4	11.3	12.2	13.1	14.1	15.1	9
10	08.0	08.8	09.7	10.6	11.5	12.5	13.5	14.6	15.7	16.8	10
11	08.8	09.7	10.7	11.6	12.7	13.8	14.9	16.0	17.3	18.5	11
12	09.6	10.6	11.6	12.7	13.8	15.0	16.2	17.5	18.8	20.2	12
13	10.4	11.5	12.6	13.8	15.0	16.3	17.6	19.0	20.4	21.9	13
14	11.2	12.4	13.6	14.8	16.1	17.5	18.9	20.4	22.0	23.6	14
15	12.0	13.2	14.5	15.9	17.3	18.8	20.3	21.9	23.5	25.2	15
16	12.8	14.1	15.5	16.9	18.4	20.0	21.6	23.3	25.1	26.9	16
17	13.6	15.0	16.5	18.0	19.6	21.3	23.0	24.8	26.7	28.6	17
18	14.4	15.9	17.4	19.0	20.7	22.5	24.3	26.2	28.2	30.3	18
19	15.2	16.8	18.4	20.1	21.9	23.8	25.7	27.7	29.8	32.0	19
20	16.0	17.6	19.4	21.2	23.0	25.0	27.0	29.2	31.4	33.6	20
21	16.8	18.5	20.3	22.2	24.2	26.3	28.4	30.6	32.9	35.3	21
22	17.6	19.4	21.3	23.3	25.3	27.5	29.7	32.1	34.5	37.0	22
23	18.4	20.3	22.3	24.3	26.5	28.8	31.1	33.5	36.1	38.7	23
24	19.2	21.2	23.2	25.4	27.7	30.0	32.5	35.0	37.6	40.4	24
25	20.0	22.1	24.2	26.5	28.8	31.3	33.8	36.5	39.2	42.1	25
26	20.8	22.9	25.2	27.5	30.0	32.5	35.2	37.9	40.8	43.7	26
27	21.6	23.8	26.1	28.6	31.1	33.8	36.5	39.4	42.3	45.4	27
28	22.4	24.7	27.1	29.6	32.3	35.0	37.9	40.8	43.9	47.1	28
29	23.2	25.6	28.1	30.7	33.4	36.3	39.2	42.3	45.5	48.8	29
30	24.0	26.5	29.0	31.7	34.6	37.5	40.6	43.7	47.0	50.5	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.30	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	Spiral angle θ_s
5°	0° 09.0′	0° 09.6′	0° 10.2′	0° 10.9′	0° 11.6′	0° 12.3′	0° 13.0′	0° 13.7′	0° 14.4′	0° 15.2′	5°
6	10.8	11.5	12.3	13.1	13.9	14.7	15.6	16.4	17.3	18.3	6
7	12.6	13.5	14.3	15.3	16.2	17.2	18.1	19.2	20.2	21.3	7
8	14.4	15.4	16.4	17.4	18.5	19.6	20.7	21.9	23.1	24.3	8
9	16.2	17.3	18.4	19.6	20.8	22.1	23.3	24.6	26.0	27.4	9
10	18.0	19.2	20.5	21.8	23.1	24.5	25.9	27.4	28.9	30.4	10
11	19.8	21.1	22.5	24.0	25.4	27.0	28.5	30.1	31.8	33.5	11
12	21.6	23.1	24.6	26.1	27.7	29.4	31.1	32.9	34.7	36.5	12
13	23.4	25.0	26.6	28.3	30.1	31.9	33.7	35.6	37.5	39.6	13
14	25.2	26.9	28.7	30.5	32.3	34.3	36.3	38.3	40.4	42.6	14
15	27.0	28.8	30.7	32.7	34.7	36.8	38.9	41.1	43.3	45.6	15
16	28.8	30.8	32.8	34.9	37.0	39.2	41.5	43.8	46.2	48.7	16
17	30.6	32.7	34.8	37.0	39.3	41.7	44.1	46.6	49.1	51.7	17
18	32.4	34.6	36.9	39.2	41.6	44.1	46.7	49.3	52.0	54.8	18
19	34.2	36.5	38.9	41.4	43.9	46.6	49.3	52.0	54.9	57.8	19
20	36.0	38.4	41.0	43.6	46.2	49.0	51.8	54.8	57.7	1° 00.8	20
21	37.8	40.4	43.0	45.7	48.6	51.5	54.4	57.5	1° 00.7	03.9	21
22	39.6	42.3	45.1	47.9	50.9	53.9	57.0	1° 00.2	03.5	06.9	22
23	41.4	44.2	47.1	50.1	53.2	56.4	59.6	03.0	06.4	10.0	23
24	43.2	46.1	49.2	52.3	55.5	58.8	1° 02.2	05.7	09.3	13.0	24
25	45.0	48.1	51.2	54.5	57.8	1° 01.3	04.8	08.5	12.2	16.1	25
26	46.8	50.0	53.3	56.6	1° 00.1	03.7	07.4	11.2	15.1	19.1	26
27	48.6	51.9	55.3	58.8	02.4	06.2	10.0	13.9	18.0	22.1	27
28	50.4	53.8	57.3	1° 01.0	04.7	08.6	12.6	16.7	20.9	25.2	28
29	52.2	55.7	59.4	03.2	07.1	11.1	15.2	19.4	23.8	28.2	29
30	54.0	57.7	1° 01.4	05.3	09.4	13.5	17.8	22.1	26.6	31.3	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s =										Spiral angle θ_s
	0.40	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	
5°	0° 16.0′	0° 16.8′	0° 17.6′	0° 18.5′	0° 19.4′	0° 20.3′	0° 21.2′	0° 22.1′	0° 23.0′	0° 24.0′	5°
6	19.2	20.2	21.2	22.2	23.2	24.3	25.4	26.5	27.6	28.8	6
7	22.4	23.5	24.7	25.9	27.1	28.4	29.6	30.9	32.3	33.6	7
8	25.6	26.9	28.2	29.6	31.0	32.4	33.9	35.3	36.9	38.4	8
9	28.8	30.3	31.8	33.3	34.9	36.5	38.1	39.8	41.5	43.2	9
10	32.0	33.6	35.3	37.0	38.7	40.5	42.3	44.2	46.1	48.0	10
11	35.2	37.0	38.8	40.7	42.6	44.6	46.6	48.6	50.7	52.8	11
12	38.4	40.3	42.3	44.4	46.5	48.6	50.8	53.0	55.3	57.6	12
13	41.6	43.7	45.9	48.1	50.3	52.7	55.0	57.4	59.9	1° 02.4	13
14	44.8	47.1	49.4	51.8	54.2	56.7	59.3	1° 01.9	1° 04.5	07.2	14
15	48.0	50.4	52.9	55.5	58.1	1° 00.8	1° 03.5	06.3	09.1	12.0	15
16	51.2	53.8	56.5	59.2	1° 02.0	04.8	07.7	10.7	13.7	16.8	16
17	54.4	57.2	1° 00.0	1° 02.9	05.8	08.9	11.9	15.1	18.3	21.6	17
18	57.6	1° 00.5	03.5	06.6	09.7	12.9	16.2	19.5	22.9	26.4	18
19	1° 00.8	03.9	07.0	10.3	13.6	17.0	20.4	23.9	27.6	31.2	19
20	04.0	07.2	10.6	14.0	17.4	21.0	24.6	28.4	32.2	36.0	20
21	07.2	10.6	14.1	17.7	21.3	25.1	28.9	32.8	36.8	40.8	21
22	10.4	14.0	17.6	21.4	25.2	29.1	33.1	37.2	41.4	45.6	22
23	13.6	17.3	21.1	25.1	29.1	33.2	37.3	41.6	46.0	50.4	23
24	16.8	20.7	24.7	28.8	32.9	37.2	41.6	46.0	50.6	55.2	24
25	20.0	24.1	28.2	32.5	36.8	41.3	45.8	50.5	55.2	2° 00.0	25
26	23.2	27.4	31.7	36.2	40.7	45.3	50.0	54.9	59.8	04.8	26
27	26.4	30.8	35.3	39.9	44.5	49.4	54.3	59.3	2° 04.4	09.6	27
28	29.6	34.1	38.8	43.5	48.4	53.4	58.5	2° 03.7	09.0	14.4	28
29	32.8	37.5	42.3	47.2	52.3	57.5	2° 02.7	08.1	13.6	19.2	29
30	36.0	40.9	45.8	50.9	56.2	2° 01.5	07.0	12.5	18.2	24.0	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.500	0.502	0.504	0.506	0.508	0.510	0.512	0.514	0.516	0.518	Spiral angle θ_s
5°	0° 25.0′	0° 25.2′	0° 25.4′	0° 25.6′	0° 25.8′	0° 26.0′	0° 26.2′	0° 26.4′	0° 26.6′	0° 26.8′	5°
6	30.0	30.2	30.5	30.7	31.0	31.2	31.5	31.7	32.0	32.2	6
7	35.0	35.3	35.6	35.8	36.1	36.4	36.7	37.0	37.3	37.6	7
8	40.0	40.3	40.6	41.0	41.3	41.6	41.9	42.3	42.6	42.9	8
9	45.0	45.4	45.7	46.1	46.4	46.8	47.2	47.6	47.9	48.3	9
10	50.0	50.4	50.8	51.2	51.6	52.0	52.4	52.8	53.3	53.7	10
11	55.0	55.4	55.9	56.3	56.8	57.2	57.7	58.1	58.6	59.0	11
12	1° 00.0	1° 00.5	1° 01.0	1° 01.4	1° 01.9	1° 02.4	1° 02.9	1° 03.4	1° 03.9	1° 04.4	12
13	05.0	05.5	06.0	06.6	07.1	07.6	08.2	08.7	09.2	09.8	13
14	10.0	10.6	11.1	11.7	12.3	12.8	13.4	14.0	14.6	15.1	14
15	15.0	15.6	16.2	16.8	17.4	18.0	18.6	19.3	19.9	20.5	15
16	20.0	20.6	21.3	21.9	22.6	23.2	23.9	24.5	25.2	25.9	16
17	25.0	25.7	26.4	27.1	27.7	28.4	29.1	29.8	30.5	31.2	17
18	30.0	30.7	31.4	32.2	32.9	33.6	34.4	35.1	35.9	36.6	18
19	35.0	35.8	36.5	37.3	38.1	38.8	39.6	40.4	41.2	42.0	19
20	40.0	40.8	41.6	42.4	43.2	44.0	44.9	45.7	46.5	47.3	20
21	45.0	45.8	46.7	47.5	48.4	49.2	50.1	51.0	51.8	52.7	21
22	50.0	50.9	51.8	52.7	53.6	54.4	55.3	56.3	57.2	58.1	22
23	55.0	55.9	56.8	57.8	58.7	59.6	2° 00.6	2° 01.5	2° 02.5	2° 03.4	23
24	2° 00.0	2° 01.0	2° 01.9	2° 02.9	2° 03.9	2° 04.8	05.8	06.8	07.8	08.8	24
25	05.0	06.0	07.0	08.0	09.0	10.0	11.1	12.1	13.1	14.2	25
26	10.0	11.0	12.1	13.1	14.2	15.2	16.3	17.4	18.4	19.5	26
27	15.0	16.1	17.2	18.3	19.3	20.4	21.5	22.7	23.8	24.9	27
28	20.0	21.1	22.2	23.4	24.5	25.6	26.8	27.9	29.1	30.2	28
29	25.0	26.2	27.3	28.5	29.7	30.8	32.0	33.2	34.4	35.6	29
30	30.0	31.2	32.4	33.6	34.8	36.0	37.3	38.5	39.7	41.0	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s =										Spiral angle θ_s
	0.520	0.522	0.524	0.526	0.528	0.530	0.532	0.534	0.536	0.538	
5°	0° 27.0′	0° 27.3′	0° 27.5′	0° 27.7′	0° 27.9′	0° 28.1′	0° 28.3′	0° 28.5′	0° 28.7′	0° 28.9′	5°
6	32.5	32.7	33.0	33.2	33.5	33.7	34.0	34.2	34.5	34.7	6
7	37.9	38.2	38.4	38.7	29.0	39.3	39.6	39.9	40.2	40.5	7
8	43.3	43.6	43.9	44.3	44.6	44.9	45.3	45.6	46.0	46.3	8
9	48.7	49.1	49.4	49.8	50.2	50.6	50.9	51.3	51.7	52.1	9
10	54.1	54.5	54.9	55.3	55.8	56.2	56.6	57.0	57.5	57.9	10
11	59.5	60.0	1° 00.4	1° 00.9	1° 01.3	1° 01.8	1° 02.3	1° 02.7	1° 03.2	1° 03.7	11
12	1° 04.9	1° 05.4	05.9	06.4	06.9	07.4	07.9	08.4	09.0	09.5	12
13	10.3	10.9	11.4	11.9	12.5	13.0	13.6	14.1	14.7	15.3	13
14	15.7	16.3	16.9	17.5	18.1	18.7	19.2	19.8	20.4	21.0	14
15	21.1	21.8	22.4	23.0	23.6	24.3	24.9	25.6	26.2	26.8	15
16	26.5	27.2	27.9	28.5	29.2	29.9	30.6	31.3	31.9	32.6	16
17	31.9	32.6	33.4	34.1	34.8	35.5	36.2	37.0	37.7	38.4	17
18	37.3	38.1	38.8	39.6	40.4	41.1	41.9	42.7	43.4	44.2	18
19	42.8	43.5	44.3	45.1	45.9	46.7	47.6	48.4	49.2	50.0	19
20	48.2	49.0	49.8	50.7	51.5	52.4	53.2	54.1	54.9	55.8	20
21	53.6	54.4	55.3	56.2	57.1	58.0	58.9	59.8	2° 00.7	2° 01 6	21
22	59.0	59.9	2° 00.8	2° 01.7	2° 02.7	2° 03.6	2° 04.5	2° 05 5	06.4	07.4	22
23	2° 04.4	2° 05.3	06.3	07.3	08.2	09.2	10.2	11.2	12 2	13 1	23
24	09.8	10.8	11.8	12.8	13.8	14.8	15.8	16.9	17.9	18.9	24
25	15.2	16.2	17.3	18.3	19.4	20.4	21.5	22.6	23.6	24.7	25
26	20.6	21.7	22.8	23.9	25.0	26.1	27.2	28.3	29.4	30.5	26
27	26.0	27.1	28.3	29.4	30.5	31.7	32 8	34.0	35.1	36.3	27
28	31.4	32.6	33.7	34.9	36.1	37.3	38.5	39.7	40.9	42.1	28
29	36.8	38.0	39.2	40.5	41.7	42.9	44.1	45.4	46.6	47.9	29
30	42.2	43.5	44.7	46.0	47.3	48.5	49.8	51.1	52.4	53.6	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.540	0.542	0.544	0.546	0.548	0.550	0.552	0.554	0.556	0.558	Spiral angle θ_s
5°	0° 29.2′	0° 29.4′	0° 29 6′	0° 29.8′	0° 30.0′	0° 30 3′	0° 30 5′	0° 30.7′	0° 30 9′	0° 31.1′	5°
6	35.0	35.3	35.5	35 8	36.0	36.3	36.6	36.8	37.1	37.4	6
7	40.8	41.1	41.4	41.7	42.0	42.4	42.7	43.0	43.3	43.6	7
8	46.7	47.0	47.4	47.7	48.1	48.4	48.8	49.1	49.5	49.8	8
9	52.5	52.9	53.3	53.7	54.1	54.5	54.9	55.2	55.6	56.1	9
10	58.3	58.8	59.2	59.6	1° 00 1	1° 00.5	1° 00 9	1° 01.4	1° 01.8	1° 02.3	10
11	1° 04.2	1° 04.6	1° 05.1	1° 05.6	06.1	06 6	07.0	07.5	08.0	08.5	11
12	10.0	10.5	11.0	11.6	12.1	12 6	13.1	13.7	14.2	14.7	12
13	15.8	16.4	16.9	17.5	18.1	18.7	19 2	19.8	20.4	21.0	13
14	21.7	22.3	22.9	23.5	24.1	24.7	25.3	25.9	26.6	27.2	14
15	27.5	28.1	28.8	29.4	30.1	30.8	31.4	32.1	32.7	33.4	15
16	33.3	34.0	34.7	35.4	36.1	36.8	37.5	38.2	38.9	39.6	16
17	39.1	39.9	40.6	41.4	42.1	42.9	43 6	44.4	45.1	45.9	17
18	45.0	45.8	46.5	47.3	48.1	48.9	49.7	50.5	51.3	52 1	18
19	50.8	51.6	52.5	53.3	54.1	55.0	55.8	56.6	57.5	58.3	19
20	56.6	57.5	58 4	59.3	2° 00.1	2° 01.0	2° 01.8	2° 02.8	2° 03.6	2° 04 5	20
21	2° 02.5	2° 03.4	2° 04.3	2° 05.2	06.1	07.0	08.0	08.9	09.8	10.8	21
22	08.3	09.3	10.2	11.2	12.1	13.1	14.1	15.0	16.0	17.0	22
23	14.1	15.1	16.1	17.1	18.1	19.1	20.2	21.2	22.2	23.2	23
24	20.0	21.0	22.0	23.1	24.1	25.2	26.2	27.3	28.3	29.4	24
25	25.8	26.9	28.0	29.0	30.1	31.2	32.3	33.4	34.6	35.7	25
26	31.6	32.7	33.9	35.0	36.1	37.3	38.4	39.6	40.7	41.9	26
27	37.4	38.6	39.8	41.0	42.1	43.3	44.5	45.7	46.9	48.1	27
28	43.3	44.5	45.7	46.9	48.2	49.4	50.6	51.8	53.1	54.3	28
29	49.1	50.4	51.6	52.9	54.2	55.4	56 7	58 0	59.3	3° 00 6	29
30	54.9	56.2	57.5	58.8	3° 00.2	3° 01.5	3° 02 8	3° 04.1	3° 05.4	06.8	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.560	0.562	0.564	0.566	0.568	0.570	0.572	0.574	0.576	0.578	Spiral angle θ_s
5°	0° 31.4′	0° 31.6′	0° 31.8′	0° 32.0′	0° 32.3′	0° 32.5′	0° 32.7′	0° 33.0′	0° 33.2′	0° 33.4′	5°
6	37.6	37.9	38.2	38.4	38.7	39.0	39.3	39.5	39.8	40 1	6
7	43.9	44.2	44.5	44.9	45.2	45.5	45.8	46.1	46.5	46.8	7
8	50.2	50.5	50.9	51.3	51.6	52.0	52.4	52.7	53.1	53.5	8
9	56.5	56.9	57.3	57.7	58.1	58.5	58.9	59.3	59.7	1° 00.1	9
10	1° 02.7	1° 03.2	1° 03.6	1° 04.1	1° 04.5	1° 05.0	1° 05.4	1° 05.9	1° 06.4	06.8	10
11	09.0	09.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	11
12	15.3	15.8	16.3	16.9	17.4	18.0	18.5	19.1	19.6	20.2	12
13	21.5	22.1	22.7	23.3	23.9	24.5	25.1	25.7	26.3	26.9	13
14	27.8	28.4	29.1	29.7	30.3	31.0	31.6	32.3	32.9	33.5	14
15	34.1	34.8	35.4	36.1	36.8	37.5	38.2	38.8	39.5	40.2	15
16	40.4	41.1	41.8	42.5	43.2	44.0	44.7	45.4	46.2	46.9	16
17	46.6	47.4	48.2	48.9	49.7	50.5	51.2	52.0	52.8	53.6	17
18	52.9	53.7	54.5	55.3	56.1	57.0	57.8	58.6	59.4	2° 00.3	18
19	59.2	2° 00.0	2° 00.9	2° 01.7	2° 02.6	2° 03.5	2° 04.3	2° 05.2	2° 06.1	06.9	19
20	2° 05.4	06.3	07.2	08.1	09.0	10.0	10.9	11.8	12.7	13.6	20
21	11.7	12.6	13.6	14.5	15.5	16.5	17.4	18.4	19.3	20.3	21
22	18.0	19.0	20.0	20.9	21.9	22.9	23.9	25.0	26.0	27.0	22
23	24.2	25.3	26.3	27.3	28.4	29.4	30.5	31.5	32.6	33.7	23
24	30.5	31.6	32.7	33.8	34.8	35.9	37.0	38.1	39.2	40.3	24
25	36.8	37.9	39.0	40.2	41.3	42.4	43.6	44.7	45.9	47.0	25
26	43.1	44.2	45.4	46.6	47.7	48.9	50.1	51.3	52.5	53.7	26
27	49.3	50.5	51.7	53.0	54.2	55.4	56.7	57 9	59.1	3° 00.4	27
28	55.6	56.8	58.1	59.4	3° 00.6	3° 01.9	3° 03.2	3° 04.5	3° 05.7	07.1	28
29	3° 01.9	3° 03.2	3° 04.5	3° 05.8	07.1	08.4	09.7	11.1	12.4	13.7	29
30	08.1	09.5	10.8	12.2	13.5	14.9	16.3	17.6	19.0	20.4	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.580	0.582	0.584	0.586	0.588	0.590	0.592	0.594	0.596	0.598	Spiral angle θ_s
5°	0° 33.6′	0° 33.9′	0° 34.1′	0° 34.3′	0° 34.6′	0° 34.8′	0° 35 1′	0° 35.2′	0° 35.5′	0° 35.8′	5°
6	40.4	40.7	40.9	41.2	41.5	41.8	42 1	42.3	42.6	42.9	6
7	47.1	47.4	47.8	48.1	48.4	48.7	49.1	49.4	49.7	50.1	7
8	53.8	54.2	54.6	54.9	55.3	55.7	56.1	56.5	56.8	57.2	8
9	1° 00.6	1° 01.1	1° 01.4	1° 01.8	1° 02.2	1° 02.7	1° 03.1	1° 03.5	1° 03.9	1° 04.4	9
10	07.3	07.7	08.2	08.7	09.2	09 6	10.1	10.6	11.0	11.5	10
11	14.0	14.5	15.0	15.6	16.1	16.6	17 1	17.6	18.2	18.7	11
12	20.7	21.3	21.9	22.4	23.0	23.5	24.1	24.7	25.3	25.8	12
13	27.5	28.1	28.7	29.3	29.9	30.5	31.1	31.7	32.4	33.0	13
14	34.2	34.8	35.5	36.2	36.8	37.5	38.1	38.8	39.5	40.1	14
15	40.9	41.6	42.3	43.0	43.7	44.4	45.1	45.9	46.6	47.3	15
16	47.7	48.4	49.1	49.9	50.6	51.4	52 2	52 9	53.7	54.4	16
17	54.4	55.2	56.0	56.8	57.6	58.4	59.2	2° 00.0	2° 00.8	2° 01.6	17
18	2° 01.1	2° 01.9	2° 02.8	2° 03.6	2° 04.5	2° 05.3	2° 06.2	07 0	07.9	08.7	18
19	07.8	08.7	09.6	10.5	11.4	12.3	13.2	14.1	15.0	15.9	19
20	14.6	15.5	16.4	17.4	18.3	19.2	20.2	21.1	22.1	23.0	20
21	21.3	22.2	23.2	24.2	25.2	26.2	27.2	28.2	29.2	30.2	21
22	28.0	29.0	30.0	31.1	32.1	33.1	34.2	35.2	36.3	37.3	22
23	34.7	35.8	36.9	37.9	39.0	40.1	41.2	42.3	43.4	44.5	23
24	41.5	42.6	43.7	44.8	45.9	47.1	48.2	49.3	50.5	51.6	24
25	48.2	49.3	50.5	51.7	52.8	54.0	55 2	56.4	57.6	58.8	25
26	54.9	56.1	57.3	58.5	59.8	3° 01.0	3° 02.2	3° 03.4	3° 04.7	3° 05.9	26
27	3° 01.6	3° 02.9	3° 04.1	3° 05.4	3° 06.7	07.9	09.2	10.5	11.8	13.1	27
28	08.3	09.7	11.0	12.3	13.6	14.9	16 2	17.5	18.9	20.2	28
29	15.1	16.4	17.8	19.1	20.5	21.9	23.2	24.6	26.0	27.4	29
30	21.8	23.2	24.6	26.0	27.4	28.8	30.2	31.6	33.1	34.5	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.600	0.602	0.604	0.606	0.608	0.610	0.612	0.614	0.616	0.618	Spiral angle θ_s
5°	0° 36.0′	0° 36.2′	0° 36.5′	0° 36.7′	0° 37.0′	0° 37.2′	0° 37.5′	0° 37.7′	0° 38.0′	0° 38.2′	5°
6	43.2	43.5	43.8	44.1	44.4	44.7	45.0	45.2	45.5	45.8	6
7	50.4	50.7	51.1	51.4	51.8	52.1	52.4	52.8	53.1	53.5	7
8	57.6	58.0	58.4	58.8	59.2	59.5	59.9	1° 00.3	1° 00.7	1° 01.1	8
9	1° 04.8	1° 05.2	1° 05.7	1° 06.1	1° 06.5	1° 07.0	1° 07.4	07.9	08.3	08.8	9
10	12.0	12.5	13.0	13.5	13.9	14.4	14.9	15.4	15.9	16.4	10
11	19.2	19.7	20.3	20.8	21.3	21.9	22.4	22.9	23.5	24.0	11
12	26.4	27.0	27.6	28.1	28.7	29.3	29.9	30.5	31.1	31.7	12
13	33.6	34.2	34.9	35.5	36.1	36.8	37.4	38.0	38.7	39.3	13
14	40.8	41.5	42.2	42.8	43.5	44.2	44.9	45.6	46.3	46.9	14
15	48.0	48.7	49.4	50.2	50.9	51.6	52.4	53.1	53.8	54.6	15
16	55.2	56.0	56.7	57.5	58.3	59.1	59.9	2° 00.6	2° 01.4	2° 02.2	16
17	2° 02.4	2° 03.2	2° 04.0	2° 04.9	2° 05.7	2° 06.5	2° 07.3	08.2	09.0	09.8	17
18	09.6	10.5	11.3	12.2	13.1	14.0	14.8	15.7	16.6	17.5	18
19	16.8	17.7	18.6	19.5	20.5	21.4	22.3	23.2	24.2	25.1	19
20	24.0	24.9	25.9	26.9	27.9	28.8	29.8	30.8	31.8	32.8	20
21	31.2	32.2	33.2	34.2	35.2	36.3	37.3	38.3	39.4	40.4	21
22	38.4	39.4	40.5	41.6	42.6	43.7	44.8	45.9	46.9	48.0	22
23	45.6	46.7	47.8	48.9	50.0	51.1	52.3	53.4	54.5	55.7	23
24	52.8	53.9	55.1	56.2	57.4	58.6	59.7	2° 00.9	3° 02.1	3° 03.3	24
25	3° 00.0	3° 01.2	3° 02.4	3° 03.6	3° 04.8	3° 06.0	3° 07.2	08.5	09.7	10.9	25
26	07.2	08.4	09.7	10.9	12.2	13.4	14.7	16.0	17.3	18.6	26
27	14.4	15.7	17.0	18.3	19.6	20.9	22.2	23.5	24.9	26.2	27
28	21.6	22.9	24.3	25.6	27.0	28.3	29.7	31.1	32.4	33.8	28
29	28.7	30.1	31.5	32.9	34.3	35.8	37.2	38.6	40.0	41.5	29
30	35.9	37.4	38.8	40.3	41.7	43.2	44.7	46.1	47.6	49.1	30

Table 10.—Deflections from the T.S. to Any Point on a Spiral—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.620	0.622	0.624	0.626	0.628	0.630	0.632	0.634	0.636	0.638	Spiral angle θ_s
5°	0° 38.4′	0° 38.7′	0° 38.9′	0° 39.2′	0° 39.4′	0° 39.7′	0° 39.9′	0° 40.2′	0° 40.5′	0° 40.7′	5°
6	46.1	46.4	46.7	47.0	47.3	47.6	47.9	48.2	48.5	48.9	6
7	53.8	54.2	54.5	54.9	55.2	55.6	55.9	56.3	56.6	57.0	7
8	1° 01.5	1° 01.9	1° 02.3	1° 02.7	1° 03.1	1° 03.5	1° 03.9	1° 04.3	1° 04.7	1° 05.1	8
9	09.2	09.6	10.1	10.5	11.0	11.4	11.9	12.4	12.8	13.3	9
10	16.9	17.4	17.9	18.4	18.9	19.4	19.9	20.4	20.9	21.4	10
11	24.6	25.1	25.7	26.2	26.8	27.3	27.9	28.4	29.0	29.6	11
12	32.3	32.9	33.5	34.1	34.7	35.3	35.9	36.5	37.1	37.7	12
13	39.9	40.6	41.2	41.9	42.5	43.2	43.9	44.5	45.2	45.8	13
14	47.6	48.3	49.0	49.7	50.4	51.1	51.8	52.6	53.3	54.0	14
15	55.3	56.1	56.8	57.6	58.3	59.1	59.8	2° 00.6	2° 01.3	2° 02.1	15
16	2° 03.0	2° 03.8	2° 04.6	2° 05.4	2° 06.2	2° 07.0	2° 07.8	08.6	09.4	10.2	16
17	10.7	11.5	12.4	13.2	14.1	14.9	15.8	16.7	17.5	18.4	17
18	18.4	19.3	20.2	21.1	22.0	22.9	23.8	24.7	25.6	26.5	18
19	26.1	27.0	27.9	28.9	29.9	30.8	31.8	32.7	33.7	34.7	19
20	33.7	34.7	35.7	36.7	37.7	38.7	39.8	40.8	41.8	42.8	20
21	41.4	42.4	43.5	44.6	45.6	46.7	47.7	48.8	49.9	50.9	21
22	49.1	50.2	51.3	52.4	53.5	54.6	55.7	56.8	57.9	59.1	22
23	56.8	57.9	59.1	2° 00.3	3° 01.4	3° 02.5	3° 03.7	3° 04.9	3° 06.0	3° 07.2	23
24	3° 04.5	3° 05.7	3° 06.9	08.0	09.3	10.5	11.7	12.9	14.1	15.3	24
25	12.2	13.4	14.6	15.9	17.1	18.4	19.7	20.9	22.2	23.5	25
26	19.8	21.1	22.4	23.7	25.0	26.3	27.6	29.0	30.3	31.6	26
27	27.5	28.9	30.2	31.6	32.9	34.3	35.6	37.0	38.4	39.7	27
28	35.2	36.6	38.0	39.4	40.8	42.2	43.6	45.0	46.4	47.9	28
29	42.9	44.3	45.8	47.2	48.7	50.1	51.6	53.1	54.5	56.0	29
30	50.6	52.1	53.6	55.0	56.5	58.1	59.6	4° 01.1	4° 02.6	4° 04.1	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s =										Spiral angle θ_s
	0.640	0.642	0.644	0.646	0.648	0.650	0.652	0.654	0.656	0.658	
5°	0° 41.0′	0° 41.2′	0° 41.5′	0° 41.7′	0° 42.0′	0° 42.3′	0° 42.5′	0° 42.8′	0° 43.0′	0° 43.3′	5°
6	49.2	49.5	49.8	50.1	50.4	50.7	51.0	51.3	51.6	52.0	6
7	57.3	57.7	58.1	58.4	58.8	59.2	59.5	59 9	1° 00.3	1° 00.6	7
8	1° 05.5	1° 05.9	1° 06.4	1° 06.8	1° 07.2	1° 07.6	1° 08.0	1° 08.4	08.9	09.3	8
9	13.7	14.2	14.7	15.1	15.6	16.1	16.5	17.0	17.5	17.9	9
10	21.9	22.4	23.0	23.5	24.0	24.5	25.0	25.5	26.1	26.6	10
11	30.1	30.7	31.2	31.8	32.4	33.0	33.5	34.1	24.7	35.3	11
12	38.3	38.9	39.5	40.2	40.8	41.4	42.0	42.7	43.3	43.9	12
13	46.5	47.2	47.8	48.5	49 2	49.9	50.5	51.2	51.9	52.6	13
14	54.7	55.4	56.1	56.9	57.6	58.3	59.0	59.8	2° 00.5	2° 01.2	14
15	2° 02.9	2° 03.6	2° 04.4	2° 05.2	2° 05.9	2° 06.7	2° 07.5	2° 08.3	09.1	09.9	15
16	11.1	11.9	12.7	13.5	14 4	15.2	16.0	16.9	17.7	18.5	16
17	19.3	20.1	21.0	21 9	22.8	23.6	24.5	25.4	26.3	27 2	17
18	27.4	28.4	29.3	30.2	31 2	32.1	33.0	34 0	34 9	35.9	18
19	35.6	36.6	37.6	38.6	39.5	40.5	41.5	42.5	43.5	44.5	19
20	43.8	44.8	45.9	46.9	47.9	49 0	50.0	51.1	52 1	53.2	20
21	52.0	53.1	54.2	55.2	56.3	57.4	58 5	59.6	3° 00 7	3° 01.8	21
22	3° 00.2	3° 01.3	3° 02.5	3° 03 6	3° 04.7	3° 05.9	3° 07.0	3° 08.2	0.93	10.5	22
23	08.4	09.6	10.7	11 9	13.1	14 3	15.5	16.7	17.9	19 1	23
24	16.6	17.8	19.0	20 3	21.5	22.8	24.0	25.2	26.5	27.8	24
25	24.7	26.0	27.3	28.6	29.9	31.2	32.5	33.8	35.1	36 4	25
26	32.9	34 3	35.6	37.0	38.3	39.6	41 0	42 3	43.7	45.1	26
27	41.1	42.5	43.9	45.3	46.7	48.1	49 5	50 9	52.3	53.7	27
28	49.3	50.7	52 2	53.6	55.1	56.5	58 0	59 4	4° 00.9	4° 02.4	28
29	57.5	59.0	4° 00.5	4° 02.0	4° 03.5	4° 05.0	4° 06.5	4° 08.0	09.5	11.0	29
30	4° 05.7	4° 07.2	08.7	10.3	11.8	13.4	15.0	16.5	18.1	19.7	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.660	0.662	0.664	0.666	0.668	0.670	0.672	0.674	0.676	0.678	Spiral angle θ_s
5°	0° 43.6′	0° 43.8′	0° 44.1′	0° 44.4′	0° 44.6′	0° 44.9′	0° 45.2′	0° 45.4′	0° 45.7′	0° 46.0′	5°
6	52.3	52.6	52.9	53.2	53.6	53.9	54.2	54.5	54.8	55.2	6
7	1° 01.0	1° 01.4	1° 01.7	1° 02.1	1° 02.5	1° 02.9	1° 03.2	1° 03.6	1° 04.0	1° 04.4	7
8	09.7	10.1	10.5	11.0	11.4	11.8	12.3	12.7	13.1	13.6	8
9	18.4	18.9	19.4	19.8	20.3	20.8	21.3	21.8	22.3	22.7	9
10	27.1	27.7	28.2	28.7	29.2	29.8	30.3	30.9	31.4	31.9	10
11	35.8	36.4	37.0	37.6	38.2	38.8	39.4	39.9	40.5	41.1	11
12	44.5	45.2	45.8	46.5	47.1	47.7	48.4	49.0	49.7	50.3	12
13	53.3	53.9	54.6	55.3	56.0	56.7	57.4	58.1	58.8	59.5	13
14	2° 02.0	2° 02.7	2° 03.4	2° 04.2	2° 04.9	2° 05.7	2° 06.4	2° 07.2	2° 08.0	2° 08.7	14
15	10.7	11.5	12.3	13.1	13.9	14.7	15.5	16.3	17.1	17.9	15
16	19.4	20.2	21.1	21.9	22.8	23.6	24.5	25.4	26.2	27.0	16
17	28.1	29.0	29.9	30.8	31.7	32.6	33.5	34.4	35.4	36.3	17
18	36.8	37.8	38.7	39.7	40.6	41.6	42.6	43.5	44.5	45.5	18
19	45.5	46.5	47.5	48.5	49.5	50.5	51.6	52.6	53.6	54.6	19
20	54.2	55.3	56.3	57.4	58.5	59.5	3° 00.6	3° 01.7	3° 02.8	3° 03.8	20
21	3° 02.9	3° 04.0	3° 05.1	3° 06.3	3° 07.4	3° 08.5	09.6	10.8	11.9	13.0	21
22	11.6	12.8	14.0	15.1	16.3	17.5	18.7	19.8	21.0	22.2	22
23	20.3	21.5	22.8	24.0	25.2	26.4	27.7	28.9	30.2	31.4	23
24	29.0	30.3	31.6	32.8	34.1	35.4	36.7	38.0	39.3	40.6	24
25	37.7	39.1	40.4	41.7	43.0	44.4	45.7	47.0	48.3	49.8	25
26	46.4	47.8	49.2	50.6	52.0	53.4	54.7	56.1	57.5	59.0	26
27	55.1	56.6	58.0	59.4	4° 00.9	4° 02.3	4° 03.8	4° 05.2	4° 06.7	4° 08.1	27
28	4° 03.9	4° 05.3	4° 06.8	4° 08.3	09.8	11.3	12.8	14.3	15.8	17.3	28
29	12.5	14.1	15.6	17.2	18.7	20.2	21.8	23.4	24.9	26.5	29
30	21.2	22.8	24.4	26.0	27.6	29.2	30.8	32.4	34.0	35.7	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.680	0.682	0.684	0.686	0.688	0.690	0.692	0.694	0.696	0.698	Spiral angle θ_s
5°	0° 46.2′	0° 46.5′	0° 46.8′	0° 47.1′	0° 47.3′	0° 47.6′	0° 47.9′	0° 48.2′	0° 48.4′	0° 48.7′	5°
6	55.5	55.8	56.1	56.5	56.8	57.1	57.5	57.8	58.1	58.5	6
7	1° 04.7	1° 05.1	1° 05.5	1° 05.9	1° 06.3	1° 06.7	1° 07.0	1° 07.4	1° 07.8	1° 08.2	7
8	14.0	14.4	14.9	15.3	15.7	16.2	16.6	17.1	17.5	18.0	8
9	23.2	23.7	24.2	24.7	25.2	25.7	26.2	26.7	27.2	27.7	9
10	32.5	33.0	33.6	34.1	34.7	35.2	35.8	36.3	36.9	37.4	10
11	41.7	42.3	42.9	43.5	44.1	44.7	45.4	46.0	46.6	47.2	11
12	51.0	51.6	52.3	52.9	53.6	54.3	54.9	55.6	56.3	56.9	12
13	2° 00.2	2° 00.9	2° 01.6	2° 02.3	2° 03.1	2° 03.8	2° 04.5	2° 05.2	2° 05.9	2° 06.7	13
14	09.5	10.2	11.0	11.8	12.5	13.3	14.1	14.9	15.6	16.4	14
15	18.7	19.5	20.4	21.2	22.0	22.8	23.6	24.5	25.3	26.1	15
16	27.9	28.8	29.7	30.6	31.5	32.3	33.2	34.1	35.0	35.9	16
17	37.2	38.1	39.1	40.0	40.9	41.9	42.8	43.7	44.7	45.6	17
18	46.4	47.4	48.4	49.4	50.4	51.4	52.4	53.4	54.4	55.4	18
19	55.7	56.7	57.8	58.8	59.8	3° 00.9	3° 01.9	3° 03.0	3° 04.1	3° 05.1	19
20	3° 04.9	3° 06.0	3° 07.1	3° 08.2	3° 09.3	10.4	11.5	12.6	13.7	14.8	20
21	14.2	15.3	16.5	17.6	18.8	19.9	21.1	22.2	23.4	24.6	21
22	23.4	24.6	25.8	27.0	28.2	29.4	30.6	31.9	33.1	34.3	22
23	32.7	33.9	35.1	36.4	37.7	38.9	40.2	41.5	42.8	44.0	23
24	41.9	43.2	44.5	45.8	47.1	48.5	49.8	51.1	52.4	53.8	24
25	51.1	52.5	53.9	55.2	56.6	57.9	59.3	4° 00.7	4° 02.1	4° 03.5	25
26	4° 00.4	4° 01.8	4° 03.2	4° 04.6	4° 06.0	4° 07.5	4° 08.9	10.4	11.8	13.2	26
27	09.6	11.1	12.5	14.0	15.5	17.0	18.5	20.0	21.5	23.0	27
28	18.8	20.4	21.9	23.4	24.9	26.5	28.0	29.6	31.1	32.7	28
29	28.1	29.6	31.2	32.8	34.4	36.0	37.6	39.2	40.8	42.4	29
30	37.3	38.9	40.6	42.2	43.9	45.5	47.2	48.8	50.5	52.2	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.700	0.702	0.704	0.706	0.708	0.710	0.712	0.714	0.716	0.718	Spiral angle θ_s
5°	0° 49.0′	0° 49.3′	0° 49.6′	0° 49.8′	0° 50.1′	0° 50.4′	0° 50.7′	0° 51.0′	0° 51.3′	0° 51.6′	5°
6	58.8	59.1	59.5	59.8	1° 00.2	1° 00.5	1° 00.8	01.2	1° 01.5	1° 01.9	6
7	1° 08.6	1° 09.0	1° 09.4	1° 09.8	10.2	10 6	11.0	11.4	11.8	12.2	7
8	18.4	18.9	19.3	19.8	20.2	20.7	21.1	21.6	22.0	22.5	8
9	28.2	28.7	29.2	29.7	30.2	30.7	31.3	31.8	32.3	32.8	9
10	38.0	38.6	39.1	39.7	40.3	40.8	41.4	42 0	42.5	43 1	10
11	47.8	48.4	49.0	49.7	50.3	50.9	51.5	52.2	52 8	53 4	11
12	57.6	58.3	59.0	59.6	2° 00.3	2° 01.0	2° 01.7	2° 02.3	2° 03.0	2° 03.7	12
13	2° 07.4	2° 08.1	2° 08.9	2° 09.6	10.3	11.1	11.8	12.5	13.3	14.0	13
14	17.2	18.0	18.8	19.6	20.4	21.1	21.9	22.7	23.5	24.3	14
15	27.0	27.8	28.7	29.5	30.4	31.2	32.1	32.9	33.8	34.6	15
16	36.8	37.7	38.6	39.5	40.4	41.3	42.2	43.1	44.0	45.0	16
17	46.6	47.5	48.5	49.5	50.4	51.4	52.3	53.3	54.3	55.3	17
18	56.4	57.4	58.4	59.4	3° 00.4	3° 01.5	3° 02 5	3° 03.5	3° 04.5	3° 05.6	18
19	3° 06.2	3° 07.2	3° 08.3	3° 09.4	10.4	11.5	12.6	13.7	14.8	15.9	19
20	16.0	17.1	18.2	19.3	20.5	21 6	22.7	23.9	25.0	26.2	20
21	25.7	26.9	28.1	29 3	30.5	31.7	32.9	34.0	35.3	36.5	21
22	35.5	36.8	38.0	39.2	40.5	41.7	43.0	44.2	45.5	46.8	22
23	45.3	46.6	47.9	49.2	50.5	51.8	53.1	54.4	55.7	57.1	23
24	55.1	56.5	57.8	59.2	4° 00.5	4° 01.9	4° 03.2	4° 04.6	4° 06.0	3° 07.4	24
25	4° 05.0	4° 06.3	4° 07.7	4° 09.1	10.5	12.0	13.4	14.8	16.2	17.6	25
26	14.7	16.2	17.6	19.1	20.5	22 0	23.5	25.0	26.4	27 9	26
27	24.5	26.0	27.5	29.0	30.6	32.1	33.6	35.2	36.7	38.2	27
28	34.3	35.8	37.4	39.0	40.6	42.2	43.7	45.3	46.9	48.5	28
29	44.1	45.7	47.3	48.9	50.6	52.2	53.9	55.5	57.2	58.8	29
30	53.8	55.5	57.2	58.9	5° 00.6	5° 02.3	5° 04.0	5° 05.7	5° 07.4	5° 09.1	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.720	0.722	0.724	0.726	0.728	0.730	0.732	0.734	0.736	0.738	Spiral angle θ_s
5°	0° 51.8′	0° 52.1′	0° 52.4′	0° 52.7′	0° 53.0′	0° 53.3′	0° 53 6′	0° 53.9′	0° 54.2′	0° 54.5′	5°
6	1° 02.2	1° 02.6	1° 02.9	1° 03.3	1° 03.6	1° 04.0	1° 04.3	1° 04.7	1° 05.0	1° 05.4	6
7	12.6	13.0	13.4	13.8	14 2	14.6	15.0	15.4	15.8	16 3	7
8	22.9	23.4	23.9	24.3	24.8	25.3	25.7	26.2	26.7	27.1	8
9	33.3	33.8	34.4	34.9	35.4	35.9	36.5	37.0	37.5	38.0	9
10	43.7	44.3	44.8	45.4	46.0	46.6	47.2	47.8	48.3	48.9	10
11	54.1	54.7	55.3	56.0	56.6	57.2	57.9	58.5	59.2	59.8	11
12	2° 04.4	2° 05.1	2° 05.8	2° 06.5	2° 07.2	2° 07.9	2° 08.6	2° 09.3	2° 10.0	2° 10.7	12
13	14.8	15.5	16.3	17.0	17.8	18.5	19.3	20.1	20.8	21.6	13
14	25.1	25.9	26.8	27.6	28.4	29.2	30.0	30.8	31.7	32.5	14
15	35.5	36.4	37.2	38.1	39.0	39.9	40.7	41.6	42.5	43.4	15
16	45.9	46.8	47.7	48.6	49.6	50.5	51.4	52 4	53 3	54 3	16
17	56.2	57.2	58.2	59.2	3° 00.2	3° 01.1	3° 02 2	3° 03.2	3° 04.2	3° 05.2	17
18	3° 06.6	3° 07.6	3° 08.7	3° 09.7	10.8	11.8	12.9	13.9	15.0	16.0	18
19	16.9	18.0	19.1	20.2	21.3	22.5	23.6	24.7	25.8	26.9	19
20	27.3	28.5	29.6	30.8	31.9	33.1	34.3	35.4	36.6	37.8	20
21	37.7	38.9	40.1	41.3	42.5	43.8	45.0	46.2	47.4	48.7	21
22	48.0	49.3	50.6	51.8	53.1	54.4	55.7	57.0	58.3	59.6	22
23	58.4	59.7	4° 01.0	4° 02.4	4° 03.8	4° 05.0	4° 06.4	4° 07.7	4° 09.1	4° 10.4	23
24	4° 08.7	4° 10.1	11.5	12.9	14.3	15.7	17.1	18.5	19.9	21.3	24
25	19.1	20.5	22.0	23.4	24.9	26.3	27.8	29.3	30.7	32 2	25
26	29.4	30.9	32.4	34.0	35.5	37.0	38.5	40.0	41.5	43.1	26
27	39.8	41.3	42.9	44.5	46.0	47.6	49.2	50.8	52.4	54.0	27
28	50.1	51.8	53.4	55.0	56.6	58.2	59 9	5° 01.5	5° 03.2	5° 04 8	28
29	5° 00.5	5° 02.2	5° 03.9	5° 05 5	5° 07.2	5° 08.9	5° 10 6	12 3	14 0	15.7	29
30	10.8	12.6	14.3	16.0	17.8	19.5	21.3	23.0	24.8	26.6	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.740	0.742	0.744	0.746	0.748	0.750	0.752	0.754	0.756	0.758	Spiral angle θ_s
5°	0° 54.8′	0° 55.1′	0° 55.4′	0° 55.7′	0° 56.0′	0° 56.3′	0° 56.6′	0° 56.9′	0° 57.2′	0° 57.5′	5°
6	1° 05.7	1° 06.1	1° 06.4	1° 06.8	1° 07.1	1° 07.5	1° 07.9	1° 08.2	1° 08.6	1° 09.0	6
7	16.7	17.1	17.5	17.9	18.3	18.8	19.2	19.6	20.0	20.4	7
8	27.6	28.1	28.6	29.0	29.5	30.0	30.5	31.0	31.5	31.9	8
9	38.6	39.1	39.6	40.2	40.7	41.3	41.8	42.3	42.9	43.4	9
10	49.5	50.1	50.7	51.3	51.9	52.5	53.1	53.7	54.3	54.9	10
11	2° 00.5	2° 01.1	2° 01.8	2° 02.4	2° 03.1	2° 03.7	2° 04.4	2° 05.1	2° 05.7	2° 06.4	11
12	11.4	12.1	12.8	13.5	14.3	15.0	15.7	16.4	17.2	17.9	12
13	22.4	23.1	23.9	24.7	25.5	26.2	27.0	27.8	28.6	29.4	13
14	33.3	34.1	35.0	35.8	36.6	37.5	38.3	39.2	40.0	40.9	14
15	44.3	45.2	46.0	46.9	47.8	48.7	49.6	50.5	51.4	52.3	15
16	55.2	56.2	57.1	58.1	59.0	3° 00.0	3° 00.9	3° 01.9	3° 02.9	3° 03.8	16
17	3° 06.1	3° 07.2	3° 08.2	3° 09.2	3° 10.2	11.2	12.2	13.3	14.3	15.3	17
18	17.1	18.2	19.2	20.3	21.4	22.5	23.5	24.6	25.7	26.8	18
19	28.0	29.2	30.3	31.4	32.5	33.7	34.8	36.0	37.1	38.3	19
20	39.0	40.2	41.3	42.5	43.7	44.9	46.1	47.3	48.5	49.8	20
21	49.9	51.2	52.4	53.7	54.9	56.2	57.4	58.7	4° 00.0	4° 01.2	21
22	4° 00.9	4° 02.2	4° 03.5	4° 04.8	4° 06.1	4° 07.4	4° 08.7	4° 10.1	11.4	12.7	22
23	11.8	13.2	14.5	15.9	17.3	18.6	20.0	21.4	22.8	24.2	23
24	22.7	24.2	25.6	27.0	28.4	29.7	31.3	32.7	34.2	35.6	24
25	33.7	35.1	36.6	38.1	39.6	41.1	42.6	44.1	45.5	47.1	25
26	44.6	46.1	47.7	49.2	50.8	52.3	53.9	55.5	57.0	58.6	26
27	55.5	57.1	58.7	5° 00.4	5° 02.0	5° 03.6	5° 05.2	5° 06.8	5° 08.4	5° 10.1	27
28	5° 06.5	5° 08.1	5° 09.8	11.5	13.1	14.8	16.5	18.2	19.8	21.5	28
29	17.4	19.1	20.8	22.6	24.3	26.0	27.8	29.5	31.3	33.0	29
30	28.1	30.1	31.9	33.8	35.5	37.2	39.0	40.8	42.7	44.5	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.760	0.762	0.764	0.766	0.768	0.770	0.772	0.774	0.776	0.778	Spiral angle θ_s
5°	0° 57.8′	0° 58.1′	0° 58 4′	0° 58.7′	0° 59 0′	0° 59.3′	0° 59.6′	0° 59 9′	1° 00.2′	1° 00.5′	5°
6	1° 09.3	1° 09.7	1° 10.0	1° 10.4	1° 10.8	1° 11.2	1° 11.5	1° 11.9	12.3	12.6	6
7	20.9	21.3	21.7	22 2	22 6	23 0	23 4	23.9	24 3	24.7	7
8	32.4	32.9	33.4	33 9	34 4	34 9	35.4	35.9	36 4	36 9	8
9	44.0	44.5	45.1	45.6	46.2	46 7	47 3	47.8	48.4	49.0	9
10	55.5	56.1	56.7	57.4	58.0	58.6	59.2	59.8	2° 00.4	2° 01.1	10
11	2° 07.1	2° 07.7	2° 08.4	2° 09.1	2° 09.7	2° 10 4	2° 11.1	2° 11 8	12.5	13.2	11
12	18.6	19.3	20.1	20.8	21.5	22 3	23.0	23.8	24 5	25.3	12
13	30.2	31.0	31.7	32.5	33.3	34.1	34.9	35 7	36.6	37 4	13
14	41.7	42.6	43.4	44.3	45.1	46.0	46.9	47.7	48.6	49.5	14
15	53.3	54.2	55.1	56.0	56.9	57.8	58.8	59.7	3° 00 6	3° 01.6	15
16	3° 04.8	3° 05.8	3° 06.7	3° 07.7	3° 08.7	3° 09.7	3° 10 7	3° 11.7	12.7	13.6	16
17	16.4	17.4	18.4	19.5	20.5	21 5	22.6	23.6	24.7	25 7	17
18	27.9	29.0	30.1	31.2	32.3	33.4	34.5	35.6	36.7	37 8	18
19	39.4	40.6	41.7	42.9	44.1	45.2	46.4	47.6	48.8	49.9	19
20	51.0	52.2	53.4	54.6	55.8	57.1	58.3	59.5	4° 00.8	4° 02.0	20
21	4° 02.5	4° 03.8	4° 05.1	4° 06.3	4° 07.6	4° 08.9	4° 10 2	4° 11.5	12.8	14.1	21
22	14.0	15.4	16.7	18.0	19.4	20.8	22 1	23.5	24.8	26 2	22
23	25.6	27.0	28.4	29.8	31.2	32.6	34.0	35.4	36.9	38.3	23
24	37.1	38.6	40.0	41.5	43.0	44.4	45.9	47.4	48.9	50.4	24
25	48.6	50.2	51.7	53.2	54.7	56.3	57.8	59.4	5° 00.9	5° 02 5	25
26	5° 00.2	5° 01 8	5° 03.3	5° 04.9	5° 06.5	5° 08 1	5° 09.7	5° 11.3	12.9	14.5	26
27	11.7	13.4	15.0	16.6	18.3	20.0	21 6	23 3	25.0	26.6	27
28	23.2	24.9	26.6	28.3	30.1	31.8	33.5	35 3	37.0	38 7	28
29	34.8	36.5	38.3	40.1	41.8	43.6	45 4	47 2	49.0	50.8	29
30	46.3	48.1	49.9	51.8	53.6	55.4	57.3	59.1	6° 01.0	6° 02.9	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.780	0.782	0.784	0.786	0.788	0.790	0.792	0.794	0.796	0.798	Spiral angle θ_s
5°	1° 00.8′	1° 01.2′	1° 01.5′	1° 01.8′	1° 02.1′	1° 02.4′	1° 02.7′	1° 03.0′	1° 03.4′	1° 03.7′	5°
6	13.0	13.4	13.8	14.1	14.5	14.9	15.3	15.7	16.0	16.4	6
7	25.2	25.6	26.1	26.5	26.9	27.4	27.8	28.3	28.7	29.2	7
8	37.3	37.8	38.3	38.9	39.4	39.9	40.4	40.9	41.4	41.9	8
9	49.5	50.1	50.6	51.2	51.8	52.3	52.9	53.5	54.1	54.6	9
10	2° 01.7	2° 02.3	2° 02.9	2° 03.6	2° 04.2	2° 04.8	2° 05.4	2° 06.1	2° 06.7	2° 07.4	10
11	13.8	14.5	15.2	15.9	16.6	17.3	18.0	18.7	19.4	20.1	11
12	26.0	26.8	27.5	28.3	29.0	29.8	30.5	31.3	32.1	32.8	12
13	38.2	39.0	39.8	40.6	41.4	42.3	43.1	43.9	44.7	45.6	13
14	50.3	51.2	52.1	53.0	53.8	54.7	55.6	56.5	57.4	58.3	14
15	3° 02.5	3° 03.4	3° 04.4	3° 05.3	3° 06.2	3° 07.2	3° 08.1	3° 09.1	3° 10.0	3° 11.0	15
16	14.6	15.6	16.6	17.6	18.7	19.7	20.7	21.7	22.7	23.7	16
17	26.8	27.9	28.9	30.0	31.1	32.2	33.2	34.3	35.4	36.4	17
18	39.0	40.1	41.2	42.3	43.5	44.6	45.7	46.9	48.0	49.2	18
19	51.1	52.3	53.5	54.7	55.9	57.1	58.3	59.5	4° 00.7	4° 01.9	19
20	4° 03.3	4° 04.5	4° 05.8	4° 07.0	4° 08.3	4° 09.5	4° 10.8	4° 12.1	13.3	14.6	20
21	15.4	16.7	18.0	19.3	20.7	22.0	23.3	24.7	26.0	27.3	21
22	27.6	28.9	30.3	31.7	33.1	34.5	35.9	37.3	38.6	40.0	22
23	39.7	41.2	42.6	44.0	45.5	46.9	48.4	49.8	51.3	52.8	23
24	51.9	53.4	54.9	56.4	57.9	59.4	5° 00.9	5° 02.4	5° 04.0	5° 05.5	24
25	5° 04.0	5° 05.6	5° 07.1	5° 08.7	5° 10.3	5° 11.9	13.4	15.0	16.6	18.2	25
26	16.2	17.8	19.4	21.0	22.7	24.3	25.9	27.6	29.2	30.9	26
27	28.3	30.0	31.7	33.3	35.1	36.8	38.5	40.2	41.9	43.6	27
28	40.4	42.2	43.9	45.7	47.5	49.2	51.0	52.8	54.5	56.3	28
29	52.6	54.4	56.2	58.0	59.8	6° 01.7	6° 03.5	6° 05.3	6° 07.2	6° 09.0	29
30	6° 04.7	6° 06.6	6° 08.5	6° 10.3	6° 12.2	14.1	16.0	17.9	19.8	21.7	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.800	0.802	0.804	0.806	0.808	0.810	0.812	0.814	0.816	0.818	Spiral angle θ_s
5°	1° 04.0′	1° 04.3′	1° 04.6′	1° 05.0′	1° 05.3′	1° 05.6′	1° 05.9′	1° 06.3′	1° 06.6′	1° 06.9′	5°
6	16.8	17.2	17.6	18.0	18.3	18.7	19.1	19.5	19.9	20.3	6
7	29.6	30.1	30.5	31.0	31.4	31.9	32.3	32.8	33.2	33.7	7
8	42.4	42.9	43.4	43.9	44.5	45.0	45.5	46.0	46.5	47.1	8
9	55.2	55.8	56.4	56.9	57.5	58.1	58.7	59.3	59.9	2° 00.4	9
10	2° 08.0	2° 08.6	2° 09.3	2° 09.9	2° 10.6	2° 11.2	2° 11.9	2° 12.5	2° 13.2	13.8	10
11	20.8	21.5	22.2	22.9	23.6	24.3	25.1	25.8	26.5	27.2	11
12	33.6	34.4	35.1	35.9	36.7	37.4	38.2	39.0	39.8	40.6	12
13	46.6	47.2	48.1	48.9	49.7	50.6	51.4	52.2	53.1	53.9	13
14	59.2	3° 00.1	3° 01.0	3° 01.9	3° 02.8	3° 03.7	3° 04.6	3° 05.5	3° 06.4	3° 07.3	14
15	3° 12.0	12.9	13.9	14.8	15.8	16.8	17.8	18.7	19.7	20.7	15
16	24.7	25.8	26.8	27.8	28.9	29.9	30.9	32.0	33.0	34.1	16
17	37.5	38.6	39.7	40.8	41.9	43.0	44.1	45.2	46.3	47.4	17
18	50.3	51.5	52.6	53.8	54.9	56.1	57.3	58.4	59.6	4° 00.8	18
19	4° 03.1	4° 04.3	4° 05.5	4° 06.8	4° 08.0	4° 09.2	4° 10.5	4° 11.7	4° 12.9	14.2	19
20	15.9	17.2	18.5	19.7	21.0	22.3	23.6	24.9	26.2	27.5	20
21	28.7	30.0	31.4	32.7	34.1	35.4	36.8	38.1	39.5	40.9	21
22	41.5	42.9	44.3	45.7	47.1	48.5	50.0	51.4	52.8	54.3	22
23	54.2	55.7	57.2	58.7	5° 00.2	5° 01.6	5° 03.1	5° 04.6	5° 06.1	5° 07.6	23
24	5° 07.0	5° 08.6	5° 10.1	5° 11.6	13.2	14.7	16.3	17.8	19.4	21.0	24
25	19.8	21.4	23.0	24.6	26.2	27.8	29.4	31.1	32.7	34.3	25
26	32.7	34.2	35.9	37.6	39.2	40.9	42.6	44.3	46.0	47.7	26
27	45.3	47.1	48.8	50.5	52.3	54.0	55.8	57.5	59.3	6° 01.0	27
28	58.1	59.9	6° 01.7	6° 03.5	6° 05.3	6° 07.1	6° 08.9	6° 10.7	6° 12.5	14.4	28
29	6° 10.9	6° 12.7	14.6	16.4	18.3	20.2	22.1	23.9	25.8	27.7	29
30	23.6	25.5	27.5	29.4	31.3	33.3	35.2	37.2	39.1	41.1	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s =										Spiral angle θ_s
	0.820	0.822	0.824	0.826	0.828	0.830	0.832	0.834	0.836	0.838	
5°	1° 07.2′	1° 07.6′	1° 07.9′	1° 08.2′	1° 08.6′	1° 08.9′	1° 09.2′	1° 09.6′	1° 09.9′	1° 10.2′	5°
6	20.7	21.1	21.5	21.9	22.3	22.7	23.1	23.5	23.9	24.3	6
7	34.1	34.6	35.1	35.5	36.0	36.5	36.9	37.4	37.9	38.3	7
8	47.6	48.1	48.6	49.2	49.7	50.2	50.8	51.3	51.8	52.4	8
9	2° 01.0	2° 01.6	2° 02.2	2° 02.8	2° 03.4	2° 04.0	2° 04.6	2° 05.2	2° 05.8	2° 06.4	9
10	14.5	15.1	15.8	16.5	17.1	17.8	18.4	19.1	19.8	20.4	10
11	27.9	28.6	29.4	30.1	30.8	31.5	32.3	33.0	33.7	34.5	11
12	41.4	42.1	42.9	43.7	44.5	45.3	46.1	46.9	47.7	48.5	12
13	54.8	55.7	56.5	57.4	58.2	59.1	3° 00.0	3° 00.8	3° 01.7	3° 02.6	13
14	3° 08.2	3° 09.2	3° 10.1	3° 11.0	3° 11.9	3° 12.9	13.8	14.7	15.6	16.6	14
15	21.7	22.7	23.6	24.6	25.6	26.6	27.6	28.6	29.6	30.6	15
16	35.1	36.2	37.2	38.3	39.3	40.4	41.4	42.5	43.6	44.6	16
17	48.5	49.7	50.8	51.9	53.0	54.2	55.3	56.4	57.5	58.7	17
18	4° 02.0	4° 03.1	4° 04.2	4° 05.5	4° 06.7	4° 07.9	4° 09.1	4° 10.3	4° 11.5	4° 12.7	18
19	15.4	16.7	17.9	19.1	20.4	21.7	22.9	24.2	25.5	26.7	19
20	28.8	30.1	31.5	32.8	34.1	35.4	36.8	38.1	39.4	40.8	20
21	42.3	43.6	45.0	46.4	47.8	49.2	50.6	52.0	53.4	54.8	21
22	55.7	57.1	58.6	5° 00.0	5° 01.5	5° 02.9	5° 04.4	5° 05.9	5° 07.3	5° 08.8	22
23	5° 09.1	5° 10.6	5° 12.1	13.7	15.2	16.7	18.2	19.7	21.3	22.8	23
24	22.5	24.1	25.7	27.3	28.8	30.4	32.0	33.6	35.2	36.8	24
25	36.0	37.6	39.2	40.9	42.5	44.2	45.8	47.5	49.2	50.8	25
26	49.4	51.1	52.8	54.5	56.2	57.9	59.7	6° 01.4	6° 03.1	6° 04.9	26
27	6° 02.8	6° 04.5	6° 06.3	6° 08.1	6° 09.9	6° 11.6	6° 13.5	15.3	17.0	18.9	27
28	16.2	18.0	19.9	21.7	23.5	25.4	27.3	29.1	31.0	32.9	28
29	29.6	31.5	33.4	35.3	37.2	39.2	41.1	43.0	44.9	46.9	29
30	43.0	45.0	47.0	48.9	50.9	52.9	54.9	56.9	58.9	7° 00.9	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s =										Spiral angle θ_s
	0.840	0.842	0.844	0.846	0.848	0.850	0.852	0.854	0.856	0.858	
5°	1° 10.6′	1° 10.9′	1° 11.2′	1° 11.6′	1° 11.9′	1° 12.3′	1° 12.6′	1° 12.9′	1° 13.3′	1° 13.6′	5°
6	24.7	25.1	25.5	25.9	26.3	26.7	27.1	27.5	27.9	28.3	6
7	38.8	39.3	39.7	40.2	40.7	41.1	41.6	42.1	42.6	43.1	7
8	52.9	53.4	54.0	54.5	55.1	55.6	56.1	56.7	57.2	57.8	8
9	2° 07.0	2° 07.6	2° 08.2	2° 08.8	2° 09.4	2° 10.0	2° 10.7	2° 11.3	2° 11.9	2° 12.5	9
10	21.1	21.8	22.5	23.1	23.8	24.5	25.2	25.7	26.5	27.2	10
11	35.2	36.0	36.7	37.4	38.2	38.9	39.7	40.4	41.2	41.9	11
12	49.3	50.1	50.9	51.7	52.6	53.4	54.2	55.0	55.8	56.7	12
13	3° 03.4	3° 04.3	3° 05.2	3° 06.1	3° 06.9	3° 07.8	3° 08.7	3° 09.6	3° 10.5	3° 11.4	13
14	17.5	18.5	19.4	20.4	21.3	22.3	23.2	24.2	25.1	26.1	14
15	31.6	32.6	33.6	34.6	35.7	36.7	37.7	38.7	39.8	40.8	15
16	45.7	46.8	47.9	49.0	50.0	51.1	52.2	53.3	54.4	55.5	16
17	59.8	4° 00.9	4° 02.1	4° 03.3	4° 04.4	4° 05.6	4° 06.7	5° 07.9	4° 09.0	4° 10.2	17
18	4° 13.9	15.1	16.3	17.5	18.8	20.0	21.2	22.4	23.7	24.9	18
19	28.0	29.3	30.6	31.8	33.1	34.4	35.7	37.0	38.3	39.6	19
20	42.1	43.4	44.8	46.1	47.5	48.8	50.2	51.6	52.9	54.3	20
21	56.2	57.6	59.0	5° 00.4	5° 01.9	5° 03.3	5° 04.7	5° 06.1	5° 07.6	5° 09.0	21
22	5° 10.3	5° 11.7	5° 13.2	14.7	16.2	17.7	19.2	20.8	22.2	23.7	22
23	24.4	25.9	27.4	29.0	30.6	32.1	33.7	35.2	36.8	38.4	23
24	38.4	40.0	41.7	43.3	44.9	46.5	48.2	49.8	51.4	53.1	24
25	52.5	54.2	55.9	57.6	59.2	6° 00.9	6° 02.6	6° 04.3	6° 06.0	6° 07.8	25
26	6° 06.6	6° 08.3	6° 10.1	6° 11.8	6° 13.6	15.4	17.1	18.9	20.7	22.4	26
27	20.7	22.5	24.3	26.1	27.9	29.8	31.6	33.4	35.3	37.1	27
28	34.7	36.6	38.5	40.4	42.3	44.2	46.1	48.0	49.9	51.8	28
29	48.8	50.8	52.7	54.7	56.6	58.6	7° 00.5	7° 02.5	7° 04.5	7° 06.5	29
30	7° 02.9	7° 04.9	7° 06.9	7° 08.9	7° 10.9	7° 13.0	15.0	17.0	19.1	21.1	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.860	0.862	0.864	0.866	0.868	0.870	0.872	0.874	0.876	0.878	Spiral angle θ_s
5°	1° 14.0′	1° 14.3′	1° 14.7′	1° 15.0′	1° 15.3′	1° 15.7′	1° 16.0′	1° 16.4′	1° 16.7′	1° 17.1′	5°
6	28.8	29.2	29.6	30.0	30.4	30.8	31.3	31.7	22.1	32.5	6
7	43.5	44.0	44.5	45.0	45.5	46.0	46.5	46.9	47.4	47.9	7
8	58.3	58.9	59.4	2° 00.0	2° 00.5	2° 01.1	2° 01.7	2° 02.2	2° 02.8	2° 03.3	8
9	2° 13.1	2° 13.7	2° 14.4	15.0	15.6	16.2	16.9	17.5	18.1	18.8	9
10	27.9	28.6	29.3	30.0	30.7	31.4	32.1	32.8	33.5	34.2	10
11	42.7	43.5	44.2	45.0	45.7	46.5	47.3	48.0	48.8	49.6	11
12	57.6	58.3	59.1	3° 00.0	3° 00.8	3° 01.6	3° 02.5	3° 03.3	3° 04.1	3° 05.0	12
13	3° 12.3	3° 13.2	3° 14.1	14.9	15.8	16.7	17.7	18.6	19.5	20.4	13
14	27.2	28.0	28.9	29.9	30.9	31.9	32.8	33.8	34.8	35.8	14
15	41.8	42.8	43.9	44.9	46.0	47.0	48.0	49.1	50.1	51.2	15
16	56.6	57.7	58.8	59.9	4° 01.0	4° 02.1	4° 03.2	4° 04.4	4° 05.5	4° 06.6	16
17	4° 11.4	4° 12.5	4° 13.7	4° 15.0	16.1	17.2	18.4	19.6	20.8	22.0	17
18	26.1	27.4	28.6	29.9	31.1	32.3	33.6	34.9	36.1	37.4	18
19	40.9	42.2	43.5	44.8	46.2	47.5	48.8	50.1	51.4	52.8	19
20	55.7	57.1	58.4	59.8	5° 01.2	5° 02.6	5° 04.0	5° 05.4	5° 06.7	5° 08.2	20
21	5° 10.4	5° 11.9	5° 13.3	5° 14.8	16.2	17.7	19.1	20.6	22.1	23.5	21
22	25.2	26.7	28.2	29.7	31.3	32.8	34.3	35.9	37.4	38.9	22
23	40.0	41.5	43.1	44.7	46.3	47.9	49.5	51.1	52.7	54.3	23
24	54.7	56.4	58.0	59.7	6° 01.3	6° 03.0	6° 04.7	6° 06.3	6° 08.0	6° 09.7	24
25	6° 09.5	6° 11.2	6° 12.9	6° 14.6	16.4	18.1	19.8	21.6	23.3	25.1	25
26	24.2	26.0	27.8	29.6	31.4	33.2	35.0	36.8	38.6	40.5	26
27	39.0	40.8	42.7	44.6	46.4	48.3	50.2	52.0	53.9	55.8	27
28	53.7	55.6	57.6	59.5	7° 01.4	7° 03.4	7° 05.3	7° 07.3	7° 09.2	7° 11.2	28
29	7° 08.5	7° 10.5	7° 12.4	7° 14.4	16.4	18.4	20.5	22.5	24.5	26.5	29
30	23.2	25.3	27.3	29.4	31.5	33.5	35.6	37.7	39.8	41.9	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio $= l/l_s =$ 0.880	0.882	0.884	0.886	0.888	0.890	0.892	0.894	0.896	0.898	Spiral angle θ_s
5°	1° 17.4′	1° 17.8′	1° 18.2′	1° 18.5′	1° 18.9′	1° 19.2′	1° 19.6′	1° 19.9′	1° 20.3′	1° 20.6′	5°
6	32.9	33.4	33.8	34.2	34.6	35.1	35.5	35.9	36.3	36.8	6
7	48.4	48.9	49.4	49.9	50.4	50.9	51.4	51.9	52.4	52.9	7
8	2° 03.9	2° 04.5	2° 05.0	2° 05.6	2° 06.2	2° 06.7	2° 07.3	2° 07.9	2° 08.4	2° 09.0	8
9	19.4	20.0	20.6	21.3	21.9	22.6	23.2	23.8	24.5	25.1	9
10	34.9	35.6	36.3	37.0	37.7	38.4	39.1	39.8	40.5	41.3	10
11	50.3	51.1	51.9	52.7	53.5	54.2	55.0	55.8	56.6	57.4	11
12	3° 05.8	3° 06.7	3° 07.5	3° 08.4	3° 09.2	3° 10.0	3° 10.9	3° 11.8	3° 12.6	3° 13.5	12
13	21.3	22.2	23.1	24.1	25.0	25.9	26.8	27.7	28 7	29.6	13
14	36.8	37.8	38.7	39.7	40.7	41.7	42.7	43.7	44.7	45.7	14
15	52.2	53.3	54.4	55.4	56.5	57.5	58.6	59.7	4° 00.7	4° 01.8	15
16	4° 07.7	4° 08.8	4° 10.0	4° 11.1	4° 12.2	4° 13.4	4° 14.5	4° 15.7	16.8	17.9	16
17	23.2	24.4	25.6	26.8	28.0	29.2	30.4	31.6	32.8	34.0	17
18	38.6	39.9	41.2	42.5	43.7	45.0	46.3	47.6	48.9	50.2	18
19	54.1	55.4	56.8	58.1	59.5	5° 00.8	5° 02.1	5° 03.5	5° 04.9	5° 06.3	19
20	5° 09.6	5° 11.0	5° 12.4	5° 13.8	5° 15.2	16.6	18.1	19.5	20.9	22.3	20
21	25.0	26.5	28.0	29.5	30.9	32.4	33.9	35.4	36.9	38.4	21
22	40.5	42.0	43.6	44.9	46.7	48.2	49.8	51.4	53.0	54.5	22
23	55.9	57.6	59.2	6° 00.8	6° 02.4	6° 04.1	6° 05.7	6° 07.3	6° 09.0	6° 10.6	23
24	6° 11.4	6° 13.1	6° 14.8	16.4	18.1	19.8	21.6	23.3	25.0	26.7	24
25	26.8	28.6	30.3	32.1	33.9	35.7	37.4	39.2	41.0	42.8	25
26	42.3	44.1	45.9	47.8	49.6	51.4	53.3	55.1	57.0	58.9	26
27	57.7	59.6	7° 01.5	7° 03.4	7° 05.5	7° 07.2	7° 09.1	7° 11.1	7° 13.0	7° 14.9	27
28	7° 13.1	7° 15.1	17.1	19.0	21.0	23.0	25.0	27.0	29.0	31.0	28
29	28.6	30.6	32.6	34.7	36.7	38.8	40.9	42.9	45.0	47.0	29
30	44.0	46.1	48.2	50.3	52.4	54.6	56.7	58.8	8° 01.1	8° 03.1	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.900	0.902	0.904	0.906	0.908	0.910	0.912	0.914	0.916	0.918	Spiral angle θ_s
5°	1° 21.0′	1° 21.4′	1° 21.7′	1° 22.1′	1° 22.4′	1° 22.8′	1° 23.2′	1° 23.5′	1° 23.9′	1° 24 3′	5°
6	37.2	37.6	38.1	38.5	38.9	39 4	39.8	40.2	40.7	41 1	6
7	53.4	53.9	54.4	54.9	55.4	55 9	56.4	57.0	57.5	58.0	7
8	2° 09.6	2° 10.2	2° 10.8	2° 11.3	2° 11.9	2° 12.5	2° 13.1	2° 13.7	2° 14.2	2° 14.8	8
9	25.8	26.4	27.1	27.8	28.4	29.1	29.7	30.4	31.0	31.7	9
10	42.0	42.7	43.4	44.2	44.9	45.6	46.3	47.1	47.8	48 5	10
11	58.2	59.0	59.8	3° 00.6	3° 01.4	3° 02.2	3° 03.0	3° 03 8	3° 04 6	3° 05.4	11
12	3° 14.4	3° 15.3	3° 16.1	17.0	17.9	18.7	19.6	20.5	21.4	22 3	12
13	30.5	31.4	32.4	33.3	34.3	35.2	36.2	37.1	38.1	39.0	13
14	46.7	47.7	48.7	49.7	50.7	51.8	52.8	53.8	54.8	55.9	14
15	4° 03.0	4° 04.0	4° 05.1	4° 06.2	4° 07.2	4° 08.3	4° 09.4	4° 10.5	4° 11.6	4° 12.7	15
16	19.1	20.3	21.4	22 6	23.7	24 9	26.1	27.2	28.4	29.6	16
17	35.3	36.5	37.8	39.0	40.2	41 5	42.7	43.9	45.1	46.3	17
18	51.4	52.7	54.0	55.3	56.6	57.9	59.2	5° 00.5	5° 01.9	5° 03.2	18
19	5° 07.6	5° 09.0	5° 10.3	5° 11.7	5° 13.1	5° 14.5	5° 15.9	17.3	18.6	20.0	19
20	23.8	25.2	26.7	28.1	29.6	31.0	32.5	34.0	35.4	36.9	20
21	39.9	41.4	42.9	44.5	46.0	47.5	49.0	50.6	52.1	53.6	21
22	56.1	57 7	59 3	6° 00.9	6° 02.5	6° 04 1	6° 05.7	6° 07.3	6° 08 9	6° 10.5	22
23	6° 12.3	6° 14.0	6° 15.6	17.2	18.9	20 5	22.2	23.9	25.6	27 3	23
24	28.4	30.1	31.9	33.6	35.3	37.1	38.8	40.6	42.3	44 1	24
25	44.6	46.4	48.2	50.0	51 8	53.5	55.4	57.2	59.0	7° 00.9	25
26	7° 00.5	7° 02.6	7° 04.5	7° 06.3	7° 08.2	7° 10.1	7° 12.0	7° 13.9	7° 15.8	17.7	26
27	16.9	18.7	20.7	22.7	24.6	26.6	28.5	30.5	32.5	34.4	27
28	33.0	35.0	37.0	39.1	41.1	43.1	45.1	47.1	49.2	51.2	28
29	49.0	51.2	53.3	55.4	57.5	59.6	8° 01.7	8° 03.8	8° 06.0	8° 08.0	29
30	8° 05.3	8° 07.4	8° 09.5	8° 11.7	8° 13.9	8° 16.1	18.2	20.4	22.6	24.8	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.920	0.922	0.924	0.926	0.928	0.930	0.932	0.934	0.936	0.938	Spiral angle θ_s
5°	1° 24.6′	1° 25.0′	1° 25.4′	1° 25.8′	1° 26.1′	1° 26.5′	1° 26.9′	1° 27.2′	1° 27.6′	1° 28.0′	5°
6	41.6	42.0	42.5	42.9	43.3	43.8	44 2	44 7	45 1	45 6	6
7	58.5	59.0	59.5	2° 00.0	2° 00.6	2° 01 1	2° 01.6	2° 02 1	2° 02.6	2° 03.2	7
8	2° 15.4	2° 16.0	2° 16.6	17.2	17.8	18.4	19.0	19.6	20 2	20.8	8
9	32.3	33.0	33.7	34.3	35.0	35.7	36.3	37.0	37.7	38.4	9
10	49 3	50.0	50.7	51.5	52.2	52.9	53.7	54 4	55.2	55.9	10
11	3° 06.2	3° 07.0	3° 07.8	3° 08.6	3° 09 4	3° 10 2	3° 11.1	3° 11.9	3° 12.7	3° 13 5	11
12	23.1	24.0	24 8	25.7	26 6	27 5	28 4	29.3	30 2	31 1	12
13	40.0	40.9	41.9	42.9	43.8	44.8	45.8	46.7	47 7	48.7	13
14	56.9	57.9	59 0	4° 00.0	4° 01.0	4° 02.1	4° 03.1	4° 04.2	4° 05.2	4° 06.3	14
15	4° 13.8	4° 14.9	4° 16.0	17.1	18.2	19.4	20.5	21.6	22.7	23.8	15
16	30 7	31 9	33.1	34.3	35.4	36.6	37.8	39 0	40.2	41.4	16
17	47.6	48.9	50.1	51.4	52.6	53.9	55.2	56 4	57.7	59.0	17
18	5° 04.5	5° 05 9	5° 07.2	5° 08.5	5° 09.8	5° 11.2	5° 12.5	5° 13.8	5° 15.2	5° 16.5	18
19	21.4	22.8	24.2	25.6	27.0	28.4	29.8	31.3	32 7	34.1	19
20	38.3	39.8	41.3	42.7	44.2	45.7	47.2	48.7	50.2	51.7	20
21	55.2	56 7	58.3	59 8	6° 01.4	6° 02.9	6° 04.5	6° 06.1	6° 07.6	6° 09.2	21
22	6° 12.1	6° 13.7	6° 15.3	6° 16.9	18.6	20.2	21.8	23.5	25.1	26 8	22
23	29.0	30.7	32.3	34.0	35.7	37.5	39.2	40 9	42 6	44 3	23
24	45.8	47.6	49.4	51.1	52.9	54.7	56.5	58.3	7° 00.1	7° 01.8	24
25	7° 02.7	7° 04.5	7° 06.4	7° 08.2	7° 10.1	7° 11.9	7° 13.8	7° 15.6	17.5	19.4	25
26	19.6	21.5	23.4	25.3	27.2	29 2	31.1	33.0	35.0	36.7	26
27	36.4	38.4	40.4	42.4	44.4	46.4	48.4	50.4	52.4	54 4	27
28	53.3	55.4	57.4	59.5	8° 01 5	8° 03.6	8° 05.7	8° 07.8	8° 09.8	8° 11.9	28
29	8° 10.1	8° 12.3	8° 14.4	8° 16.5	18.7	20 8	23.0	25.1	27.3	29 5	29
30	27.0	29.2	31.4	33.6	35 8	38.0	40.3	42.5	44.7	47.0	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s =										Spiral angle θ_s
	0.940	0.942	0.944	0.946	0.948	0.950	0.952	0.954	0.956	0.958	
5°	1° 28.4′	1° 28.7′	1° 29.1′	1° 29.5′	1° 29.9′	1° 30.2′	1° 30.6′	1° 31.0′	1° 31.4′	1° 31.8′	5°
6	46.0	46.5	46.9	47.4	47.8	48.3	48.8	49.2	49.7	50.1	6
7	2° 03.7	2° 04.2	2° 04.8	2° 05.3	2° 05.8	2° 06.3	2° 06.9	2° 07.4	2° 07.9	2° 08.5	7
8	21.4	22.0	22.5	23.2	23.8	24.4	25.0	25.6	26.2	26.8	8
9	39.0	39.7	40.4	41.1	41.8	42.4	43.1	43.8	44.5	45.2	9
10	56.7	57.5	58.2	59.0	59.7	3° 00.5	3° 01.2	3° 02.0	3° 02.8	3° 03.5	10
11	3° 14.3	3° 15.2	3° 16.0	3° 16.8	3° 17.6	18.5	19.3	20.2	21.0	21.9	11
12	32.0	32.9	33.8	34.7	35.6	36.5	37.4	38.4	39.3	40.2	12
13	49.7	50.6	51.6	52.6	53.6	54.6	55.6	56.5	57.5	58.5	13
14	4° 07.3	4° 08.4	4° 09.4	4° 10.5	4° 11.5	4° 12.6	4° 13.7	4° 14.7	4° 15.8	4° 16.9	14
15	25.0	26.1	27.2	28.4	29.5	30.6	31.8	32.9	34.0	35.2	15
16	42.6	43.8	45.0	46.2	47.4	48.6	49.9	51.1	52.3	53.5	16
17	5° 00.3	5° 01.6	5° 02.8	5° 04.1	5° 05.4	5° 06.7	5° 08.0	5° 09.3	5° 10.5	5° 11.8	17
18	17.9	19.2	20.6	22.0	23.3	24.7	26.0	27.4	28.8	30.2	18
19	35.5	37.0	38.4	39.8	41.3	42.7	44.1	45.6	47.0	48.5	19
20	53.2	54.7	56.2	57.7	59.2	6° 00.7	6° 02.2	6° 03.7	6° 05.2	6° 06.8	20
21	6° 10.8	6° 12.3	6° 13.9	6° 15.5	6° 17.1	18.7	20.3	21.9	23.5	25.1	21
22	28.4	30.1	31.7	33.4	35.0	36.7	38.4	40.0	41.7	43.4	22
23	46.0	47.7	49.5	51.2	52.9	54.7	56.4	58.2	59.9	7° 01.7	23
24	7° 03.6	7° 05.4	7° 07.2	7° 09.0	7° 10.8	7° 12.7	7° 14.5	7° 16.3	7° 18.1	20.0	24
25	21.2	23.1	25.0	26.9	28.8	30.7	32.6	34.5	36.4	38.3	25
26	38.8	40.8	42.8	44.7	46.7	48.6	50.6	52.6	54.6	56.5	26
27	56.4	58.5	8° 00.5	8° 02.5	8° 04.6	8° 06.6	8° 08.6	8° 10.7	8° 12.8	8° 14.8	27
28	8° 14.0	8° 16.1	18.2	20.3	22.4	24.6	26.7	28.8	30.9	33.1	28
29	31.6	33.8	35.8	38.1	40.3	42.5	44.7	46.9	49.1	51.3	29
30	49.2	51.4	53.7	56.0	58.2	9° 00.5	9° 02.7	9° 05.0	9° 07.3	9° 09.6	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.960	0.962	0.964	0.966	0.968	0.970	0.972	0.974	0.976	0.978	Spiral angle θ_s
5°	1° 32.2′	1° 32.5′	1° 32.9′	1° 33.3′	1° 33.7′	1° 34 1′	1° 34.5′	1° 34.9′	1° 35.3′	1° 35.7′	5°
6	50.6	51.1	51.5	52.0	52.4	52.9	53.4	53.8	54 3	54.8	6
7	2° 09.0	2° 09.6	2° 10 1	2° 10.6	2° 11.2	2° 11.7	2° 12.3	2° 12.8	2° 13.4	2° 13.9	7
8	27.4	28.1	28.7	29.3	29.9	30.5	31.2	31.8	32.4	33.0	8
9	45.9	46.6	47.2	47.9	48.6	49.3	50.0	50.7	51.4	52.1	9
10	3° 04.3	3° 05.1	3° 05.8	3° 06.6	3° 07.4	3° 08.1	3° 08.9	3° 09.7	3° 10.5	3° 11.3	10
11	22.7	23.6	24.4	25.2	26.1	26.9	27.8	28.7	29.5	30 4	11
12	41.1	42.0	43.0	43.9	44.8	45.7	46.7	47.6	48.5	49.5	12
13	59.5	4° 00.5	4° 01.5	4° 02.5	4° 03.5	4° 04.5	4° 05.5	4° 06.6	4° 07.6	4° 08.6	13
14	4° 17.9	19.0	20.1	21.2	22.3	23.3	24.4	25.5	26.6	27.7	14
15	36.3	37.5	38.6	39.8	41.0	42.1	43.3	44.5	45.6	46.8	15
16	54.7	56.0	57.2	58.4	59.7	5° 00.9	5° 02.2	5° 03.4	5° 04.6	5° 05.9	16
17	5° 13.1	5° 14.4	5° 15.8	5° 17.1	5° 18.4	19.7	21.0	22.3	23.7	25.0	17
18	31.5	32.9	34.3	35.7	37.1	38.5	39.9	41.3	42.7	44.1	18
19	49.9	51.4	52.9	54.3	55.8	57.2	58.7	6° 00.2	6° 01.7	6° 03.1	19
20	6° 08.3	6° 09.8	6° 11.4	6° 12.9	6° 14.5	6° 16.0	6° 17.6	19.1	20.7	22.2	20
21	26.7	28.3	29.9	31.5	33.2	34.8	36.4	38.0	39.7	41.3	21
22	45.1	46.8	48.5	50.1	51.8	53.5	55.2	57.0	58.7	7° 00.4	22
23	7° 03.4	7° 05.2	7° 07.0	7° 08.7	7° 10.5	7° 12.3	7° 14.1	7° 15.9	7° 17.6	19.4	23
24	21.8	23.6	25.5	27.3	29.2	31.0	32.9	34.8	36.6	38.5	24
25	40.2	42.1	44.0	45.9	47.8	49.8	51.7	53.7	55.6	57.5	25
26	58.5	8° 00.5	8° 02.5	8° 04.5	8° 06.5	8° 08.5	8° 10.5	8° 12.5	8° 14.6	8° 16.6	26
27	8° 16.7	18.9	21 0	23.1	25.2	27.2	29.3	31.4	33.5	35 6	27
28	35.2	37.3	39.5	41.6	43.8	46.0	48.1	50.3	52.4	54.6	28
29	53.5	55.8	58.0	9° 00.2	9° 02.4	9° 04.7	9° 06.9	9° 09.1	9° 11.4	9° 13.7	29
30	9° 11.9	9° 14.2	9° 16.5	18.7	21.1	23.4	25.7	28.0	30.3	32.7	30

TABLE 10.—DEFLECTIONS FROM THE T.S. TO ANY POINT ON A SPIRAL—(*Continued*)

Spiral angle θ_s	Distance ratio = l/l_s = 0.980	0.982	0.984	0.986	0.988	0.990	0.992	0.994	0.996	0.998	1.000	Spiral angle θ_s
5°	1° 36.0′	1° 36.4′	1° 36.8′	1° 37.2′	1° 37.6′	1° 38.0′	1° 38.4′	1° 38.8′	1° 39.2′	1° 39.6′	1° 40.0′	5°
6	55.3	55.7	56.2	56.7	57.1	57.6	58.1	58 6	59.0	59 5	2° 00 0	6
7	2° 14.5	2° 15.0	2° 15.6	2° 16.1	2° 16.7	2° 17.2	2° 17.8	2° 18.3	2° 18 9	2° 19.4	20.0	7
8	33.6	34.3	34.9	35.5	36.2	36.8	37.4	38.1	38.7	39.3	40.0	8
9	52.8	53.6	54.3	55.0	55.7	56.4	57.1	57.8	58.5	59.3	3° 00.0	9
10	3° 12.0	3° 12.8	3° 13.6	3° 14.4	3° 15.2	3° 16.0	3° 16.8	3° 17.6	3° 18.4	3° 19 2	20 0	10
11	31.2	32.1	33.0	33.8	34.7	35.6	36.4	37.3	38.2	39.1	39.9	11
12	50.4	51.4	52.3	53.3	54.2	55.1	56.1	57.0	58.0	59.0	59.9	12
13	4° 09.6	4° 10.6	4° 11.7	4° 12.7	4° 13.7	4° 14.7	4° 15.8	4° 16.8	4° 17.8	4° 18.8	4° 19 9	13
14	28.8	29.9	31.0	32.1	33.2	34.3	35.4	36.5	37.6	38.7	39.9	14
15	48.0	49.1	50.3	51.5	52 7	53.9	55.1	56.3	57.4	58.6	59.8	15
16	5° 07.1	5° 08.4	5° 09.7	5° 10.9	5° 12.2	5° 13.4	5° 14.7	5° 15.9	5° 17.2	5° 18.5	5° 19 8	16
17	26.3	27.6	29.0	30.3	31.7	33.0	34.3	35 7	37 0	38.4	39.7	17
18	45.5	46.9	48.3	49.7	51.1	52.6	54.0	55.4	56.8	58.3	59 7	18
19	6° 04.6	6° 06.1	6° 07.6	6° 09.1	6° 10.6	6° 12.1	6° 13.6	6° 15.1	6° 16.6	6° 18.1	6° 19.6	19
20	23.8	25.4	26.9	28.5	30.1	31.7	33.2	34.8	36.4	38.0	39.6	20
21	42.9	44.6	46.2	47.9	49.5	51 2	52.9	54.5	56 2	57 9	59 5	21
22	7° 02.1	7° 03.8	7° 05.5	7° 07.3	7° 09.0	7° 10.7	7° 12.5	7° 14.2	7° 16.0	7° 17.7	7° 19.4	22
23	21.2	23.0	24.8	26.6	28.4	30.2	32.1	33.9	35.7	37.5	39 4	23
24	40.4	42.2	44.1	46.0	47.9	49.8	51.7	53.6	55.5	57.4	59.3	24
25	59.5	8° 01.4	8° 03.4	8° 05.4	8° 07.3	8° 09.3	8° 11.3	8° 13.2	8° 15.2	8° 17.2	8° 19 2	25
26	8° 18.6	20.6	22.7	24 7	26.7	28.8	30 8	32.9	35.0	37.0	39 1	26
27	37.7	39.8	41.9	44.1	46.2	48.3	50 4	52.6	54.7	56.8	59 0	27
28	56.8	59.0	9° 01.2	9° 03.4	9° 05.6	9° 07.8	9° 10.0	9° 12 2	9° 14.4	9° 16 6	9° 18.9	28
29	9° 15.9	9° 18.2	20.4	22.7	25.0	27.3	29 5	31 8	34.1	36 4	38 7	29
30	35.1	37.3	39.7	42.0	44.4	46.7	49.1	51.5	53.8	56.2	58.6	30

TABLE 11.—TANGENTS AND EXTERNALS FOR THE UNIT DOUBLE-SPIRAL CURVE.

Length of each spiral = 100 feet.

Spiral angle = $\frac{1}{2}\Delta$.

Δ	T_s	E_s	Δ	T_s	E_s	Δ	T_s	E_s
4° 00′	100.028	1.164	24° 00′	101.041	7.115	30° 00′	101.644	8.990
6 00	.064	.746	10	.056	.166	10	.663	9.043
8 00	.114	2.332	20	.071	.218	20	.682	.096
10 00	.178	.918	30	.085	.269	30	.700	.149
			40	.101	.321	40	.720	.203
11 00	.216	3.212	50	.116	.372	50	.739	.256
12 00	.257	.507						
13 00	.302	.801	25 00	.131	.424	31 00	.758	.309
14 00	.350	4.098	10	.146	.476	10	.777	.362
15 00	.402	.396	20	.162	.527	20	.797	.416
16 00	.458	.693	30	.177	.579	30	.817	.469
17 00	.518	.992	40	.193	.631	40	.837	.523
18 00	.581	5.292	50	.209	.682	50	.857	.576
19 00	.648	.593						
20 00	.719	.894	26 00	.225	.734	32 00	.877	.629
			10	.241	.786	10	.897	.683
20 10	.731	.944	20	.257	.837	20	.918	.737
20	.744	.995	30	.274	.889	30	.938	.790
30	.756	6.045	40	.290	.941	40	.959	.844
40	.769	.096	50	.307	.993	50	.980	.898
50	.781	.146						
			27 00	.324	8.045	33 00	102.001	.952
21 00	.794	.197	10	.341	.097	10	.022	10.006
10	.807	.247	20	.358	.149	20	.043	.060
20	.820	.298	30	.375	.201	30	.064	.114
30	.833	.348	40	.392	.253	40	.085	.168
40	.846	.399	50	.410	.306	50	.106	.223
50	.860	.450						
22 00	.873	.502	28 00	.427	.358	34 00	.128	.277
10	.886	.553	10	.444	.410	10	.150	.331
20	.900	.604	20	.462	.463	20	.171	.386
30	.913	.655	30	.480	.515	30	.193	.440
40	.927	.706	40	.497	.568	40	.215	.495
50	.941	.757	50	.515	.620	50	.238	.549
23 00	.955	.808	29 00	.534	.673	35 00	.260	.604
10	.969	.859	10	.552	.726	10	.282	.659
20	.983	.910	20	.570	.778	20	.305	.713
30	.997	.961	30	.588	.831	30	.327	.768
40	101.012	7.013	40	.607	.884	40	.350	.823
50	.026	.064	50	.625	.937	50	.373	.878

TABLE 11.—TANGENTS AND EXTERNALS FOR THE UNIT DOUBLE-SPIRAL CURVE—(*Continued*).
Length of each spiral = 100 feet.
Spiral angle = $\frac{1}{2}\Delta$.

Δ	T_s	E_s	Δ	T_s	E_s	Δ	T_s	E_s
36° 00′	102 396	10 933	42° 00′	103.310	12.962	48° 00′	104 397	15 094
10	419	.988	10	.338	13 020	10	430	.155
20	.442	11.043	20	.366	.077	20	.463	.216
30	.465	.098	30	.394	.135	30	.496	.277
40	.489	153	40	.422	.193	40	.530	.338
50	.513	.209	50	.450	.251	50	.564	.400
37 00	.537	.264	43 00	.479	309	49 00	.598	.461
10	.561	319	10	.508	.367	10	.632	.523
20	.585	375	20	.536	.425	20	.666	.584
30	.609	431	30	.565	.484	30	.701	.646
40	.633	.486	40	.594	.542	40	.735	.708
50	.657	.542	50	.624	.600	50	.770	.770
38 00	.682	.598	44 00	.653	.659	50 00	.805	.832
10	.707	.654	10	.682	.718	10	.840	.894
20	.731	.710	20	.712	.777	20	875	.956
30	.756	.766	30	.742	.836	30	.910	16.018
40	.781	.822	40	.772	.895	40	.946	.081
50	.807	.879	50	.802	.954	50	.981	.143
39 00	.831	.935	45 00	.832	14.013	51 00	105.017	.206
10	.857	.991	10	.862	.072	10	.053	.269
20	.882	12.048	20	.892	.132	20	.088	.332
30	.908	.105	30	.923	.191	30	.124	.394
40	.934	.161	40	.953	.251	40	.160	.458
50	.960	.218	50	.984	.310	50	.197	.521
40 00	.987	.275	46 00	104.015	.370	52 00	.233	.584
10	103.012	.332	10	.046	.430	10	.269	.647
20	.039	.389	20	.077	.490	20	.306	.711
30	.065	.446	30	.108	.549	30	.343	.774
40	.092	.503	40	.140	.610	40	.379	.838
50	.118	.560	50	.171	.670	50	.416	.902
41 00	.146	.617	47 00	.203	.730	53 00	.453	.966
10	.173	.674	10	.235	.790	10	.491	17.030
20	.200	.732	20	.267	.851	20	.528	.094
30	.227	.789	30	.299	.911	30	.566	.158
40	.255	.847	40	.332	.972	40	.604	.223
50	.282	.904	50	.364	15.033	50	.642	.287

TABLE 11.—TANGENTS AND EXTERNALS FOR THE UNIT DOUBLE-SPIRAL CURVE—(*Continued*).

Length of each spiral = 100 feet.

Spiral angle = $\frac{1}{2}\Delta$.

Δ	T_s	E_s	Δ	T_s	E_s	Δ	T_s	E_s
54° 00′	105.680	17.352	60° 00′	107.174	19.761	66° 00′	108.910	22.355
10	718	.417	10	.218	.830	10	.962	.430
20	757	.481	20	.263	.900	20	109.014	.505
30	.796	.546	30	.309	.970	30	.067	.580
40	.835	.611	40	.354	20.040	40	.119	.656
50	.874	.677	50	.400	.110	50	.172	.731
55 00	.913	.742	61 00	.446	.180	67 00	.225	.807
10	.953	.807	10	.491	.250	10	278	.883
20	.992	.872	20	.537	.321	20	.332	.959
30	106.032	.938	30	.584	.391	30	.386	23.036
40	.072	18 004	40	.630	.462	40	.440	.112
50	.113	.070	50	.677	.533	50	.494	.189
56 00	.153	.137	62 00	.724	.604	68 00	.548	.266
10	.194	.203	10	.771	.675	10	.602	.343
20	.234	.269	20	.818	.746	20	.657	.420
30	.275	.336	30	.865	.818	30	.712	.498
40	.316	.402	40	913	.889	40	.767	.575
50	.358	.469	50	961	.961	50	.822	.653
57 00	.399	.536	63 00	108.009	21.033	69 00	.877	.731
10	.441	.603	10	.057	.105	10	.933	.809
20	.482	.670	20	.105	.177	20	.989	.888
30	.524	.737	30	.154	.250	30	110.045	.966
40	.566	.805	40	.203	.322	40	.101	24.045
50	.609	.872	50	.252	.395	50	.158	.124
58 00	.651	.940	64 00	.301	.468	70 00	.215	.203
10	.694	19.008	10	.351	.541	10	.272	.282
20	.737	.075	20	.400	.614	20	.330	.362
30	.779	.143	30	.450	.688	30	.387	.442
40	.823	.211	40	.501	.761	40	.445	.522
50	.866	.280	50	.551	.835	50	.503	.602
59 00	.909	.348	65 00	.602	.909	71 00	.561	.682
10	.953	.416	10	.653	.983	10	.620	.762
20	.997	.485	20	.704	22.057	20	.679	.843
30	107.041	.554	30	.755	.131	30	.738	.924
40	.084	.623	40	.807	.206	40	.797	25.005
50	.129	.696	50	.858	.280	50	.856	.086

TABLE 11.—TANGENTS AND EXTERNALS FOR THE UNIT DOUBLE-SPIRAL CURVE—(*Continued*).

Length of each spiral = 100 feet.

Spiral angle = $\frac{1}{2}\Delta$.

Δ	T_s	E_s	Δ	T_s	E_s	Δ	T_s	E_s
72° 00′	110.916	25 167	78° 00′	113.239	28.243	84° 00′	115 929	31.639
10	.976	.249	10	.308	.332	10	116.009	.739
20	111.036	.330	20	.378	.423	20	.090	.838
30	.097	.412	30	.448	.513	30	.171	.939
40	.158	.495	40	.519	.603	40	.253	32.039
50	.219	.577	50	.589	.694	50	.335	.140
73 00	.280	.660	79 00	.660	.785	85 00	.417	.241
10	.342	.743	10	.732	.876	10	.500	.342
20	.403	.826	20	.803	.968	20	.583	.444
30	.466	.909	30	.875	29 059	30	.666	.546
40	.528	.993	40	.947	.151	40	.750	.648
50	.591	26.077	50	114.019	.244	50	.834	.750
74 00	.654	.161	80 00	.092	.336	86 00	.918	.854
10	.717	.245	10	.165	.429	10	117.003	.957
20	.780	.329	20	.238	.522	20	.088	33.061
30	.844	.414	30	.312	.615	30	.173	.164
40	.907	.499	40	.386	.709	40	.259	.269
50	.972	.584	50	.460	.803	50	.345	.373
75 00	112.036	.669	81 00	.534	.897	87 00	.432	.478
10	.100	.754	10	.609	.991	10	.519	.583
20	.165	.840	20	.684	30.086	20	.606	.689
30	.230	.926	30	.760	.181	30	.694	.795
40	.296	27.012	40	.835	.276	40	.782	.901
50	.361	.098	50	.912	.371	50	.870	34.008
76 00	.427	.185	82 00	.988	.467	88 00	.959	.115
10	.493	.272	10	115.065	.563	10	118.048	.222
20	.560	.359	20	.142	.660	20	.138	.330
30	.626	.446	30	.219	.756	30	.228	.438
40	.693	.534	40	.297	.853	40	.318	.546
50	.760	.622	50	.375	.950	50	.409	.655
						89 00	.500	.764
77 00	.828	.710	83 00	.453	31.048			
10	.896	.798	10	.531	.146	10	.591	.873
20	.964	.887	20	.610	.244	20	.683	.983
30	113.032	.975	30	.689	.342	30	.775	35.093
40	.101	28.064	40	.769	.441	40	.868	.203
50	.170	.154	50	.849	.540	50	.961	.313

TABLE 11.—TANGENTS AND EXTERNALS FOR THE UNIT DOUBLE-SPIRAL CURVE—(*Continued*).

Length of each spiral = 100 feet.

Spiral angle = $\frac{1}{2}\Delta$.

Δ	T_s	E_s	Δ	T_s	E_s	Δ	T_s	E_s
90° 00′	119.054	35.424	96° 00′	122.698	39.688	102° 00′	126 980	44.544
10	.148	.535	10	.808	.814	10	127.109	.689
20	.242	.647	20	.918	941	20	.239	.834
30	.336	.759	30	123.029	40.068	30	370	.980
40	.431	.872	40	.140	.195	40	.501	45.127
50	.526	.985	50	.252	.323	50	.633	.274
91 00	.622	36 098	97 00	.364	.452	103 00	765	.422
10	.718	.212	10	.477	.581	10	.898	.570
20	.815	.326	20	.590	.710	20	128.032	.719
30	.911	.440	30	.704	.840	30	.166	.869
40	120 009	.555	40	.818	.971	40	.301	46.019
50	.107	.670	50	.933	41.102	50	.437	.170
92 00	.205	.786	98 00	124.048	.233	104 00	.573	.321
10	.304	.902	10	.164	.365	10	.710	.473
20	.403	37.019	20	.280	.498	20	.848	.626
30	.502	.136	30	.397	.631	30	.986	.779
40	.603	.253	40	.515	.764	40	129 125	.933
50	.703	.371	50	.633	.898	50	.265	47.088
93 00	.804	.489	99 00	.751	42.033	105 00	.405	.243
10	.905	.608	10	.870	.168	10	.546	.399
20	121.007	.727	20	.990	.303	20	.688	.556
30	.109	.846	30	125.110	.439	30	.830	.713
40	.212	.966	40	.231	.576	40	.973	.871
50	.315	38.086	50	.352	.713	50	130.117	48.030
94 00	.419	.207	100 00	.474	.851	106 00	.261	.189
10	.523	.328	10	.596	.989	10	.406	.349
20	.629	.449	20	.719	43.127	20	.552	.509
30	.732	.571	30	.843	.266	30	.698	.670
40	.838	.693	40	.967	.406	40	.846	.832
50	.944	.816	50	126.091	.546	50	.993	.995
95 00	122.050	.939	101 00	.216	.687	107 00	131.142	49.158
10	.157	39.063	10	.342	.828	10	.291	.322
20	.264	.187	20	.468	.970	20	.441	.487
30	.372	.311	30	.595	44.113	30	.592	.652
40	.480	.436	40	.723	.256	40	.744	.818
50	.589	.562	50	.851	.400	50	.896	.985

TABLE 12.—FUNCTIONS OF THE SPIRAL WHICH IS COMPOUNDED WITH A 1° CIRCULAR CURVE

Note: The spiral is compounded at the S.C. of the circular curve whose central portion remains unchanged, while its ends are sharpened to provide clearance for the spiral.

θ_s = spiral angle	a = distance from P.C. to T.S. (ft.)	l_s = length of spiral (ft.)	$\frac{100}{l_s}$	y = offset from P.C. to spiral (ft.)	D_s = degree-of-curve of spiral 100 ft. from the T.S. (deg.)	φ = spiral deflection for the 1st 100 ft. of spiral (min.)	θ_s = spiral angle
4°	200.0′	600.0′	0.16667	0.52′	0.22222°	2.22222′	4°
5	250.0	750.0	0.13333	0.81	0.17778	1.77778	5
6	300.0	899.9	0.11112	1.17	0.14818	1.48181	6
7	350.0	1,049.8	0.09526	1.59	0.12703	1.27032	7
8	400.0	1,199.7	0.08335	2.07	0.11117	1.11166	8
9	450.0	1,349.6	0.07410	2.62	0.09882	0.98824	9
10	500.0	1,499.6	0.06669	3.24	0.08894	0.88936	10
11	550.0	1,649.3	0.06063	3.92	0.08088	0.80877	11
12	599.9	1,799.1	0.05558	4.67	0.07415	0.74148	12
13	649.9	1,948.8	0.05131	5.48	0.06846	0.68460	13
14	699.9	2,098.5	0.04765	6.36	0.06358	0.63582	14
15	749.9	2,248.2	0.04448	7.31	0.05935	0.59354	15
16	799.9	2,397.7	0.04171	8.33	0.05566	0.55662	16
17	849.8	2,547.3	0.03926	9.41	0.05240	0.52398	17
18	899.8	2,696.9	0.03708	10.56	0.04950	0.49496	18
19	949.8	2,846.3	0.03513	11.78	0.04690	0.46905	19
20	999.8	2,995.6	0.03338	13.06	0.04457	0.44575	20
21	1,049.7	3,145.0	0.03180	14.41	0.04246	0.42463	21
22	1,099.7	3,294.2	0.03036	15.83	0.04055	0.40547	22
23	1,149.6	3,443.3	0.02904	17.32	0.03880	0.38798	23
24	1,199.5	3,592.5	0.02784	18.88	0.03719	0.37192	24
25	1,249.4	3,741.5	0.02673	20.51	0.03572	0.35717	25
26	1,299.3	3,890.4	0.02570	22.21	0.03436	0.34357	26
27	1,349.3	4,039.2	0.02476	23.98	0.03310	0.33098	27
28	1,399.2	4,188.1	0.02388	25.83	0.03193	0.31927	28
29	1,449.1	4,336.7	0.02306	27.75	0.03084	0.30840	29
30	1,499.0	4,485.2	0.02230	29.74	0.02982	0.29825	30

For any other degree-of-curve $(=D)$;
divide the values of a, l_s, and y by D;
multiply $\frac{100}{l_s}$ by D;
multiply D_s and ϕ by D^2.

TABLE 13.—FUNCTIONS OF THE SPIRAL WHICH IS COMPOUNDED WITH A CIRCULAR CURVE OF RADIUS 10,000 FT.

Note: The spiral is compounded at the S.C. of the circular curve whose central portion remains unchanged, while its ends are sharpened to provide clearance for the spiral.

θ_s = spiral angle	a = distance from P.C. to T.S. (ft.)	l_s = length of spiral (ft.)	$\frac{100}{l_s}$	y = offset from P.C. to spiral (ft.)	D_s = degree-of-curve of spiral 100 ft. from the T.S. (deg.)	φ = spiral deflection for the 1st 100 ft. of spiral (min.)	θ_s = spiral angle
4°	349.1′	1,047.2′	0.09549	0.91′	0.07295°	0.72951	4°
5	436.3	1,309.0	0.07639	1.41	0.05836	0.58361	5
6	523.6	1,570.6	0.06367	2.04	0.04864	0.48645	6
7	610.9	1,832.2	0.05458	2.78	0.04170	0.41702	7
8	698.1	2,093.9	0.04776	3.61	0.03649	0.36494	8
9	785.4	2,355.5	0.04246	4.57	0.03244	0.32442	9
10	872.7	2,617.3	0.03821	5.65	0.02920	0.29196	10
11	959.9	2,878.6	0.03474	6.84	0.02655	0.26550	11
12	1,047.0	3,140.0	0.03184	8.15	0.02434	0.24341	12
13	1,134.3	3,401.3	0.02940	9.56	0.02247	0.22474	13
14	1,221.6	3,662.6	0.02730	11.10	0.02087	0.20873	14
15	1,308.8	3,923.8	0.02549	12.76	0.01948	0.19485	15
16	1,396.1	4,184.8	0.02390	14.54	0.01827	0.18273	16
17	1,483.2	4,445.9	0.02249	16.42	0.01720	0.17201	17
18	1,570.4	4,707.0	0.02125	18.43	0.01625	0.16249	18
19	1,657.7	4,967.7	0.02013	20.56	0.01540	0.15398	19
20	1,745.0	5,228.3	0.01913	22.79	0.01463	0.14633	20
21	1,832.1	5,489.1	0.01822	25.15	0.01394	0.13940	21
22	1,919.3	5,749.5	0.01739	27.63	0.01331	0.13311	22
23	2,006.4	6,009.7	0.01664	30.23	0.01274	0.12737	23
24	2,093.5	6,270.1	0.01595	32.95	0.01221	0.12209	24
25	2,180.6	6,301.5	0.01532	35.80	0.01172	0.11725	25
26	2,267.7	6,790.0	0.01473	38.76	0.01128	0.11279	26
27	2,355.0	7,049.7	0.01419	41.85	0.01086	0.10865	27
28	2,442.1	7,309.6	0.01368	45.08	0.01048	0.10481	28
29	2,529.2	7,569.0	0.01321	48.43	0.01012	0.10124	29
30	2,616.2	7,828.1	0.01279	51.91	0.00979	0.09791	30

For any other radius (R);

divide **the values of a, l_s and y by $\frac{10,000}{R}$;**

multiply **$\frac{100}{l_s}$ by $\frac{10,000}{R}$;**

multiply **D_s and ϕ by $\left(\frac{10,000}{R}\right)^2$.**

Table 14.—Offsets Locating a Modified 200-ft. Spiral

Deg. of curve D	Perpendicular offsets from the tangent (ft.)					Radial offsets from the circle (ft.)					
	At T.S. P.C. −100′	P.C. −80′	P.C. −60′	P.C. −40′	P.C. −20′	At P.C. 0	P.C. +20′	P.C. +40′	P.C. +60′	P.C. +80′	At S.C. P.C. +100′
5° 00′	0.00	0.01	0.05	0.16	0.37	0.73	1.08	1.29	1.40	1.44	1.45
5 30	0.00	0.01	0.05	0.17	0.41	0.80	1.19	1.43	1.55	1.59	1.60
6 00	0.00	0.01	0.06	0.19	0.45	0.87	1.29	1.55	1.68	1.73	1.74
6 30	0.00	0.01	0.06	0.21	0.49	0.95	1.40	1.68	1.83	1.88	1.89
7 00	0.00	0.01	0.07	0.22	0.52	1.02	1.51	1.81	1.97	2.03	2.04
7 30	0.00	0.01	0.07	0.24	0.56	1.09	1.62	1.94	2.11	2.17	2.18
8 00	0.00	0.01	0.07	0.25	0.59	1.16	1.73	2.08	2.26	2.32	2.33
8 30	0.00	0.01	0.08	0.27	0.63	1.23	1.84	2.21	2.40	2.46	2.47
9 00	0.00	0.01	0.08	0.28	0.67	1.31	1.95	2.34	2.54	2.61	2.62
9 30	0.00	0.01	0.09	0.30	0.71	1.38	2.05	2.47	2.67	2.75	2.76
10 00	0.00	0.01	0.09	0.31	0.74	1.45	2.16	2.60	2.82	2.90	2.91
10 30	0.00	0.01	0.10	0.33	0.78	1.53	2.27	2.72	2.96	3.04	3.05
11 00	0.00	0.01	0.10	0.35	0.82	1.60	2.38	2.85	3.10	3.19	3.20
11 30	0.00	0.01	0.11	0.36	0.86	1.67	2.48	2.98	3.24	3.33	3.34
12 00	0.00	0.01	0.11	0.38	0.89	1.74	2.59	3.11	3.38	3.48	3.49
12 30	0.00	0.01	0.12	0.39	0.93	1.82	2.70	3.24	3.51	3.62	3.63
13 00	0.00	0.02	0.12	0.41	0.97	1.89	2.80	3.37	3.65	3.76	3.77
13 30	0.00	0.02	0.13	0.42	1.00	1.96	2.91	3.50	3.79	3.90	3.92
14 00	0.00	0.02	0.13	0.44	1.04	2.03	3.02	3.62	3.93	4.04	4.06
14 30	0.00	0.02	0.13	0.45	1.08	2.10	3.13	3.75	4.07	4.19	4.21
15 00	0.00	0.02	0.14	0.47	1.12	2.18	3.23	3.88	4.21	4.33	4.35
15 30	0.00	0.02	0.14	0.49	1.15	2.25	3.34	4.01	4.35	4.48	4.50
16 00	0.00	0.02	0.15	0.50	1.19	2.32	3.45	4.14	4.49	4.62	4.64
16 30	0.00	0.02	0.15	0.52	1.22	2.39	3.56	4.27	4.63	4.77	4.79
17 00	0.00	0.02	0.16	0.53	1.26	2.47	3.67	4.40	4.77	4.91	4.93
17 30	0.00	0.02	0.16	0.55	1.30	2.54	3.77	4.53	4.91	5.05	5.07
18 00	0.00	0.02	0.17	0.56	1.34	2.61	3.88	4.66	5.05	5.20	5.22
18 30	0.00	0.02	0.17	0.58	1.37	2.68	3.99	4.78	5.19	5.34	5.36
19 00	0.00	0.02	0.18	0.59	1.41	2.75	4.09	4.91	5.33	5.48	5.50
19 30	0.00	0.02	0.18	0.61	1.44	2.82	4.20	5.04	5.47	5.63	5.65
20 00	0.00	0.02	0.19	0.63	1.48	2.90	4.31	5.17	5.61	5.77	5.79
20 30	0.00	0.02	0.19	0.64	1.52	2.97	4.42	5.30	5.75	5.92	5.94
21 00	0.00	0.02	0.19	0.66	1.56	3.04	4.52	5.42	5.89	6.06	6.08
21 30	0.00	0.02	0.20	0.67	1.59	3.11	4.63	5.55	6.03	6.20	6.22
22 00	0.00	0.03	0.20	0.69	1.63	3.18	4.74	5.68	6.17	6.34	6.37
22 30	0.00	0.03	0.21	0.70	1.66	3.25	4.85	5.81	6.30	6.48	6.51
23 00	0.00	0.03	0.21	0.72	1.70	3.33	4.95	5.94	6.44	6.62	6.65
23 30	0.00	0.03	0.22	0.73	1.74	3.40	5.06	6.07	6.58	6.77	6.80
24 00	0.00	0.03	0.22	0.75	1.78	3.47	5.16	6.19	6.72	6.91	6.94
24 30	0.00	0.03	0.23	0.76	1.81	3.54	5.27	6.32	6.85	7.05	7.08
25 00	0.00	0.03	0.23	0.78	1.85	3.61	5.37	6.44	6.99	7.19	7.22

Note: The middle portion of the circular curve is shifted inward by the amount of the offset at the S.C.

TABLE 15 (PART 1).—EXTRA AREA OF SPIRAL PORTIONS OF WIDENED PAVEMENT

Area of both portions $= \frac{l_s f}{9} - B$ (sq. yd.),

where l_s = length of each ℄ spiral;

f = maximum central widening.

Based on normal width of pavement = **16 ft.**

Values of B:

θ_s	f = 1.0′	1.5′	2.0′	2.5′	3.0′	3.5′	4.0′	4.5′	θ_s
5°	0.11	0.17	0.23	0.29	0.35	0.42	0.49	0.56	5°
10	0.22	0.33	0.45	0.58	0.71	0.84	0.98	1.13	10
15	0.33	0.50	0.68	0.87	1.06	1.26	1.47	1.69	15
20	0.43	0.66	0.91	1.16	1.42	1.69	1.97	2.25	20
25	0.54	0.83	1.13	1.44	1.77	2.11	2.46	2.82	25
30	0.65	1.00	1.36	1.73	2.12	2.53	2.95	3.38	30
35	0.76	1.16	1.58	2.02	2.48	2.95	3.44	3.95	35
40	0.87	1.33	1.81	2.31	2.83	3.37	3.93	4.51	40
45	0.97	1.49	2.04	2.60	3.18	3.79	4.42	5.07	45
50	1.08	1.66	2.26	2.89	3.54	4.21	4.91	5.64	50
55	1.19	1.83	2.49	3.18	3.89	4.64	5.40	6.20	55
60	1.30	1.99	2.71	3.47	4.25	5.06	5.90	6.76	60
65	1.41	2.16	2.94	3.76	4.60	5.48	6.39	7.33	65
70	1.52	2.32	3.17	4.04	4.96	5.90	6.88	7.89	70

θ_s	f = 5.0′	5.5′	6.0′	6.5′	7.0′	8.0′	9.0′	10.0′	θ_s
5°	0.64	0.72	0.80	0.88	0.96	1.14	1.32	1.52	5°
10	1.28	1.43	1.59	1.75	1.92	2.27	2.65	3.04	10
15	1.91	2.15	2.39	2.63	2.88	3.41	3.97	4.56	15
20	2.55	2.86	3.18	3.51	3.85	4.55	5.29	6.08	20
25	3.19	3.58	3.97	4.39	4.81	5.69	6.62	7.60	25
30	3.83	4.29	4.77	5.26	5.77	6.83	7.94	9.11	30
35	4.47	5.01	5.57	6.14	6.73	7.96	9.26	10.63	35
40	5.11	5.72	6.36	7.02	7.69	9.10	10.59	12.15	40
45	5.75	6.44	7.16	7.89	8.65	10.24	11.91	13.67	45
50	6.38	7.16	7.95	8.77	9.61	11.38	13.23	15.19	50
55	7.02	7.87	8.75	9.65	10.58	12.51	14.56	16.71	55
60	7.66	8.59	9.54	10.53	11.54	13.65	15.88	18.23	60
65	8.30	9.30	10.34	11.40	12.50	14.79	17.21	19.75	65
70	8.94	10.02	11.13	12.28	13.46	15.93	18.53	21.27	70

Area of intermediate circular portion is given in Table 16.

Table 15 (Part 2).—Extra Area of Spiral Portions of Widened Pavement

Area of both portions $= \dfrac{l_s f}{9} - B$ (sq. yd.),

where l_s = length of each ℄ spiral;

f = maximum central widening.

Based on normal width of pavement = **18 ft.**

Values of B:

θ_s	f = 1.0′	1.5′	2.0′	2.5′	3.0′	3.5′	4.0′	4.5′	θ_s
5°	0.12	0.19	0.25	0.32	0.39	0.47	0.54	0.62	5°
10	0.24	0.37	0.50	0.64	0.78	0.93	1.09	1.24	10
15	0.36	0.56	0.76	0.96	1.18	1.40	1.63	1.87	15
20	0.48	0.74	1.01	1.28	1.57	1.87	2.17	2.49	20
25	0.61	0.93	1.26	1.61	1.96	2.33	2.72	3.11	25
30	0.73	1.11	1.51	1.93	2.36	2.80	3.26	3.73	30
35	0.85	1.30	1.76	2.25	2.75	3.27	3.80	4.35	35
40	0.97	1.48	2.02	2.57	3.14	3.73	4.34	4.97	40
45	1.09	1.67	2.27	2.89	3.53	4.20	4.89	5.60	45
50	1.21	1.85	2.52	3.21	3.93	4.67	5.43	6.22	50
55	1.33	2.04	2.77	3.53	4.32	5.13	5.97	6.84	55
60	1.54	2.33	3.02	3.85	4.71	5.60	6.52	7.46	60
65	1.66	2.41	3.27	4.18	5.10	6.07	7.06	8.08	65
70	1.78	2.60	3.52	4.50	5.49	6.53	7.60	8.70	70

θ_s	f = 5.0′	5.5′	6.0′	6.5′	7.0′	8.0′	9.0′	10.0′	θ_s
5°	0.70	0.79	0.87	0.96	1.05	1.24	1.44	1.65	5°
10	1.41	1.57	1.74	1.92	2.10	2.48	2.88	3.30	10
15	2.11	2.36	2.62	2.88	3.16	3.72	4.32	4.94	15
20	2.81	3.15	3.49	3.84	4.21	4.96	5.76	6.59	20
25	3.51	3.93	4.36	4.81	5.26	6.21	7.20	8.24	25
30	4.22	4.72	5.24	5.77	6.31	7.45	8.64	9.89	30
35	4.92	5.51	6.11	6.73	7.36	8.69	10.08	11.54	35
40	5.62	6.29	6.98	7.69	8.42	9.93	11.52	13.19	40
45	6.33	7.08	7.85	8.65	9.47	11.17	12.96	14.83	45
50	7.03	7.87	8.73	9.61	10.52	12.41	14.40	16.48	50
55	7.73	8.65	9.60	10.57	11.57	13.65	15.84	18.13	55
60	8.44	9.44	10.47	11.53	12.62	14.89	17.28	19.78	60
65	9.14	10.23	11.34	12.49	13.67	15.13	18.72	21.43	65
70	9.84	11.01	12.21	13.46	14.72	16.37	19.16	23.08	70

Area of intermediate circular portion is given in Table 16.

TABLE 15 (PART 3).—EXTRA AREA OF SPIRAL PORTIONS OF WIDENED PAVEMENT

Area of both portions $= \frac{l_s f}{9} - B$ (sq. yd.),

where l_s = length of each ℄ spiral;

f = maximum central widening.

Based on normal width of pavement = **20 ft.**

Values of B:

θ_s	f = 1.0′	1.5′	2.0′	2.5′	3.0′	3.5′	4.0′	4.5′	θ_s
5°	0.13	0.20	0.28	0.35	0.43	0.51	0.60	0.68	5°
10	0.27	0.41	0.56	0.71	0.86	1.02	1.19	1.36	10
15	0.40	0.61	0.83	1.06	1.29	1.54	1.78	2.04	15
20	0.54	0.82	1.11	1.41	1.73	2.05	2.38	2.72	20
25	0.67	1.02	1.39	1.77	2.16	2.56	2.97	3.40	25
30	0.80	1.23	1.67	2.12	2.59	3.07	3.57	4.08	30
35	0.94	1.43	1.95	2.47	3.02	3.58	4.16	4.76	35
40	1.07	1.64	2.22	2.83	3.45	4.10	4.76	5.44	40
45	1.21	1.84	2.50	3.18	3.88	4.61	5.35	6.12	45
50	1.34	2.05	2.78	3.54	4.31	5.12	5.95	6.80	50
55	1.47	2.25	3.06	3.89	4.75	5.63	6.54	7.48	55
60	1.61	2.46	3.34	4.24	5.18	6.14	7.14	8.16	60
65	1.74	2.66	3.61	4.60	5.61	6.65	7.73	8.84	65
70	1.88	2.87	3.89	4.95	6.04	7.17	8.33	9.52	70

θ_s	f = 5.0′	5.5′	6.0′	6.5′	7.0′	8.0′	9.0′	10.0′	θ_s
5°	0.77	0.86	0.95	1.05	1.14	1.34	1.56	1.79	5°
10	1.53	1.72	1.90	2.09	2.28	2.69	3.11	3.55	10
15	2.30	2.57	2.85	3.14	3.43	4.03	4.67	5.33	15
20	3.07	3.43	3.80	4.18	4.57	5.38	6.22	7.11	20
25	3.84	4.29	4.75	5.23	5.71	6.72	7.78	8.89	25
30	4.61	5.15	5.70	6.27	6.86	8.07	9.34	10.67	30
35	5.37	6.00	6.65	7.32	8.00	9.41	10.89	12.44	35
40	6.14	6.86	7.60	8.36	9.14	10.76	12.45	14.22	40
45	6.91	7.72	8.55	9.41	10.28	12.10	14.01	16.00	45
50	7.68	8.58	9.50	10.45	11.42	13.44	15.56	17.78	50
55	8.44	9.43	10.45	11.50	12.57	14.79	17.12	19.55	55
60	9.21	10.29	11.40	12.54	13.71	16.13	18.67	21.33	60
65	9.98	11.15	12.35	13.59	14.85	17.48	20.23	23.11	65
70	10.75	12.01	13.30	14.63	16.00	18.82	21.79	24.89	70

Area of intermediate circular portion is given in Table 16.

TABLE 15 (PART 4).—EXTRA AREA OF SPIRAL PORTIONS OF WIDENED PAVEMENT

Area of both portions $= \frac{l_s f}{9} - B$ (sq. yd.),

where l_s = length of each ℄ spiral;

f = maximum central widening.

Based on normal width of pavement = **22 ft.**

Values of B:

θ_s	f = 1.0′	1.5′	2.0′	2.5′	3.0′	3.5′	4.0′	4.5′	θ_s
5°	0.15	0.22	0.30	0.39	0.47	0.56	0.65	0.74	5°
10	0.29	0.45	0.61	0.77	0.94	1.11	1.29	1.48	10
15	0.44	0.67	0.91	1.16	1.41	1.67	1.94	2.21	15
20	0.59	0.90	1.22	1.54	1.88	2.23	2.59	2.95	20
25	0.74	1.12	1.52	1.93	2.35	2.79	3.23	3.69	25
30	0.88	1.35	1.82	2.32	2.82	3.34	3.88	4.43	30
35	1.03	1.57	2.13	2.70	3.29	3.90	4.52	5.17	35
40	1.18	1.79	2.43	3.09	3.76	4.46	5.17	5.91	40
45	1.32	2.02	2.73	3.47	4.23	5.01	5.82	6.64	45
50	1.47	2.24	3.04	3.86	4.70	5.57	6.46	7.38	50
55	1.62	2.47	3.34	4.24	5.17	6.13	7.11	8.12	55
60	1.76	2.69	3.65	4.63	5.64	6.69	7.76	8.86	60
65	1.91	2.91	3.95	5.02	6.11	7.24	8.40	9.60	65
70	2.06	3.14	4.25	5.40	6.58	7.80	9.05	10.33	70

θ_s	f = 5.0′	5.5′	6.0′	6.5′	7.0′	8.0′	9.0′	10.0′	θ_s
5°	0.83	0.93	1.05	1.13	1.23	1.45	1.67	1.91	5°
10	1.66	1.86	2.09	2.26	2.47	2.90	3.35	3.81	10
15	2.50	2.79	3.14	3.39	3.70	4.34	5.02	5.72	15
20	3.33	3.72	4.18	4.52	4.93	5.79	6.69	7.63	20
25	4.16	4.64	5.23	5.65	6.17	7.24	8.36	9.53	25
30	4.99	5.57	6.27	6.78	7.40	8.69	10.04	11.44	30
35	5.83	6.50	7.32	7.90	8.63	10.14	11.71	13.35	35
40	6.66	7.43	8.36	9.03	9.86	11.58	13.38	15.26	40
45	7.49	8.36	9.41	10.16	11.10	13.03	15.05	17.16	45
50	8.32	9.29	10.45	11.29	12.33	14.48	16.73	19.07	50
55	9.15	10.22	11.50	12.42	13.56	15.93	18.40	20.98	55
60	9.99	11.15	12.54	13.55	14.80	17.38	20.07	22.88	60
65	10.82	12.07	13.59	14.68	16.03	18.82	21.74	24.79	65
70	11.65	13.00	14.13	15.81	17.26	20.27	23.42	26.70	70

Area of intermediate circular portion is given in Table 16.

TABLE 16 (PART 1).—EXTRA AREA OF CIRCULAR PORTION OF WIDENED PAVEMENT (IN SQUARE YARDS) PER LINEAL FOOT OF NOMINAL ℄ CURVE

16-ft. pavement

Deg. of curve D	f = constant central widening = 1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	Deg. of curve D
5°	.110	.220	.331	.441	.550	.660	.770	.880	.989	1.099	5°
6	.110	.220	.330	.440	.549	.659	.768	.878	.987	1.096	6
7	.110	.220	.329	.439	.548	.658	.767	.876	.985	1.093	7
8	.110	.219	.329	.438	.547	.656	.765	.874	.983	1.091	8
9	.110	.219	.328	.437	.546	.655	.764	.872	.980	1.088	9
10	.109	.219	.328	.437	.545	.654	.762	.870	.978	1.086	10
11	.109	.218	.327	.436	.544	.652	.761	.868	.976	1.083	11
12	.109	.218	.327	.435	.543	.651	.759	.867	.974	1.081	12
13	.109	.218	.326	.434	.542	.650	.757	.865	.972	1.078	13
14	.109	.217	.326	.434	.541	.649	.756	.863	.969	1.076	14
15	.109	.217	.325	.433	.540	.647	.754	.861	.967	1.073	15
16	.108	.217	.324	.432	.539	.646	.753	.859	.965	1.071	16
17	.108	.216	.324	.431	.538	.645	.751	.857	.963	1.068	17
18	.108	.216	.323	.430	.537	.643	.750	.855	.961	1.066	18
19	.108	.216	.323	.430	.536	.642	.748	.854	.959	1.063	19
20	.108	.215	.322	.429	.535	.641	.747	.852	.956	1.061	20
30	.106	.212	.317	.421	.525	.628	.731	.833	.935	1.035	30
40	.105	.208	.311	.413	.515	.615	.715	.814	.913	1.010	40
50	.103	.205	.306	.406	.505	.602	.700	.796	.891	.985	50
60	.101	.201	.300	.398	.494	.589	.684	.777	.869	.960	60
70	.100	.198	.295	.390	.484	.576	.669	.759	.847	.935	70
80	.098	.194	.289	.382	.474	.563	.653	.740	.825	.909	80
90	.096	.191	.284	.375	.464	.551	.637	.721	.804	.884	90
100	.095	.187	.278	.367	.454	.538	.622	.703	.782	.859	100

18-ft. pavement

Deg. of curve D	f = constant central widening = 1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	Deg. of curve D
5°	.110	.220	.330	.440	.550	.660	.769	.879	.988	1.098	5°
6	.110	.220	.330	.439	.549	.658	.768	.877	.986	1.095	6
7	.110	.220	.329	.438	.548	.657	.766	.875	.984	1.092	7
8	.110	.219	.328	.438	.547	.655	.764	.873	.981	1.089	8
9	.109	.219	.328	.437	.546	.654	.763	.871	.979	1.087	9
10	.109	.218	.327	.436	.544	.653	.761	.869	.976	1.084	10
11	.109	.218	.327	.435	.543	.651	.759	.867	.974	1.081	11
12	.109	.218	.326	.434	.542	.650	.757	.865	.972	1.079	12
13	.109	.217	.325	.433	.541	.649	.756	.863	.969	1.076	13
14	.109	.217	.325	.432	.540	.647	.754	.861	.967	1.073	14
15	.108	.216	.324	.432	.539	.646	.752	.859	.965	1.070	15
16	.108	.216	.324	.431	.538	.644	.751	.857	.962	1.068	16
17	.108	.216	.323	.430	.537	.643	.749	.855	.960	1.065	17
18	.108	.215	.322	.429	.535	.642	.747	.853	.958	1.062	18
19	.108	.215	.322	.428	.534	.640	.746	.851	.955	1.060	19
20	.107	.214	.321	.427	.533	.639	.744	.849	.953	1.057	20
30	.106	.211	.315	.419	.522	.625	.727	.828	.929	1.030	30
40	.104	.207	.309	.410	.511	.611	.710	.808	.906	1.003	40
50	.102	.203	.303	.402	.500	.597	.693	.788	.882	.975	50
60	.100	.199	.297	.393	.489	.583	.676	.768	.859	.948	60
70	.098	.195	.291	.385	.478	.569	.659	.748	.835	.921	70
80	.096	.191	.284	.376	.466	.555	.642	.728	.812	.894	80
90	.095	.187	.278	.368	.455	.541	.625	.707	.788	.867	90
100	.093	.183	.272	.359	.444	.527	.608	.687	.764	.840	100

TABLE 16 (PART 2).—EXTRA AREA OF CIRCULAR PORTION OF WIDENED PAVEMENT (IN SQUARE YARDS) PER LINEAL FOOT OF NOMINAL ℄ CURVE

	Deg. of curve D	f = constant central widening =										Deg. of curve D
		1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	
20-ft. pavement	5°	.111	.220	.330	.440	.549	.659	.769	.878	.987	1.097	5°
	6	.110	.220	.329	.439	.548	.658	.767	.876	.985	1.094	6
	7	.110	.219	.329	.438	.547	.656	.765	.874	.982	1.091	7
	8	.109	.219	.328	.437	.546	.655	.763	.872	.980	1.088	8
	9	.109	.218	.327	.436	.545	.653	.761	.869	.977	1.085	9
	10	.109	.218	.327	.435	.543	.652	.759	.867	.975	1.082	10
	11	.109	.218	.326	.434	.542	.650	.758	.865	.972	1.079	11
	12	109	.217	.325	.433	.541	.649	.756	.863	.970	1.076	12
	13	.108	.217	.325	.432	.540	.647	.754	.861	.967	1.073	13
	14	.108	.216	.324	.431	.539	.645	.752	.858	.965	1.070	14
	15	.108	.216	.323	.430	.537	.644	.750	.856	.962	1.067	15
	16	.108	.215	.323	.430	.536	.642	.748	.854	.960	1.065	16
	17	.108	.215	.322	.429	.535	.641	.747	.852	.957	1.062	17
	18	.107	.215	.321	.428	.534	.639	.745	.850	.954	1.059	18
	19	.107	.214	.321	.427	.533	.638	.743	.848	.952	1.056	19
	20	.107	.214	.320	.426	.531	.636	.741	.845	.949	1.053	20
	30	.105	.209	.313	.417	.519	.621	.723	.824	.924	1.024	30
	40	.103	.205	.307	.407	.507	.606	.704	.802	.899	.995	40
	50	.101	.201	.300	.398	.495	.591	.686	.780	.873	.966	50
	60	.099	.197	.293	.389	.483	.576	.668	.759	.848	.937	60
	70	.097	.192	.287	.379	.471	.561	.649	.737	.823	.907	70
	80	.095	.188	.280	.370	.459	.546	.631	.715	.798	.878	80
	90	.093	.184	.273	.361	.446	.531	.613	.693	.772	.849	90
	100	.091	.180	.266	.351	.434	.515	.595	.672	.747	.820	100
		f = constant central widening =										
		1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	
22-ft. pavement	5°	.110	.220	.330	.439	.549	.659	.768	.877	.986	1.096	5°
	6	.110	.219	.329	.438	.548	.657	.766	.875	.984	1.092	6
	7	.110	.219	.328	.437	.546	.655	.764	.873	.981	1.089	7
	8	.109	.218	.328	.436	.545	.654	.762	.870	.978	1.086	8
	9	.109	.218	.327	.435	.544	.652	.760	.868	.976	1.083	9
	10	.109	.218	.326	.434	.542	.650	.758	.866	.973	1.080	10
	11	.109	.217	.325	.433	.541	.649	.756	.863	.970	1.077	11
	12	.108	.217	.325	.432	.540	.647	.754	.861	.968	1.074	12
	13	.108	.216	.324	.431	.539	.645	.752	.859	.965	1.071	13
	14	.108	.216	.323	.430	.537	.644	.750	.856	.962	1.068	14
	15	.108	.215	.322	.429	.536	.642	.748	.854	.959	1.065	15
	16	.108	.215	.322	.428	.535	.641	.746	.852	.957	1.061	16
	17	.107	.214	.321	.427	.533	.639	.744	.849	.954	1.058	17
	18	.107	.214	.320	.426	.532	.637	.742	.847	.951	1.055	18
	19	.107	.213	.320	.425	.531	.636	.740	.845	.949	1.052	19
	20	.107	.213	.319	.424	.529	.634	.738	.842	.946	1.049	20
	30	.104	.208	.312	.414	.516	.618	.719	.819	.919	1.018	30
	40	.102	.204	.304	.404	.503	.602	.699	.796	.892	.987	40
	50	.100	.199	.297	.394	.490	.585	.679	.773	.865	.956	50
	60	.098	.194	.290	.384	.477	.569	.660	.749	.838	.925	60
	70	.095	.190	.282	.374	.464	.553	.640	.726	.811	.894	70
	80	.093	.185	.275	.364	.451	.536	.620	.703	.784	.863	80
	90	.091	.180	.268	.354	.438	.520	.601	.679	.757	.832	90
	100	.089	.176	.261	.344	.425	.504	.581	.656	.729	.801	100

TABLE 17.—AREAS OF WATERWAY CALCULATED BY TALBOT'S FORMULA

a = area of waterway in square feet.
A = drainage area in acres.

$$a = C\sqrt[4]{A^3}.$$

Drainage area		Areas of waterway in square feet						
		Mountainous land	Hilly land		Rolling land		Flat land	
Acres	Square miles	$C = 1.00$	$C = 0.80$	$C = 0.60$	$C = 0.50$	$C = 0.40$	$C = 0.30$	$C = 0.20$
1	0.0016	1.0	0.8	0.6	0.5	0.4	0.3	0.2
2	.0031	1.7	1.4	1.0	.8	.7	.5	.3
4	.0062	2.8	2.2	1.7	1.4	1.1	.8	.6
6	.0094	3.8	3.0	2.3	1.9	1.5	1.1	.8
8	.0125	4.8	3.8	2.9	2.4	1.9	1.4	1.0
10	.018	3.6	4.5	3.4	2.8	2.2	1.7	1.2
15	.023	7.6	5.1	4.6	3.8	3.0	2.3	1.5
20	.031	9.5	7.6	5.7	4.7	3.8	2.8	1.9
25	.039	11.2	9.0	6.7	5.6	4.5	3.4	2.2
30	.047	12.8	10.2	7.7	6.4	5.1	3.8	2.6
35	.05	14.4	11.5	8.6	7.2	5.8	4.3	2.9
40	.06	15.9	12.7	9.5	8.0	6.4	4.8	3.2
45	.07	17.4	13.9	10.4	8.7	7.0	5.2	3.5
50	.08	18.8	15.0	11.3	9.4	7.5	5.6	3.8
55	.09	20	16	12	10	8	6	4
60	.09	22	17	13	11	9	6	4
65	.10	23	18	14	11	10	7	5
70	.11	24	19	14	12	10	7	5
75	.12	26	21	16	13	10	8	5
80	.13	27	22	16	13	11	8	5
85	.13	28	22	17	14	11	8	6
90	.14	29	23	17	14	12	9	6
95	.15	30	24	18	15	12	9	6
100	.16	32	26	19	16	13	10	6
150	.23	43	34	26	21	17	13	9
200	.31	53	42	32	27	21	16	11
250	.39	63	50	38	32	25	19	13
300	.47	72	58	43	36	29	22	14
350	.54	81	65	49	41	32	24	16
400	.63	89	71	53	44	36	27	18
450	.70	98	78	59	49	39	29	20
500	.78	106	85	64	53	42	32	21
550	.86	114	91	68	57	46	34	23
600	.94	121	97	73	61	48	36	24
650	1.02	129	103	77	65	52	39	26
700	1.09	136	109	82	68	54	41	27
750	1.17	143	114	86	72	57	43	29
800	1.25	150	120	90	75	60	45	30
850	1.33	157	126	94	79	63	47	31
900	1.41	164	131	98	82	66	49	33
950	1.48	171	137	103	86	68	51	34
1,000	1.56	178	142	107	89	71	53	36
1,100	1.72	191	153	115	95	76	57	38
1,200	1.88	204	163	122	102	82	61	41
1,300	2.03	216	173	130	108	86	65	43
1,400	2.2	229	183	137	114	92	69	46
1,500	2.3	241	193	145	121	96	72	48
1,600	2.5	253	202	152	126	101	76	51
1,700	2.7	265	212	159	132	106	80	53
1,800	2.8	276	221	166	138	110	83	55
1,900	3.0	288	230	173	144	115	86	58
2,000	3.1	299	239	179	150	120	90	60

TABLE 17.—AREAS OF WATERWAY CALCULATED BY TALBOT'S FORMULA—(*Continued*)

a = area of waterway in square feet.

A = drainage area in acres.

$$a = C\sqrt[4]{A^3}.$$

Drainage area		Area of waterway in square feet						
		Mountainous land	Hilly land		Rolling land		Flat land	
Acres	Square miles	$C = 1.00$	$C = 0.80$	$C = 0.60$	$C = 0.50$	$C = 0.40$	$C = 0.30$	$C = 0.20$
2,500	3.91	354	283	212	177	142	106	71
3,000	4.69	405	324	243	203	162	122	81
3,500	5.47	455	364	273	228	182	137	91
4,000	6.25	503	402	302	252	201	151	101
5,000	7.81	595	476	357	298	238	179	119
6,000	9.37	682	546	409	341	273	205	136
7,000	10.94	765	612	459	383	306	230	153
8,000	12.50	846	677	508	423	338	254	169
9,000	14.06	924	739	554	462	370	277	185
10,000	15.62	1,000	800	600	500	400	300	200
20,000	31.2	1,682	1,346	1,009	841	673	505	336
30,000	46.9	2,280	1,824	1,368	1,140	912	684	456
40,000	62.5	2,828	2,262	1,697	1,414	1,131	848	566
50,000	78.1	3,344	2,675	2,006	1,672	1,338	1,003	669
60,000	93.7	3,834	3,067	2,300	1,917	1,534	1,150	767
70,000	109	4,304	3,443	2,582	2,152	1,722	1,291	861
80,000	125	4,757	3,806	2,854	2,378	1,903	1,427	951
90,000	141	5,196	4,157	3,118	2,598	2,078	1,559	1,039
100,000	156	5,623	4,498	3,374	2,811	2,249	1,687	1,125
110,000	172	6,040	4,832	3,624	3,020	2,416	1,812	1,208
120,000	187	6,447	5,158	3,868	3,224	2,579	1,934	1,289
130,000	203	6,846	5,477	4,108	3,423	2,738	2,054	1,369
140,000	219	7,238	5,790	4,343	3,619	2,895	2,171	1,448
150,000	234	7,622	6,098	4,573	3,811	3,049	2,287	1,524
160,000	250	8,000	6,400	4,800	4,000	3,200	2,400	1,600
170,000	266	8,372	6,698	5,024	4,187	3,349	2,512	1,675
180,000	281	8,739	6,991	5,243	4,370	3,496	2,622	1,748
190,000	297	9,100	7,280	5,460	4,550	3,640	2,730	1,820
200,000	312	9,457	7,566	5,674	4,729	3,783	2,837	1,891
210,000	328	9,810	7,848	5,886	4,905	3,924	2,943	1,962
220,000	344	10,158	8,126	6,095	5,079	4,063	3,047	2,032
230,000	359	10,503	8,402	6,302	5,252	4,201	3,151	2,101
240,000	375	10,843	8,674	6,506	5,422	4,337	3,253	2,169
250,000	391	11,180	8,944	6,708	5,590	4,472	3,354	2,236
260,000	406	11,514	9,211	6,908	5,757	4,606	3,454	2,303
270,000	422	11,845	9,476	7,107	5,923	4,738	3,554	2,369
280,000	437	12,172	9,738	7,303	6,086	4,869	3,652	2,434
290,000	453	12,497	9,998	7,498	6,249	4,999	3,749	2,499
300,000	469	12,819	10,255	7,691	6,410	5,128	3,846	2,564
310,000	484	13,138	10,510	7,883	6,569	5,255	3,941	2,628
320,000	500	13,454	10,763	8,072	6,727	5,382	4,036	2,691
330,000	516	13,768	11,014	8,261	6,884	5,507	4,130	2,754
340,000	531	14,080	11,264	8,448	7,040	5,632	4,224	2,816
350,000	547	14,390	11,512	8,634	7,195	5,756	4,317	2,878
360,000	562	14,697	11,758	8,818	7,349	5,879	4,409	2,939
370,000	578	15,002	12,002	9,001	7,501	6,001	4,501	3,000
380,000	594	15,305	12,244	9,183	7,653	6,122	4,592	3,061
390,000	609	15,606	12,485	9,364	7,803	6,242	4,682	3,121
400,000	625	15,905	12,724	9,543	7,953	6,362	4,772	3,181
500,000	781	18,804	15,043	11,282	9,402	7,522	5,641	3,761

TABLE 18.—VOLUME OF 100-FOOT EARTHWORK SECTIONS (CUBIC YARDS).

A = Sum of end areas in square feet.

A	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	A
0	0.0	0.2	0.4	0.6	0.7	0.9	1.1	1.3	1.5	1 7	0
1	1.9	2.0	2.2	2.4	2.6	2.8	3.0	3.1	3.3	3.6	1
2	3.7	3.9	4.1	4.3	4.4	4.6	4.8	5.0	5.2	5.4	2
3	5.6	5.7	5.9	6.1	6.3	6.5	6.7	6.9	7.0	7.2	3
4	7.4	7.6	7.8	8.0	8.1	8.3	8.5	8.7	8.9	9.1	4
5	9.3	9.4	9.6	9.8	10.0	10.2	10.4	10.6	10.7	10.9	5
6	11.1	11.3	11.5	11.7	11.9	12.0	12.2	12.4	12.5	12.8	6
7	13 0	13.1	13.3	13.5	13.7	13.9	14.1	14.3	14.4	14.6	7
8	14.8	15.0	15.2	15.4	15.6	15.7	15.9	16.1	16.3	16.5	8
9	16.7	16.9	17.0	17.2	17.4	17.6	17.8	18.0	18.1	18.3	9
10	18.5	18.7	18.9	19.1	19.3	19.4	19.6	19.8	20.0	20.2	10
11	20.4	20.6	20.7	20.9	21.1	21.3	21.5	21.7	21.9	22.0	11
12	22.2	22.4	22.6	22.8	23.0	23.1	23.3	23.5	23.7	23.9	12
13	24.1	24.3	24.4	24.6	24.8	25.0	25.2	25.4	25.6	25.7	13
14	25.9	26.1	26.3	26.5	26.7	26.8	27.0	27.2	27.4	27.6	14
15	27.8	28.0	28.1	28.3	28.5	28.7	28.9	29.1	29.3	29.4	15
16	29.6	29.8	30.0	30.2	30.4	30.6	30.7	30.9	31.1	31.3	16
17	31.5	31.7	31.9	32.0	32.2	32.4	32.6	32.8	33.0	33.1	17
18	33.3	33.5	33.7	33.9	34.1	34.3	34.4	34.6	34.8	35.0	18
19	35.2	35.4	35.6	35.7	35.9	36.1	36.3	36.5	36.7	36.9	19
20	37.0	37.2	37.4	37.6	37.8	38.0	38.1	38.3	38.5	38.7	20
21	38.9	39.1	39.3	39.4	39.6	39.8	40.0	40.2	40.4	40.6	21
22	40.7	40.9	41.1	41.3	41.5	41.7	41.9	42.0	42.2	42.4	22
23	42.6	42.8	43.0	43.1	43.3	43.5	43.7	43.9	44.1	44.3	23
24	44.4	44.6	44.8	45.0	45.2	45.4	45.6	45.7	45.9	46.1	24
25	46.3	46.5	46.7	46.9	47.0	47.2	47.4	47.6	47.8	48.0	25
26	48.1	48.3	48.5	48.7	48.9	49.1	49.3	49.4	49.6	49.8	26
27	50.0	50.2	50.4	50.6	50.7	50.9	51.1	51.3	51.5	51.7	27
28	51.9	52.0	52.2	52.4	52.6	52.8	53.0	53.1	53.3	53.5	28
29	53.7	53.9	54.1	54.3	54.4	54.6	54.8	55.0	55.2	55.4	29
30	55.6	55.7	55.9	56.1	56.3	56.5	56.7	56.9	57.0	57.2	30
31	57.4	57.6	57.8	58.0	58.1	58.3	58.5	58.7	58.9	59.1	31
32	59.3	59.4	59.6	59.8	60.0	60.2	60.4	60.6	60.7	60.9	32
33	61.1	61.3	61.5	61.7	61.9	62.0	62.2	62.4	62.6	62.8	33
34	63.0	63.1	63.3	63.5	63.7	63.9	64.1	64.3	64.4	64.6	34
35	64.8	65.0	65.2	65.4	65.6	65.7	65.9	66.1	66.3	66.5	35
36	66.7	66.9	67.0	67.2	67.4	67.6	67.8	68.0	68.1	68.3	36
37	68.5	68.7	68.9	69.1	69.3	69.4	69.6	69.8	70.0	70.2	37
38	70.4	70.6	70.7	70.9	71.1	71.3	71.5	71.7	71.9	72.0	38
39	72.2	72.4	72.6	72.8	73.0	73.1	73.3	73.5	73.7	73.9	39
40	74.1	74.3	74.4	74.6	74.8	75.0	75.2	75.4	75.6	75.7	40
41	75.9	76.1	76.3	76.5	76.7	79.6	77.0	77.2	77.4	77.6	41
42	77.8	78.0	78.1	78.3	78.5	78.7	78.9	79.1	79.3	79.4	42
43	79.6	79.8	80.0	80.2	80.4	80.6	80.7	80.9	81.1	81.3	43
44	81.5	81.7	81.9	82.0	82.2	82.4	82.6	82.8	83.0	83.1	44
45	83.3	83.5	83.7	83.9	84.1	84.3	84.4	84.6	84.8	85.0	45
46	85.2	85.4	85.6	85.7	85.9	86.1	86.3	86.5	86.7	86.9	46
47	87.0	87.2	87.4	87.6	87.8	88.0	88.1	88.3	88.5	88.7	47
48	88.9	89.1	89.3	89.4	89.6	89.8	90.0	90.2	90.4	90.6	48
49	90.7	90.9	91.1	91.3	91.5	91.7	91.9	92.0	92.2	92.4	49
50	92.6	92.8	93.0	93.1	93.3	93.5	93.7	93.9	94.1	94.3	50

TABLE 18.—VOLUME OF 100-FOOT EARTHWORK SECTIONS (CUBIC YARDS)—(*Continued*).

A = Sum of end areas in square feet.

A	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	A
51	94.4	94.6	94.8	95.0	95.2	95.4	95.6	95.7	95.9	96.1	51
52	96.3	96.5	96.7	96.9	97.0	97.2	97.4	97.6	97.8	98.0	52
35	98.1	98.3	98.5	98.7	98.9	99.1	99.3	99.4	99.6	99.8	53
54	100.0	100.2	100.4	100.6	100.7	100.9	101.1	101.1	101.5	101.7	54
55	101.9	102.0	102.2	102.4	102.6	102.8	103.0	103.1	103.3	103.5	55
56	103.7	103.9	104.1	104.3	104.4	104.6	104.8	105.0	105.2	105.2	56
57	105.6	105.7	105.9	106.1	106.3	106.5	106.7	106.9	107.0	107.2	57
58	107.4	107.6	107.8	108.0	108.1	108.3	108.5	108.7	108.9	109.1	58
59	109.3	109.4	109.6	109.8	110.0	110.2	110.4	110.6	110.7	110.9	59
60	111.1	111.3	111.5	111.7	111.9	112.0	112.2	112.4	112.6	112.8	60
61	113.0	113.1	113.3	113.5	113.7	113.9	114.1	114.3	114.4	114.6	61
62	114.8	115.0	115.2	115.4	115.6	115.7	115.9	116.1	116.3	116 5	62
63	116.7	116.9	117.0	117.2	117.4	117.6	117.8	118.0	118.1	118 3	63
64	118.5	118.7	118.9	119.1	119.3	119.4	119.6	119.8	120 0	120.2	64
65	120.4	120.6	120.7	120.9	121.1	121.3	121.5	121.7	121.9	122.0	65
66	122.2	122.2	122.6	122.8	123.0	123.1	123.3	123.5	123.7	123.9	66
67	124.1	124.3	124.4	124.6	124.8	125.0	125.2	125.4	125.6	125.7	67
68	125.9	126.1	126.3	126.5	126.7	126.9	127.0	127.2	127.4	127.6	68
69	127.8	128.0	128.1	128.3	128.5	128.7	128.9	129.1	129.3	129.4	69
70	129.6	129.8	130.0	130.2	130.4	130.6	130.7	130.9	131.1	131.3	70
71	131.5	131.7	131.9	132.0	132.2	132.4	132.6	132.8	133.0	133.1	71
72	133.3	133.5	133.7	133.9	134.1	134.3	134.4	134.6	134.8	135.0	72
73	135.2	135.4	135.6	135.7	135.9	136.1	136.3	136.5	136.7	136.9	73
74	137.0	137.2	137.4	137.6	137.8	138.0	138.1	138.3	138.5	138.7	74
75	138.9	139.1	139.3	139.4	139.6	139.8	140.0	140.2	140.4	140.6	75
76	140.7	140.9	141.1	141.3	141 5	141.7	141.9	142.0	142.2	142.4	76
77	142.6	142.8	143.0	143.1	143.3	143.5	143.7	143.9	144.1	144.3	77
78	144.4	144.6	144.8	145.0	145.2	145.4	145.6	145.7	145.9	146.1	78
79	146.3	146.5	146.7	146.9	147.0	147.2	147.4	147.6	147.8	148.0	79
80	148.1	148.3	148.5	148.7	149.9	149.1	149.3	149.4	149.6	149.8	80
81	150.0	150.2	150.4	150.6	150.7	150.9	151.1	151.3	151.5	151.7	81
82	151.9	152.0	152.2	152.4	152.6	152.8	153.0	153.1	153.3	153.5	82
83	153 7	153.9	154.0	154.3	154.4	154.6	154.8	155.0	155.2	155.4	83
84	155.6	155.7	155.9	156.1	156.3	156.5	156.7	156.9	157.0	157.2	84
85	157.4	157.6	157.8	158.0	158.1	158.3	158.5	158.7	158.9	159.1	85
86	159.3	159.4	159.6	159.8	160.0	160.2	160.4	160.6	160.7	160.9	86
87	161.1	161.3	161.5	161.7	161.9	162.0	162.2	162.4	162.6	162.8	87
88	163.0	163.1	163.3	163.5	163.7	163.9	164.1	164.3	164.4	164.6	88
89	164.8	165.0	165.2	165.4	165.6	165.7	165.9	166.1	166.3	166.5	89
90	166.7	166.9	167.0	167.2	167.4	167.6	167.8	168.0	168.1	168.3	90
91	168.5	168.7	168.9	169.1	169.3	169.4	169.6	169.8	170.0	170.1	91
92	170.4	170.6	170.7	170.9	171.1	171.3	171.5	171.7	171.9	172.0	92
93	172.2	172.4	172.6	172.8	173.0	173.1	173.3	173.5	173.7	173.9	93
94	174.1	174.3	174.4	174.6	174.8	175.0	175.2	175.4	175.6	175.7	94
95	175.9	176.1	176.3	176.5	176.7	176.9	177.0	177.2	177.4	177.6	95
96	177.8	178.0	178.1	178.3	178.5	178.7	178.9	179.1	179.3	179.4	96
97	179.6	179.8	180.0	180.2	180.4	180.6	180.7	180.9	181.1	181.3	97
98	181.5	181.7	181.9	182.0	182.2	182.4	182.6	182.8	183.0	183.1	98
99	183.3	183.5	183.7	183.9	184.1	184.3	184.4	184.6	184.8	185.0	99
100	185.2	185.4	185.6	185.7	185.9	186.1	186.3	186.5	186.7	186.9	100

TABLE 18.—VOLUME OF 100-FOOT EARTHWORK SECTIONS (CUBIC YARDS)—(*Continued*).

A = Sum of end areas in square feet.

A	V	A	V	A	V	A	V	A	V
101	187.0	152	281.5	203	376.0	254	470.4	305	564.8
102	188.9	153	283.3	204	377.8	255	472.2	306	566.7
103	190.7	154	285.2	205	379.6	256	474.1	307	568.5
104	192.6	155	287.0	206	381.5	257	475.9	308	570.4
105	194.4	156	288.9	207	383.3	258	477.8	309	572.2
106	196.3	157	290.7	208	385.2	259	479.6	310	574.1
107	198.1	158	292.6	209	387.0	260	481.5	311	575.9
108	200.0	159	294.4	210	388.9	261	483.3	312	577.8
109	201.9	160	296.3	211	390.7	262	485.2	313	579.6
110	203.7	161	298.1	212	392.6	263	487.0	314	581.5
111	205.6	162	300.0	213	394.4	264	488.9	315	583.3
112	207.4	163	301.9	214	396.3	265	490.7	316	585.2
113	209.3	164	303.7	215	398.1	266	492.6	317	587.0
114	211.1	165	305.6	216	400.0	267	494.4	318	588.9
115	213.0	166	307.4	217	401.9	268	496.3	319	590.7
116	214.8	167	309.3	218	403.7	269	498.1	320	592.6
117	216.7	168	311.1	219	405.6	270	500.0	321	594.4
118	218.6	169	313.0	220	407.4	271	501.9	322	596.3
119	220.4	170	314.8	221	409.3	272	503.7	323	598.1
120	222.2	171	316.7	222	411.1	273	505.6	324	600.0
121	224.1	172	318.5	223	413.0	274	507.4	325	601.9
122	226.0	173	320.4	224	414.8	275	509.3	326	603.7
123	227.8	174	322.2	225	416.7	276	511.1	327	605.6
124	229.6	175	324.1	226	418.5	277	513.0	328	607.4
125	231.5	176	326.0	227	420.4	278	514.8	329	609.3
126	233.3	177	327.8	228	422.2	279	516.7	330	611.1
127	235.2	178	329.6	229	424.1	280	518.5	331	613.0
128	237.0	179	331.5	230	425.9	281	520.4	332	614.8
129	238.9	180	333.3	231	427.8	282	522.2	333	616.7
130	240.7	181	332.2	232	429.6	283	524.1	334	618.5
131	242.6	182	337.0	233	431.5	284	525.9	335	620.4
132	244.4	183	338.9	234	433.3	285	527.8	336	622.2
133	246.3	184	340.7	235	435.2	286	529.6	337	624.1
134	248.1	185	342.6	236	437.0	287	531.5	338	625.9
135	250.0	186	344.4	237	438.9	288	533.3	339	627.8
136	251.9	187	346.3	238	440.7	289	535.2	340	629.6
137	253.7	188	348.1	239	442.6	290	537.0	341	631.5
138	255.6	189	350.0	240	444.4	291	538.9	342	633.3
139	257.4	190	351.9	241	446.3	292	540.7	343	635.2
140	259.3	191	353.7	242	448.1	293	542.6	344	637.0
141	261.1	192	355.6	243	450.0	294	544.4	345	638.9
142	263.0	193	357.4	244	451.9	295	546.3	346	640.7
143	264.8	194	359.3	245	453.7	296	548.1	347	642.6
144	266.7	195	361.1	246	455.6	297	550.0	348	644.4
145	268.5	196	363.0	247	457.4	298	551.9	349	646.3
146	270.4	197	364.8	248	459.3	299	553.7	350	648.1
147	272.2	198	366.7	249	461.1	300	555.6	351	650.0
148	274.1	199	368.5	250	463.0	301	557.4	352	651.9
149	275.9	200	370.4	251	464.8	302	559.3	353	653.7
150	277.8	201	372.2	252	466.7	303	561.1	354	655.6
151	279.6	202	374.1	253	468.5	304	563.0	355	657.4

Table 18.—Volume of 100-foot Earthwork Sections (Cubic Yards)—(*Continued*).

A = Sum of end areas in square feet.

A	V	A	V	A	V	A	V	A	V
356	659.3	407	753.7	458	848.1	509	942.6	560	1,037.0
357	661.1	408	755.6	459	850.0	510	944.4	561	1,038.9
358	663.0	409	757.4	460	851.9	511	946.3	562	1,040.7
359	664.8	410	759.3	461	853.7	512	948.3	563	1,042.6
360	666.7	411	761.2	462	855.6	513	950.0	564	1,044.4
361	668.5	412	763.0	463	857.4	514	951.9	565	1,046.3
362	670.4	413	764.8	464	859.3	515	953.7	566	1,048.1
363	672.2	414	766.7	465	861.1	516	955.6	567	1,050.0
364	674.1	415	768.5	466	863.0	517	957.4	568	1,051.9
365	675.9	416	770.4	467	864.8	518	959.3	569	1,053.7
366	677.8	417	772.2	468	866.7	519	961.1	570	1,055.6
367	679.6	418	774.1	469	868.5	520	963.0	571	1,057.4
368	681.5	419	775.9	470	870.4	521	964.8	572	1,059.3
369	683.3	420	777.8	471	872.2	522	966.7	573	1,061.1
370	685.2	421	779.6	472	874.1	523	968.6	574	1,063.0
371	687.0	422	781.5	473	875.9	524	970.4	575	1,064.8
372	688.9	423	783.3	474	877.8	525	972.2	576	1,066.7
373	690.7	424	785.2	475	879.6	526	974.1	577	1,068.5
374	692.6	425	787.0	476	881.5	527	975.9	578	1,070.4
375	694.4	426	788.9	477	883.3	528	977.8	579	1,072.2
376	696.3	427	790.7	478	885.2	529	979.6	580	1,074.1
377	698.1	428	792.6	479	887.0	530	981.5	581	1,075.9
378	700.0	429	794.4	480	888.9	531	983.3	582	1,077.8
379	701.9	430	796.3	481	890.7	532	985.2	583	1,079.6
380	703.7	431	798.1	482	892.6	533	987.0	584	1,081.5
381	705.6	432	800.0	483	894.4	534	988.9	585	1,083.3
382	707.4	433	801.9	484	896.3	535	990.7	586	1,085.2
383	709.3	434	803.7	485	898.1	536	992.6	587	1,087.0
384	711.1	435	805.6	486	900.0	537	994.4	588	1,088.9
385	713.0	436	807.4	487	901.9	538	996.3	589	1,090.7
386	714.8	437	809.3	488	903.7	539	998.1	590	1,092.6
387	716.7	438	811.1	489	905.6	540	1,000.0	591	1,094.6
388	718.5	439	813.0	490	907.4	541	1,001.9	592	1,096.3
389	720.4	440	814.8	491	909.3	542	1,003.7	593	1,098.1
390	722.2	441	816.7	492	911.1	543	1,005.6	594	1,100.0
391	724.1	442	818.5	493	913.0	544	1,007.4	595	1,101.9
392	725.9	443	820.4	494	914.8	545	1,009.3	596	1,103.7
393	727.8	444	822.2	495	916.7	546	1,011.1	597	1,105.6
394	729.6	445	824.1	496	918.5	547	1,013.0	598	1,107.4
395	731.5	446	825.9	497	920.4	548	1,014.8	599	1,109.3
396	733.3	447	827.8	498	922.2	549	1,016.7	600	1,111.1
397	735.2	448	829.6	499	924.1	550	1,018.5	601	1,113.0
398	737.0	449	831.5	500	925.9	551	1,020.4	602	1,114.8
399	738.9	450	833.4	501	927.8	552	1,022.2	603	1,116.7
400	740.7	451	835.2	502	929.6	553	1,024.1	604	1,118.5
401	742.6	452	837.0	503	931.5	554	1,025.9	605	1,120.3
402	744.4	453	838.9	504	933.3	555	1,027.8	606	1,122.2
403	746.3	454	840.7	505	935.2	556	1,029.6	607	1,124.1
404	748.1	455	842.6	506	937.0	557	1,031.5	608	1,125.9
405	750.0	456	844.4	507	938.9	558	1,033.3	609	1,127.8
406	751.9	457	846.3	508	940.7	559	1,035.2	610	1,129.6

TABLE 19.—PRISMOIDAL CORRECTION (CUBIC YARDS) FOR 100-FOOT SECTIONS

$D - D'$	$X - X'$								
	1	2	3	4	5	6	7	8	9
0.2	0.06	0.12	0.19	0.25	0.31	0.37	0.43	0.49	0.56
0.4	0.12	0.25	0.37	0.49	0.62	0.74	0.86	0.99	1.11
0.6	0.19	0.37	0.56	0.74	0.93	1.11	1.30	1.48	1.67
0.8	0.25	0.49	0.74	0.99	1.23	1.48	1.73	1.98	2.22
1.0	0.31	0.62	0.93	1.23	1.54	1.85	2.16	2.47	2.78
1.2	0.37	0.74	1.11	1.48	1.85	2.22	2.59	2.96	3.33
1.4	0.43	0.86	1.30	1.73	2.16	2.59	3.02	3.46	3.89
1.6	0.49	0.99	1.48	1.98	2.47	2.96	3.46	3.95	4.44
1.8	0.56	1.11	1.67	2.22	2.78	3.33	3.89	4.44	5.00
2.0	0.62	1.23	1.85	2.47	3.09	3.70	4.32	4.94	5.56
2.2	0.68	1.36	2.04	2.72	3.40	4.07	4.75	5.43	6.11
2.4	0.74	1.48	2.22	2.96	3.70	4.44	5.19	5.93	6.67
2.6	0.80	1.60	2.41	3.21	4.01	4.81	5.62	6.42	7.22
2.8	0.86	1.73	2.59	3.46	4.32	5.19	6.05	6.91	7.78
3.0	0.93	1.85	2.78	3.70	4.63	5.56	6.48	7.41	8.33
3.2	0.99	1.98	2.96	3.95	4.94	5.93	6.91	7.90	8.89
3.4	1.05	2.10	3.15	4.20	5.25	6.30	7.35	8.40	9.44
3.6	1.11	2.22	3.33	4.44	5.56	6.67	7.78	8.89	10.00
3.8	1.17	2.35	3.52	4.69	5.86	7.04	8.21	9.38	10.56
4.0	1.23	2.47	3.70	4.94	6.17	7.41	8.64	9.88	11.11
4.2	1.30	2.59	3.89	5.19	6.48	7.78	9.07	10.37	11.67
4.4	1.36	2.72	4.07	5.43	6.79	8.15	9.51	10.86	12.22
4.6	1.42	2.84	4.26	5.68	7.10	8.52	9.94	11.36	12.78
4.8	1.48	2.96	4.44	5.93	7.41	8.89	10.37	11.85	13.33
5.0	1.54	3.09	4.63	6.17	7.72	9.26	10.80	12.35	13.89
5.2	1.60	3.21	4.81	6.42	8.02	9.63	11.23	12.84	14.44
5.4	1.67	3.33	5.00	6.67	8.33	10.00	11.67	13.33	15.00
5.6	1.73	3.46	5.19	6.91	8.64	10.37	12.10	13.83	15.56
5.8	1.79	3.58	5.37	7.16	8.95	10.74	12.53	14.32	16.11
6.0	1.85	3.70	5.56	7.41	9.26	11.11	12.96	14.81	16.67
6.2	1.91	3.83	5.74	7.65	9.57	11.48	13.40	15.31	17.22
6.4	1.98	3.95	5.93	7.90	9.88	11.85	13.83	15.80	17.78
6.6	2.04	4.07	6.11	8.15	10.19	12.22	14.26	16.30	18.33
6.8	2.10	4.20	6.30	8.40	10.49	12.59	14.69	16.79	18.89
7.0	2.16	4.32	6.48	8.64	10.80	12.96	15.12	17.28	19.44
7.2	2.22	4.44	6.67	8.89	11.11	13.33	15.56	17.78	20.00
7.4	2.28	4.57	6.85	9.14	11.42	13.70	15.99	18.27	20.56
7.6	2.35	4.69	7.04	9.38	11.73	14.07	16.42	18.77	21.11
7.8	2.41	4.81	7.22	9.63	12.04	14.44	16.85	19.26	21.67
8.0	2.47	4.94	7.41	9.88	12.35	14.81	17.28	19.75	22.22
8.2	2.53	5.06	7.59	10.12	12.65	15.19	17.72	20.25	22.78
8.4	2.59	5.19	7.78	10.37	12.96	15.56	18.15	20.74	23.33
8.6	2.65	5.31	7.96	10.62	13.27	15.93	18.58	21.23	23.89
8.8	2.72	5.43	8.15	10.86	13.58	16.30	19.01	21.73	24.44
9.0	2.78	5.56	8.33	11.11	13.89	16.67	19.44	22.22	25.00
9.2	2.84	5.68	8.52	11.36	14.20	17.04	19.88	22.72	25.56
9.4	2.90	5.80	8.70	11.60	14.51	17.41	20.31	23.21	26.11
9.6	2.96	5.93	8.89	11.85	14.81	17.78	20.74	23.70	26.67
9.8	3.02	6.05	9.07	12.10	15.12	18.15	21.17	24.20	27.22
10.0	3.09	6.17	9.26	12.35	15.43	18.52	21.60	24.69	27.78

Table 20.—Natural Sines and Cosines

′	0° Sine	0° Cosine	′	′	0° Sine	0° Cosine	′	′	0° Sine	0° Cosine	′
0	.00000	1	60	21	.00611	.99998	39	41	.01193	.99993	19
1	.00029	1	59	22	.00640	.99998	38	42	.01222	.99993	18
2	.00058	1	58	23	.00669	.99998	37	43	.01251	.99992	17
3	.00087	1	57	24	.00698	.99998	36	44	.01280	.99992	16
4	.00116	1	56	25	.00727	.99997	35	45	.01309	.99991	15
5	.00145	1	55	26	.00756	.99997	34	46	.01338	.99991	14
6	.00175	1	54	27	.00785	.99997	33	47	.01367	.99991	13
7	.00204	1	53	28	.00814	.99997	32	48	.01396	.99990	12
8	.00233	1	52	29	.00844	.99996	31	49	.01425	.99990	11
9	.00262	1	51	30	.00873	.99996	30	50	.01454	.99989	10
10	.00291	1	50	31	.00902	.99996	29	51	.01483	.99989	9
11	.00320	.99999	49	32	.00931	.99996	28	52	.01513	.99989	8
12	.00349	.99999	48	33	.00960	.99995	27	53	.01542	.99988	7
13	.00378	.99999	47	34	.00989	.99995	26	54	.01571	.99988	6
14	.00407	.99999	46	35	.01018	.99995	25	55	.01600	.99987	5
15	.00436	.99999	45	36	.01047	.99995	24	56	.01629	.99987	4
16	.00465	.99999	44	37	.01076	.99994	23	57	.01658	.99986	3
17	.00495	.99999	43	38	.01105	.99994	22	58	.01687	.99986	2
18	.00524	.99999	42	39	.01134	.99994	21	59	.01716	.99985	1
19	.00553	.99998	41	40	.01164	.99993	20	60	.01745	.99985	0
20	.00582	.99998	40								
′	Cosine 89°	Sine 89°	′	′	Cosine 89°	Sine 89°	′	′	Cosine 89°	Sine 89°	′

From Davis "Manual of Surveying."

TABLE 20.—NATURAL SINES AND COSINES—(*Continued*)

′	1° Sine	1° Cosine	2° Sine	2° Cosine	3° Sine	3° Cosine	4° Sine	4° Cosine	′
0	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosine 88°	Sine 88°	Cosine 87°	Sine 87°	Cosine 86°	Sine 86°	Cosine 85°	Sine 85°	′

TABLE 20.—NATURAL SINES AND COSINES—(*Continued*)

′	5° Sine	5° Cosine	6° Sine	6° Cosine	7° Sine	7° Cosine	8° Sine	8° Cosine	′
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	0
′	84° Cosine	84° Sine	83° Cosine	83° Sine	82° Cosine	82° Sine	81° Cosine	81° Sine	′

TABLE 20.—NATURAL SINES AND COSINES—(*Continued*)

′	9° Sine	9° Cosine	10° Sine	10° Cosine	11° Sine	11° Cosine	12° Sine	12° Cosine	′
0	.15643	.98769	.17365	.98481	.19081	.98163	.20791	.97815	60
1	.15672	.98764	.17393	.98476	.19109	.98157	.20820	.97809	59
2	.15701	.98760	.17422	.98471	.19138	.98152	.20848	.97803	58
3	.15730	.98755	.17451	.98466	.19167	.98146	.20877	.97797	57
4	.15758	.98751	.17479	.98461	.19195	.98140	.20905	.97791	56
5	.15787	.98746	.17508	.98455	.19224	.98135	.20933	.97784	55
6	.15816	.98741	.17537	.98450	.19252	.98129	.20962	.97778	54
7	.15845	.98737	.17565	.98445	.19281	.98124	.20990	.97772	53
8	.15873	.98732	.17594	.98440	.19309	.98118	.21019	.97766	52
9	.15902	.98728	.17623	.98435	.19338	.98112	.21047	.97760	51
10	.15931	.98723	.17651	.98430	.19366	.98107	.21076	.97754	50
11	.15959	.98718	.17680	.98425	.19395	.98101	.21104	.97748	49
12	.15988	.98714	.17708	.98420	.19423	.98096	.21132	.97742	48
13	.16017	.98709	.17737	.98414	.19452	.98090	.21161	.97735	47
14	.16046	.98704	.17766	.98409	.19481	.98084	.21189	.97729	46
15	.16074	.98700	.17794	.98404	.19509	.98079	.21218	.97723	45
16	.16103	.98695	.17823	.98399	.19538	.98073	.21246	.97717	44
17	.16132	.98690	.17852	.98394	.19566	.98067	.21275	.97711	43
18	.16160	.98689	.17880	.98389	.19595	.98061	.21303	.97705	42
19	.16189	.98681	.17909	.98383	.19623	.98056	.21331	.97698	41
20	.16218	.98676	.17937	.98378	.19652	.98050	.21360	.97692	40
21	.16246	.98671	.17966	.98373	.19680	.98044	.21388	.97686	39
22	.16275	.98667	.17995	.98368	.19709	.98039	.21417	.97680	38
23	.16304	.98662	.18023	.98362	.19737	.98033	.21445	.97673	37
24	.16333	.98657	.18052	.98357	.19766	.98027	.21474	.97667	36
25	.16361	.98652	.18081	.98352	.19794	.98021	.21502	.97661	35
26	.16390	.98648	.18109	.98347	.19823	.98016	.21530	.97655	34
27	.16419	.98643	.18138	.98341	.19851	.98010	.21559	.97648	33
28	.16447	.98638	.18166	.98336	.19880	.98004	.21587	.97642	32
29	.16476	.98633	.18195	.98331	.19908	.97987	.21616	.97636	31
30	.16505	.98629	.18224	.98325	.19937	.97992	.21644	.97630	30
31	.16533	.98624	.18252	.98320	.19965	.97987	.21672	.97623	29
32	.16562	.98619	.18281	.98315	.19994	.97981	.21701	.97617	28
33	.16591	.98614	.18309	.98310	.20022	.97975	.21729	.97611	27
34	.16620	.98609	.18338	.98304	.20031	.97969	.21758	.97604	26
35	.16648	.98604	.18367	.98299	.20079	.97963	.21786	.97598	25
36	.16677	.98600	.18395	.98294	.20108	.97958	.21814	.97592	24
37	.16706	.98595	.18424	.98288	.20136	.97952	.21843	.97585	23
38	.16734	.98590	.18452	.98283	.20165	.97946	.21871	.97579	22
39	.16763	.98585	.18481	.98277	.20193	.97940	.21899	.97573	21
40	.16792	.98580	.18509	.98272	.20222	.97934	.21928	.97566	20
41	.16820	.98575	.18538	.98267	.20250	.97928	.21956	.97560	19
42	.16849	.98570	.18567	.98261	.20279	.97922	.21985	.97553	18
43	.16878	.98565	.18595	.98256	.20307	.97916	.22013	.97547	17
44	.16906	.98561	.18624	.98250	.20336	.97910	.22041	.97541	16
45	.16935	.98556	.18652	.98245	.20364	.97905	.22070	.97534	15
46	.16964	.98551	.18681	.98240	.20393	.97899	.22098	.97528	14
47	.16992	.98546	.18710	.98234	.20421	.97893	.22126	.97521	13
48	.17021	.98541	.18738	.98229	.20450	.97887	.22155	.97515	12
49	.17050	.98536	.18767	.98223	.20478	.97881	.22183	.97508	11
50	.17078	.98531	.18795	.98218	.20507	.97875	.22212	.97502	10
51	.17107	.98526	.18824	.98212	.20535	.97869	.22240	.97496	9
52	.17136	.98521	.18852	.98207	.20563	.97863	.22268	.97489	8
53	.17164	.98516	.18881	.98201	.20592	.97857	.22297	.97483	7
54	.17193	.98511	.18910	.98196	.20620	.97851	.22325	.97476	6
55	.17222	.98506	.18938	.98190	.20649	.97845	.22353	.97470	5
56	.17250	.98501	.18967	.98185	.20677	.97839	.22382	.97463	4
57	.17279	.98496	.18995	.98179	.20706	.97833	.22410	.97457	3
58	.17308	.98491	.19024	.98174	.20734	.97827	.22438	.97450	2
59	.17336	.98486	.19052	.98168	.20763	.97821	.22467	.97444	1
60	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	0
′	Cosine 80°	Sine 80°	Cosine 79°	Sine 79°	Cosine 78°	Sine 78°	Cosine 77°	Sine 77°	′

TABLE 20.—NATURAL SINES AND COSINES—(*Continued*)

′	13° Sine	13° Cosine	14° Sine	14° Cosine	15° Sine	15° Cosine	16° Sine	16° Cosine	′
0	.22495	.97437	.24192	.97030	.25882	.96593	.27564	.96126	60
1	.22523	.97430	.24220	.97023	.25910	.96585	.27592	.96118	59
2	.22552	.97424	.24249	.97015	.25938	.96578	.27620	.96110	58
3	.22580	.97417	.24277	.97008	.25966	.96570	.27648	.96102	57
4	.22608	.97411	.24305	.97001	.25994	.96562	.27676	.96094	56
5	.22637	.97404	.24333	.96994	.26022	.96555	.27704	.96086	55
6	.22665	.97398	.24362	.96987	.26050	.96547	.27731	.96078	54
7	.22693	.97391	.24390	.96980	.26079	.96540	.27759	.96070	53
8	.22722	.97384	.24418	.96973	.26107	.96532	.27787	.96062	52
9	.22750	.97378	.24446	.96966	.26135	.96524	.27815	.96054	51
10	.22778	.97371	.24474	.96959	.26163	.96517	.27843	.96046	50
11	.22807	.97365	.24503	.96952	.26191	.96509	.27871	.96037	49
12	.22835	.97358	.24531	.96945	.26219	.96502	.27899	.96029	48
13	.22863	.97351	.24559	.96937	.26247	.96494	.27927	.96021	47
14	.22892	.97345	.24587	.96930	.26275	.96486	.27955	.96013	46
15	.22920	.97338	.24615	.96923	.26303	.96479	.27983	.96005	45
16	.22948	.97331	.24644	.96916	.26331	.96471	.28011	.95997	44
17	.22977	.97325	.24672	.96909	.26359	.96463	.28039	.95989	43
18	.23005	.97318	.24700	.96902	.26387	.96456	.28067	.95981	42
19	.23033	.97311	.24728	.96894	.26415	.96448	.28095	.95972	41
20	.23062	.97304	.24756	.96887	.26443	.96440	.28123	.95964	40
21	.23090	.97298	.24784	.96880	.26471	.96433	.28150	.95956	39
22	.23118	.97291	.24813	.96873	.26500	.96425	.28178	.95948	38
23	23146	.97284	.24841	.96866	.26528	.96417	.28206	.95940	37
24	.23175	.97278	.24869	.96858	.26556	.96410	.28234	.95931	36
25	.23203	.97271	.24897	.96851	.26584	.96402	.28262	.95923	35
26	.23231	.97264	.24925	.96844	.26612	.96394	.28290	.95915	34
27	.23260	.97257	.24954	.96837	.26640	.96386	.28318	.95907	33
28	.23288	.97251	.24982	.96829	.26668	.96379	.28346	.95898	32
29	.23316	.97244	.25010	.96822	.26696	.96371	.28374	.95890	31
30	.23345	.97237	.25038	.96815	.26724	.96363	.28402	.95882	30
31	.23373	.97230	.25066	.96807	.26752	.96355	.28429	.95874	29
32	.23401	.97223	.25094	.96800	.26780	.96347	.28457	.95865	28
33	.23429	.97217	.25122	.96793	.26808	.96340	.28485	.95857	27
34	.23458	.97210	.25151	.96786	.26836	.96332	.28513	.95849	26
35	.23486	.97203	.25179	.96778	.26864	.96324	.28541	.95841	25
36	.23514	.97196	.25207	.96771	.26892	.96316	.28569	.95832	24
37	.23542	.97189	.25235	.96764	.26920	.96308	.28597	.95824	23
38	.23571	.97182	.25263	.96756	.26948	.96301	.28625	.95816	22
39	.23599	.97176	.25291	.96749	.26976	.96293	.28652	.95807	21
40	.23627	.97169	.25320	.96742	.27004	.96285	.28680	.95799	20
41	.23656	.97162	.25348	.96734	.27032	.96277	.28708	.95791	19
42	.23684	.97155	.25376	.96727	.27060	.96269	.28736	.95782	18
43	.23712	.97148	.25404	.96719	.27088	.96261	.28764	.95774	17
44	.23740	.97141	.25432	.96712	.27116	.96253	.28792	.95766	16
45	.23769	.97134	.25460	.96705	.27144	.96246	.28820	.95757	15
46	.23797	.97127	.25488	.96697	.27172	.96238	.28847	.95749	14
47	.23825	.97120	.25516	.96690	.27200	.96230	.28875	.95740	13
48	.23853	.97113	.25545	.96682	.27228	.96222	.28903	.95732	12
49	.23882	.97106	.25573	.96675	.27256	.96214	.28931	.95724	11
50	.23910	.97100	.25601	.96667	.27284	.96206	.28959	.95715	10
51	.23938	.97093	.25629	.96660	.27312	.96198	.28987	.95707	9
52	.23966	.97086	.25657	.96653	.27340	.96190	.29015	.95698	8
53	.23995	.97079	.25685	.96645	.27368	.96182	.29042	.95690	7
54	.24023	.97072	.25713	.96638	.27396	.96174	.29070	.95681	6
55	.24051	.97065	.25741	.96630	.27424	.96166	.29098	.95673	5
56	.24079	.97058	.25769	.96623	.27452	.96158	.29126	.95664	4
57	.24108	.97051	.25798	.96615	.27480	.96150	.29154	.95656	3
58	.24136	.97044	.25826	.96608	.27508	.96142	.29182	.95647	2
59	.24164	.97037	.25854	.96600	.27536	.96134	.29209	.95639	1
60	.24192	.97030	.25882	.96593	.27564	.96126	.29237	.95630	0
′	Cosine 76°	Sine 76°	Cosine 75°	Sine 75°	Cosine 74°	Sine 74°	Cosine 73°	Sine 73°	′

TABLE 20.—NATURAL SINES AND COSINES—(*Continued*)

′	17°		18°		19°		20°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	60
1	.29265	.95622	.30929	.95097	.32584	.94542	.34229	.93959	59
2	.29293	.95613	.30957	.95088	.32612	.94533	.34257	.93949	58
3	.29321	.95605	.30985	.95079	.32639	.94523	.34284	.93939	57
4	.29348	.95596	.31012	.95070	.32667	.94514	.34311	.93929	56
5	.29376	.95588	.31040	.95061	.32694	.94504	.34339	.93919	55
6	.29404	.95579	.31068	.95052	.32722	.94495	.34366	.93909	54
7	.29432	.95571	.31095	.95043	.32749	.94485	.34393	.93899	53
8	.29460	.95562	.31123	.95033	.32777	.94476	.34421	.93889	52
9	.29487	.95554	.31151	.95024	.32804	.94466	.34448	.93879	51
10	.29515	.95545	.31178	.95015	.32832	.94457	.34475	.93869	50
11	.29543	.95536	.31206	.95006	.32859	.94447	.34503	.93859	49
12	.29571	.95528	.31233	.94997	.32887	.94438	.34530	.93849	48
13	.29599	.95519	.31261	.94988	.32914	.94428	.34557	.93839	47
14	.29626	.95511	.31289	.94979	.32942	.94418	.34584	.93829	46
15	.29654	.95502	.31316	.94970	.32969	.94409	.34612	.93819	45
16	.29682	.95493	.31344	.94961	.32997	.94399	.34639	.93809	44
17	.29710	.95485	.31372	.94952	.33024	.94390	.34666	.93799	43
18	.29737	.95476	.31399	.94943	.33051	.94380	.34694	.93789	42
19	.29765	.95467	.31427	.94933	.33079	.94370	.34721	.93779	41
20	.29793	.95459	.31454	.94924	.33106	.94361	.34748	.93769	40
21	.29821	.95450	.31482	.94915	.33134	.94351	.34775	.93759	39
22	.29849	.95441	.31510	.94906	.33161	.94342	.34803	.93748	38
23	.29876	.95433	.31537	.94897	.33189	.94332	.34830	.93738	37
24	.29904	.95424	.31565	.94888	.33216	.94322	.34857	.93728	36
25	.29932	.95415	.31593	.94878	.33244	.94313	.34884	.93718	35
26	.29960	.95407	.31620	.94869	.33271	.94303	.34912	.93708	34
27	.29987	.95398	.31648	.94860	.33298	.94293	.34939	.93698	33
28	.30015	.95389	.31675	.94851	.33326	.94284	.34966	.93688	32
29	.30043	.95380	.31703	.94842	.33353	.94274	.34993	.93677	31
30	.30071	.95372	.31730	.94832	.33381	.94264	.35021	.93667	30
31	.30098	.95363	.31758	.94823	.33408	.94254	.35048	.93657	29
32	.30126	.95354	.31786	.94814	.33436	.94245	.35075	.93647	28
33	.30154	.95345	.31813	.94805	.33463	.94235	.35102	.93637	27
34	.30182	.95337	.31841	.94795	.33490	.94225	.35130	.93626	26
35	.30209	.95328	.31868	.94786	.33518	.94215	.35157	.93616	25
36	.30237	.95319	.31896	.94777	.33545	.94206	.35184	.93606	24
37	.30265	.95310	.31923	.94768	.33573	.94196	.35211	.93596	23
38	.30292	.95301	.31951	.94758	.33600	.94186	.35239	.93585	22
39	.30320	.95293	.31979	.94749	.33627	.94176	.35266	.93575	21
40	.30348	.95284	.32006	.94740	.33655	.94167	.35293	.93565	20
41	.30376	.95275	.32034	.94730	.33682	.94157	.35320	.93555	19
42	.30403	.95266	.32061	.94721	.33710	.94147	.35347	.93544	18
43	.30431	.95257	.32089	.94712	.33737	.94137	.35375	.93534	17
44	.30459	.95248	.32116	.94702	.33764	.94127	.35402	.93524	16
45	.30486	.95240	.32144	.94693	.33792	.94118	.35429	.93514	15
46	.30514	.95231	.32171	.94684	.33819	.94108	.35456	.93503	14
47	.30542	.95222	.32199	.94674	.33846	.94098	.35484	.93493	13
48	.30570	.95213	.32227	.94665	.33874	.94088	.35511	.93483	12
49	.30597	.95204	.32254	.94656	.33901	.94078	.35538	.93472	11
50	.30625	.95195	.32282	.94646	.33929	.94068	.35565	.93462	10
51	.30653	.95186	.32309	.94637	.33956	.94058	.35592	.93452	9
52	.30680	.95177	.32337	.94627	.33983	.94049	.35619	.93441	8
53	.30708	.95168	.32364	.94618	.34011	.94039	.35647	.93431	7
54	.30736	.95159	.32392	.94609	.34038	.94029	.35674	.93420	6
55	.30763	.95150	.32419	.94599	.34065	.94019	.35701	.93410	5
56	.30791	.95142	.32447	.94590	.34093	.94009	.35728	.93400	4
57	.30819	.95133	.32474	.94580	.34120	.93999	.35755	.93389	3
58	.30846	.95124	.32502	.94571	.34147	.93989	.35782	.93379	2
59	.30874	.95115	.32529	.94561	.34175	.93979	.35810	.93368	1
60	.30902	.95106	.32557	.94552	.34202	.93969	.35837	.93358	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	72°		71°		70°		69°		

TABLE 20.—NATURAL SINES AND COSINES—(*Continued*)

′	21° Sine	21° Cosine	22° Sine	22° Cosine	23° Sine	23° Cosine	24° Sine	24° Cosine	′
0	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine 68°	Sine 68°	Cosine 67°	Sine 67°	Cosine 66°	Sine 66°	Cosine 65°	Sine 65°	′

TABLE 20.—NATURAL SINES AND COSINES—(*Continued*)

′	25° Sine	25° Cosine	26° Sine	26° Cosine	27° Sine	27° Cosine	28° Sine	28° Cosine	′
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	9
52	.43628	.89981	45192	.89206	.46742	.88404	.48277	.87575	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	0
′	64° Cosine	64° Sine	63° Cosine	63° Sine	62° Cosine	62° Sine	61° Cosine	61° Sine	′

TABLE 20.—NATURAL SINES AND COSINES—(*Continued*)

′	29° Sine	29° Cosine	30° Sine	30° Cosine	31° Sine	31° Cosine	32° Sine	32° Cosine	′
0	.48481	.87462	.50000	.86603	.51504	.85717	.52992	.84805	60
1	.48506	.87448	.50025	.86588	.51529	.85702	.53017	.84789	59
2	.48532	.87434	.50050	.86573	.51554	.85687	.53041	.84774	58
3	.48557	.87420	.50076	.86559	.51579	.85672	.53066	.84759	57
4	.48583	.87406	.50101	.86544	.51604	.85657	.53091	.84743	56
5	.48608	.87391	.50126	.86530	.51628	.85642	.53115	.84728	55
6	.48634	.87377	.50151	.86515	.51653	.85627	.53140	.84712	54
7	.48659	.87363	.50176	.86501	.51678	.85612	.53164	.84697	53
8	.48684	.87349	.50201	.86486	.51703	.85597	.53189	.84681	52
9	.48710	.87335	.50227	.86471	.51728	.85582	.53214	.84666	51
10	.48735	.87321	.50252	.86457	.51753	.85567	.53238	.84650	50
11	.48761	.87306	.50277	.86442	.51778	.85551	.53263	.84635	49
12	.48786	.87292	.50302	.86427	.51803	.85536	.53288	.84619	48
13	.48811	.87278	.50327	.86413	.51828	.85521	.53312	.84604	47
14	.48837	.87264	.50352	.86398	.51852	.85506	.53337	.84588	46
15	.48862	.87250	.50377	.86384	.51877	.85491	.53361	.84573	45
16	.48888	.87235	.50403	.86369	.51902	.85476	.53386	.84557	44
17	.48913	.87221	.50428	.86354	.51927	.85461	.53411	.84542	43
18	.48938	.87207	.50453	.86340	.51952	.85446	.53435	.84526	42
19	.48964	.87193	.50478	.86325	.51977	.85431	.53460	.84511	41
20	.48989	.87178	.50503	.86310	.52002	.85416	.53484	.84495	40
21	.49014	.87164	.50528	.86295	.52026	.85401	.53509	.84480	39
22	.49040	.87150	.50553	.86281	.52051	.85385	.53534	.84464	38
23	.49065	.87136	.50578	.86266	.52076	.85370	.53558	.84448	37
24	.49090	.87121	.50603	.86251	.52101	.85355	.53583	.84433	36
25	.49116	.87107	.50628	.86237	.52126	.85340	.53607	.84417	35
26	.49141	.87093	.50654	.86222	.52151	.85325	.53632	.84402	34
27	.49166	.87079	.50679	.86207	.52175	.85310	.53656	.84386	33
28	.49192	.87064	.50704	.86192	.52200	.85294	.53681	.84370	32
29	.49217	.87050	.50729	.86178	.52225	.85279	.53705	.84355	31
30	.49242	.87036	.50754	.86163	.52250	.85264	.53730	.84339	30
31	.49268	.87021	.50779	.86148	.52275	.85249	.53754	.84324	29
32	.49293	.87007	.50804	.86133	.52299	.85234	.53779	.84308	28
33	.49318	.86993	.50829	.86119	.52324	.85218	.53804	.84292	27
34	.49344	.86978	.50854	.86104	.52349	.85203	.53828	.84277	26
35	.49369	.86964	.50879	.86089	.52374	.85188	.53853	.84261	25
36	.49394	.86949	.50904	.86074	.52399	.85173	.53877	.84245	24
37	.49419	.86935	.50929	.86059	.52423	.85157	.53902	.84230	23
38	.49445	.86921	.50954	.86045	.52448	.85142	.53926	.84214	22
39	.49470	.86906	.50979	.86030	.52473	.85127	.53951	.84198	21
40	.49495	.86892	.51004	.86015	.52498	.85112	.53975	.84182	20
41	.49521	.86878	.51029	.86000	.52522	.85096	.54000	.84167	19
42	.49546	.86863	.51054	.85985	.52547	.85081	.54024	.84151	18
43	.49571	.86849	.51079	.85970	.52572	.85066	.54049	.84135	17
44	.49596	.86834	.51104	.85956	.52597	.85051	.54073	.84120	16
45	.49622	.86820	.51129	.85941	.52621	.85035	.54097	.84104	15
46	.49647	.86805	.51154	.85926	.52646	.85020	.54122	.84088	14
47	.49672	.86791	.51179	.85911	.52671	.85005	.54146	.84072	13
48	.49697	.86777	.51204	.85896	.52696	.84989	.54171	.84057	12
49	.49723	.86762	.51229	.85881	.52720	.84974	.54195	.84041	11
50	.49748	.86748	.51254	.85866	.52745	.84959	.54220	.84025	10
51	.49773	.86733	.51279	.85851	.52770	.84943	.54244	.84009	9
52	.49798	.86719	.51304	.85836	.52794	.84928	.54269	.83994	8
53	.49824	.86704	.51329	.85821	.52819	.84913	.54293	.83978	7
54	.49849	.86690	.51354	.85806	.52844	.84897	.54317	.83962	6
55	.49874	.86675	.51379	.85792	.52869	.84882	.54342	.83946	5
56	.49899	.86661	.51404	.85777	.52893	.84866	.54366	.83930	4
57	.49924	.86646	.51429	.85762	.52918	.84851	.54391	.83915	3
58	.49950	.86632	.51454	.85747	.52943	.84836	.54415	.83899	2
59	.49975	.86617	.51479	.85732	.52967	.84820	.54440	.83883	1
60	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	0
′	60° Cosine	60° Sine	59° Cosine	59° Sine	58° Cosine	58° Sine	57° Cosine	57° Sine	′

TABLE 20.—NATURAL SINES AND COSINES—(*Continued*)

′	33° Sine	33° Cosine	34° Sine	34° Cosine	35° Sine	35° Cosine	36° Sine	36° Cosine	′
0	.54464	.83867	.55919	.82904	.57358	.81915	.58779	.80902	60
1	.54488	.83851	.55943	.82887	.57381	.81899	.58802	.80885	59
2	.54513	.83835	.55968	.82871	.57405	.81882	.58826	.80867	58
3	.54537	.83819	.55992	.82855	.57429	.81865	.58849	.80850	57
4	.54561	.83804	.56016	.82839	.57453	.81848	.58873	.80833	56
5	.54586	.83788	.56040	.82822	.57477	.81832	.58896	.80816	55
6	.54610	.83772	.56064	.82806	.57501	.81815	.58920	.80799	54
7	.54635	.83756	.56088	.82790	.57524	.81798	.58943	.80782	53
8	.54659	.83740	.56112	.82773	.57548	.81782	.58967	.80765	52
9	.54683	.83724	.56136	.82757	.57572	.81765	.58990	.80748	51
10	.54708	.83708	.56160	.82741	.57596	.81748	.59014	.80730	50
11	.54732	83692	.56184	.82724	.57619	.81731	.59037	.80713	49
12	.54756	.83676	.56208	.82708	.57643	.81714	.59061	.80696	48
13	.54781	.83660	.56232	.82692	.57667	.81698	.59084	.80679	47
14	.54805	.83645	.56256	.82675	.57691	.81681	.59108	.80662	46
15	.54829	.83629	.56280	.82659	.57715	.81664	.59131	.80644	45
16	.54854	.83613	.56305	.82643	.57738	.81647	.59154	.80627	44
17	.54878	.83597	.56329	.82626	.57762	.81631	.59178	.80610	43
18	.54902	.83581	.56353	.82610	.57786	.81614	.59201	.80593	42
19	.54927	.83565	.56377	.82593	.57810	.81597	.59225	.80576	41
20	.54951	.83549	.56401	.82577	.57833	.81580	.59248	.80558	40
21	.54975	.83533	.56425	.82561	.57857	.81563	.59272	.80541	39
22	.54999	.83517	.56449	.82544	.57881	.81546	.59295	.80524	38
23	.55024	.83501	.56473	.82528	.57904	.81530	.59318	.80507	37
24	.55048	.83485	.56497	.82511	.57928	.81513	.59342	.80489	36
25	.55072	.83469	.56521	.82495	.57952	.81496	.59365	.80472	35
26	.55097	.83453	.56545	.82478	.57976	.81479	.59389	.80455	34
27	.55121	.83437	.56569	.82462	.57999	.81462	.59412	.80438	33
28	.55145	.83421	.56593	.82446	.58023	.81445	.59436	.80420	32
29	.55169	.83405	.56617	.82429	.58047	.81428	.59459	.80403	31
30	.55194	.83389	.56641	.82413	.58070	.81412	.59482	.80386	30
31	.55218	.83373	.56665	.82396	.58094	.81395	.59506	.80368	29
32	.55242	.83356	.56689	.82340	.58118	.81378	.59529	.80351	28
33	.55266	.83340	.56713	.82363	.58141	.81361	.59552	.80334	27
34	.55291	.83324	.56736	.82347	.58165	.81344	.59576	.80316	26
35	.55315	.83308	.56760	.82330	.58189	.81327	.59599	.80299	25
36	.55339	.83292	.56784	.82314	.58212	.81310	.59622	.80282	24
37	.55363	.83276	.56808	.82297	.58236	.81293	.59646	.80264	23
38	.55388	.83260	.56832	.82281	.58260	.81276	.59669	.80247	22
39	.55412	.83244	.56856	.82264	.58283	.81259	.59693	.80230	21
40	.55436	.83228	.56880	.82248	.58307	.81242	.59716	.80212	20
41	.55460	.83212	.56904	.82231	.58330	.81225	.59739	.80195	19
42	.55484	.83195	.56928	.82214	.58354	.81208	.59763	.80178	18
43	.55509	.83179	.56952	.82198	.58378	.81191	.59786	.80160	17
44	.55533	.83163	.56976	.82181	.58401	.81174	.59809	.80143	16
45	.55557	.83147	.57000	.82165	.58425	.81157	.59832	.80125	15
46	.55581	.83131	.57024	.82148	.58449	.81140	.59856	.80108	14
47	.55605	.83115	.57047	.82132	.58472	.81123	.59879	.80091	13
48	.55630	.83098	.57071	.82115	.58496	.81106	.59902	.80073	12
49	.55654	.83082	.57095	.82098	.58519	.81089	.59926	.80056	11
50	.55678	.83066	.57119	.82082	.58543	.81072	.59949	.80038	10
51	.55702	.83050	.57143	.82065	.58567	.81055	.59972	.80021	9
52	.55726	.83034	.57167	.82048	.58590	.81038	.59995	.80003	8
53	.55750	.83017	.57191	.82032	.58614	.81021	.60019	.79986	7
54	.55775	.83001	.57215	.82015	.58637	.81004	.60042	.79968	6
55	.55799	.82985	.57238	.81999	.58661	.80987	.60065	.79951	5
56	.55823	.82969	.57262	.81982	.58684	.80970	.60089	.79934	4
57	.55847	.82953	.57286	.81965	.58708	.80953	.60112	.79916	3
58	.55871	.82936	.57310	.81949	.58731	.80936	.60135	.79899	2
59	.55895	.82920	.57334	.81932	.58755	.80919	.60158	.79881	1
60	.55919	.82904	.57358	.81915	.58779	.80902	.60182	.79864	0
′	56° Cosine	56° Sine	55° Cosine	55° Sine	54° Cosine	54° Sine	53° Cosine	53° Sine	′

TABLE 20.—NATURAL SINES AND COSINES—(*Continued*)

′	37° Sine	37° Cosine	38° Sine	38° Cosine	39° Sine	39° Cosine	40° Sine	40° Cosine	′
0	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	60
1	.60205	.79846	.61589	.78783	.62955	.77696	.64301	.76586	59
2	.60228	.79829	.61612	.78765	.62977	.77678	.64323	.76567	58
3	.60251	.79811	.61635	.78747	.63000	.77660	.64346	.76548	57
4	.60274	.79793	.61658	.78729	.63022	.77641	.64368	.76530	56
5	.60298	.79776	.61681	.78711	.63045	.77623	.64390	.76511	55
6	.60321	.79758	.61704	.78694	.63068	.77605	.64412	.76492	54
7	.60344	.79741	.61726	.78676	.63090	.77586	.64435	.76473	53
8	.60367	.79723	.61749	.78658	.63113	.77568	.64457	.76455	52
9	.60390	.79706	.61772	.78640	.63135	.77550	.64479	.76436	51
10	.60414	.79688	.61795	.78622	.63158	.77531	.64501	.76417	50
11	.60437	.79671	.61818	.78604	.63180	.77513	.64524	.76398	49
12	.60460	.79653	.61841	.78586	.63203	.77494	.64546	.76380	48
13	.60483	.79635	.61864	.78568	.63225	.77476	.64568	.76361	47
14	.60506	.79618	.61887	.78550	.63248	.77458	.64590	.76342	46
15	.60529	.79600	.61909	.78532	.63271	.77439	.64612	.76323	45
16	.60553	.79583	.61932	.78514	.63293	.77421	.64635	.76304	44
17	.60576	.79565	.61955	.78496	.63316	.77402	.64657	.76286	43
18	.60599	.79547	.61978	.78478	.63338	.77384	.64679	.76267	42
19	.60622	.79530	.62001	.78460	.63361	.77366	.64701	.76248	41
20	.60645	.79512	.62024	.78442	.63383	.77347	.64723	.76229	40
21	.60668	.79494	.62046	.78424	.63406	.77329	.64746	.76210	39
22	.60691	.79477	.62069	.78405	.63428	.77310	.64768	.76192	38
23	.60714	.79459	.62092	.78387	.63451	.77292	.64790	.76173	37
24	.60738	.79441	.62115	.78369	.63473	.77273	.64812	.76154	36
25	.60761	.79424	.62138	.78351	.63496	.77255	.64834	.76135	35
26	.60784	.79406	.62160	.78333	.63518	.77236	.64856	.76116	34
27	.60807	.79388	.62183	.78315	.63540	.77218	.64878	.76097	33
28	.60830	.79371	.62206	.78297	.63563	.77199	.64901	.76078	32
29	.60853	.79353	.62229	.78279	.63585	.77181	.64923	.76059	31
30	.60876	.79335	.62251	.78261	.63608	.77162	.64945	.76041	30
31	.60899	.79318	.62274	.78243	.63630	.77144	.64967	.76022	29
32	.60922	.79300	.62297	.78225	.63653	.77125	.64989	.76003	28
33	.60945	.79282	.62320	.78206	.63675	.77107	.65011	.75984	27
34	.60968	.79264	.62342	.78188	.63698	.77088	.65033	.75965	26
35	.60991	.79247	.62365	.78170	.63720	.77070	.65055	.75946	25
36	.61015	.79229	.62388	.78152	.63742	.77051	.65077	.75927	24
37	.61038	.79211	.62411	.78134	.63765	.77033	.65100	.75908	23
38	.61061	.79193	.62433	.78116	.63787	.77014	.65122	.75889	22
39	.61084	.79176	.62456	.78098	.63810	.76996	.65144	.75870	21
40	.61107	.79158	.62479	.78079	.63832	.76977	.65166	.75851	20
41	.61130	.79140	.62502	.78061	.63854	.76959	.65188	.75832	19
42	.61153	.79122	.62524	.78043	.63877	.76940	.65210	.75813	18
43	.61176	.79105	.62547	.78025	.63899	.76921	.65232	.75794	17
44	.61199	.79087	.62570	.78007	.63922	.76903	.65254	.75775	16
45	.61222	.79069	.62592	.77988	.63944	.76884	.65276	.75756	15
46	.61245	.79051	.62615	.77970	.63966	.76866	.65298	.75738	14
47	.61268	.79033	.62638	.77952	.63989	.76847	.65320	.75719	13
48	.61291	.79016	.62660	.77934	.64011	.76828	.65342	.75700	12
49	.61314	.78998	.62683	.77916	.64033	.76810	.65364	.75680	11
50	.61337	.78980	.62706	.77897	.64056	.76791	.65386	.75661	10
51	.61360	.78962	.62728	.77879	.64078	.76772	.65408	.75642	9
52	.61383	.78944	.62751	.77861	.64100	.76754	.65430	.75623	8
53	.61406	.78926	.62774	.77843	.64123	.76735	.65452	.75604	7
54	.61429	.78908	.62796	.77824	.64145	.76717	.65474	.75585	6
55	.61451	.78891	.62819	.77806	.64167	.76698	.65496	.75566	5
56	.61474	.78873	.62842	.77788	.64190	.76679	.65518	.75547	4
57	.61497	.78855	.62864	.77769	.64212	.76661	.65540	.75528	3
58	.61520	.78837	.62887	.77751	.64234	.76642	.65562	.75509	2
59	.61543	.78819	.62909	.77733	.64256	.76623	.65584	.75490	1
60	.61566	.78801	.62932	.77715	.64279	.76604	.65606	.75471	0
′	52° Cosine	52° Sine	51° Cosine	51° Sine	50° Cosine	50° Sine	49° Cosine	49° Sine	′

TABLE 20.—NATURAL SINES AND COSINES—(*Continued*)

′	41° Sine	41° Cosine	42° Sine	42° Cosine	43° Sine	43° Cosine	44° Sine	44° Cosine	′
0	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.65694	.75395	.66999	.74227	.68285	.73056	.69549	.71853	56
5	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	48° Cosine	48° Sine	47° Cosine	47° Sine	46° Cosine	46° Sine	45° Cosine	45° Sine	′

TABLE 21.—NATURAL TANGENTS AND COTANGENTS

′	0° Tan.	0° Co-tan.	1° Tan.	1° Co-tan.	2° Tan.	2° Co-tan.	3° Tan.	3° Co-tan.	′
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.750	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.870	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.920	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00814	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04832	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	89°		88°		87°		86°		

From Davis "Manual of Surveying."

TABLE 21.—NATURAL TANGENTS AND COTANGENTS—
(Continued)

′	4° Tan.	4° Co-tan.	5° Tan.	5° Co-tan.	6° Tan.	6° Co-tan.	7° Tan.	7° Co-tan.	′
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33154	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	85°		84°		83°		82°		

TABLE 21.—NATURAL TANGENTS AND COTANGENTS—
(*Continued*)

′	8°		9°		10°		11°		′
′	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	′
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	7.01174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.00552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18383	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.83605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91235	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	81°		80°		79°		78°		

TABLE 21.—NATURAL TANGENTS AND COTANGENTS—(*Continued*)

′	12° Tan.	12° Co-tan.	13° Tan.	13° Co-tan.	14° Tan.	14° Co-tan.	15° Tan.	15° Co-tan.	′
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27920	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03075	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	77° Co-tan.	77° Tan.	76° Co-tan.	76° Tan.	75° Co-tan.	75° Tan.	74° Co-tan.	74° Tan.	′

Table 21.—Natural Tangents and Cotangents—(Continued)

′	16° Tan.	16° Co-tan.	17° Tan.	17° Co-tan.	18° Tan.	18° Co-tan.	19° Tan.	19° Co-tan.	′
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32011	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35019	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33160	3.01489	.35117	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94590	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	73° Co-tan.	73° Tan.	72° Co-tan.	72° Tan.	71° Co-tan.	71° Tan.	70° Co-tan.	70° Tan.	′

TABLE 21.—NATURAL TANGENTS AND COTANGENTS—(*Continued*)

	20°		21°		22°		23°		
′	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	′
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37124	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38862	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	69°		68°		67°		66°		

TABLE 21.—NATURAL TANGENTS AND COTANGENTS—(*Continued*)

	24°		25°		26°		27°		
′	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	′
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45537	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97680	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52984	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	65°		64°		63°		62°		

TABLE 21.—NATURAL TANGENTS AND COTANGENTS—(*Continued*)

	28°		29°		30°		31°		
′	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	′
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1. 6209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65534	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60021	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56500	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58904	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76630	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1 62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1·76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan	Tan.	
	61°		60°		59°		58°		

TABLE 21.—NATURAL TANGENTS AND COTANGENTS—(*Continued*)

′	32° Tan.	32° Co-tan.	33° Tan.	33° Co-tan.	34° Tan.	34° Co-tan.	35° Tan.	35° Co-tan.	′
0	.62487	1.60033	.64941	1.53086	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65023	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72166	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	57° Co-tan.	57° Tan.	56° Co-tan.	56° Tan.	55° Co-tan.	55° Tan.	54° Co-tan.	54° Tan.	′

TABLE 21.—NATURAL TANGENTS AND COTANGENTS—
(*Continued*)

′	36° Tan.	36° Co-tan.	37° Tan.	37° Co-tan.	38° Tan.	38° Co-tan.	39° Tan.	39° Co-tan.	′
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36133	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Co-tan. 53°	Tan. 53°	Co-tan. 52°	Tan. 52°	Co-tan. 51°	Tan. 51°	Co-tan. 50°	Tan. 50°	′

TABLE 21.—NATURAL TANGENTS AND COTANGENTS—(*Continued*)

′	40° Tan.	40° Co-tan.	41° Tan.	41° Co-tan.	42° Tan.	42° Co-tan.	43° Tan.	43° Co-tan.	′
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85307	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Co-tan. 49°	Tan. 49°	Co-tan. 48°	Tan. 48°	Co-tan. 47°	Tan. 47°	Co-tan. 46°	Tan. 46°	′

TABLE 21.—NATURAL TANGENTS AND COTANGENTS—(*Continued*)

′	44° Tan.	44° Co-tan.	′	′	44° Tan.	44° Co-tan.	′	′	44° Tan.	44° Co-tan.	′
0	.96569	1.03553	60	21	.97756	1.02295	39	41	.98901	1.01112	19
1	.96625	1.03493	59	22	.97813	1.02236	38	42	.98958	1.01053	18
2	.96681	1.03433	58	23	.97870	1.02176	37	43	.99016	1.00994	17
3	.96738	1.03372	57	24	.97927	1.02117	36	44	.99073	1.00935	16
4	.96794	1.03312	56	25	.97984	1.02057	35	45	.99131	1.00876	15
5	.96850	1.03252	55	26	.98041	1.01998	34	46	.99189	1.00818	14
6	.96907	1.03192	54	27	.98098	1.01939	33	47	.99247	1.00759	13
7	.96963	1.03132	53	28	.98155	1.01879	32	48	.99304	1.00701	12
8	.97020	1.03072	52	29	.98213	1.01820	31	49	.99362	1.00642	11
9	.97076	1.03012	51	30	.98270	1.01761	30	50	.99420	1.00583	10
10	.97133	1.02952	50	31	.98327	1.01702	29	51	.99478	1.00525	9
11	.97189	1.02892	49	32	.98384	1.01642	28	52	.99536	1.00467	8
12	.97246	1.02832	48	33	.98441	1.01583	27	53	.99594	1.00408	7
13	.97302	1.02772	47	34	.98499	1.01524	26	54	.99652	1.00350	6
14	.97359	1.02713	46	35	.98556	1.01465	25	55	.99710	1.00291	5
15	.97416	1.02653	45	36	.98613	1.01406	24	56	.99768	1.00233	4
16	.97472	1.02593	44	37	.98671	1.01347	23	57	.99826	1.00175	3
17	.97529	1.02533	43	38	.98728	1.01288	22	58	.99884	1.00116	2
18	.97586	1.02474	42	39	.98786	1.01229	21	59	.99942	1.00058	1
19	.97643	1.02414	41	40	.98843	1.01170	20	60	1	1	0
20	.97700	1.02355	40								
′	Co-tan. 45°	Tan. 45°	′	′	Co-tan. 45°	Tan. 45°	′	′	Co-tan. 45°	Tan. 45°	′

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS

M.	0° Vers.	0° Exsec.	1° Vers.	1° Exsec.	2° Vers.	2° Exsec.	3° Vers.	3° Exsec.	M.
0	.00000	.00000	.00015	.00015	.00061	.00061	.00137	.00137	0
1	000	000	016	016	062	062	139	139	1
2	000	000	016	016	063	063	140	140	2
3	000	000	017	017	064	064	142	142	3
4	000	000	017	017	065	065	143	143	4
5	.00000	.00000	.00018	.00018	.00066	.00066	.00145	.00145	5
6	000	000	018	018	067	067	146	147	6
7	000	000	019	019	068	068	148	148	7
8	000	000	020	020	069	069	149	150	8
9	000	000	020	020	070	070	151	151	9
10	.00000	.00000	.00021	.00021	.00071	.00072	.00153	.00153	10
11	001	001	021	021	073	073	154	155	11
12	001	001	022	022	074	074	156	156	12
13	001	001	023	023	075	075	158	158	13
14	001	001	023	023	076	076	159	159	14
15	.00001	.00001	.00024	.00024	.00077	.00077	.00161	.00161	15
16	001	001	024	024	078	078	162	163	16
17	001	001	025	025	079	079	164	164	17
18	001	001	026	026	081	081	166	166	18
19	002	002	026	026	082	082	167	168	19
20	.00002	.00002	.00027	.00027	.00083	.00083	.00169	.00169	20
21	002	002	028	028	084	084	171	171	21
22	002	002	028	028	085	085	173	173	22
23	002	002	029	029	087	087	174	175	23
24	002	002	030	030	088	088	176	176	24
25	.00003	.00003	.00031	.00031	.00089	.00089	.00178	.00178	25
26	003	003	031	031	090	090	179	180	26
27	003	003	032	032	091	091	181	182	27
28	003	003	033	033	093	093	183	183	28
29	004	004	034	034	094	094	185	185	29
30	.00004	.00004	.00034	.00034	.00095	.00095	.00187	.00187	30
31	004	004	035	035	096	097	188	189	31
32	004	004	036	036	098	098	190	190	32
33	005	005	037	037	099	099	192	192	33
34	005	005	037	037	100	100	194	194	34
35	.00005	.00005	.00038	.00038	.00102	.00102	.00196	.00196	35
36	005	005	039	039	103	103	197	198	36
37	006	006	040	040	104	104	199	200	37
38	006	006	041	041	106	106	201	201	38
39	006	006	041	041	107	107	203	203	39
40	.00007	.00007	.00042	.00042	.00108	.00108	.00205	.00205	40
41	007	007	043	043	110	110	207	207	41
42	007	007	044	044	111	111	208	209	42
43	008	008	045	045	112	113	210	211	43
44	008	008	046	046	114	114	212	213	44
45	.00009	.00009	.00047	.00047	.00115	.00115	.00214	.00215	45
46	009	009	048	048	117	117	216	216	46
47	009	009	048	048	118	118	218	218	47
48	010	010	049	049	119	120	220	220	48
49	010	010	050	050	121	121	222	222	49
50	.00011	.00011	.00051	.00051	.00122	.00122	.00224	.00224	50
51	011	011	052	052	124	124	226	226	51
52	011	011	053	053	125	125	228	228	52
53	012	012	054	054	127	127	230	230	53
54	012	012	055	055	128	128	232	232	54
55	.00013	.00013	.00056	.00056	.00130	.00130	.00234	.00234	55
56	013	013	057	057	131	131	236	236	56
57	014	014	058	058	133	133	238	238	57
58	014	014	059	059	134	134	240	240	58
59	015	015	060	060	136	136	242	242	59
60	.00015	.00015	.00061	.00061	.00137	.00137	.00244	.00244	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—
(*Continued*)

M.	4° Vers.	4° Exsec.	5° Vers.	5° Exsec.	6° Vers.	6° Exsec.	7° Vers.	7° Exsec.	M.
0	.00244	.00244	.00381	.00382	.00548	.00551	.00745	.00751	0
1	246	246	383	385	551	554	749	755	1
2	248	248	386	387	554	557	752	758	2
3	250	250	388	390	557	560	756	762	3
4	252	252	391	392	560	563	760	765	4
5	.00254	.00254	.00393	.00395	.00563	.00566	.00763	.00769	5
6	256	257	396	397	566	569	767	773	6
7	258	259	398	400	569	573	770	776	7
8	260	261	401	403	572	576	774	780	8
9	262	263	404	405	576	579	778	784	9
10	.00264	.00265	.00406	.00408	.00579	.00582	.00781	.00787	10
11	266	267	409	411	582	585	785	791	11
12	269	269	412	413	585	588	789	795	12
13	271	271	414	416	588	592	792	799	13
14	273	274	417	419	591	595	796	802	14
15	.00275	.00276	.00420	.00421	.00594	.00598	.00800	.00806	15
16	277	278	422	424	598	601	803	810	16
17	279	280	425	427	601	604	807	813	17
18	281	282	428	429	604	608	811	817	18
19	284	284	430	432	607	611	814	821	19
20	.00286	.00287	.00433	.00435	.00610	.00614	.00818	.00825	20
21	288	289	436	438	614	617	822	828	21
22	290	291	438	440	617	621	825	832	22
23	292	293	441	443	620	624	829	836	23
24	295	296	444	446	623	627	833	840	24
25	.00297	.00298	.00447	.00449	.00626	.00630	.00837	.00844	25
26	299	300	449	451	630	634	840	848	26
27	301	302	452	454	633	637	844	851	27
28	304	305	455	457	636	640	848	855	28
29	306	307	458	460	640	644	852	859	29
30	.00308	.00309	.00460	.00463	.00643	.00647	.00856	.00863	30
31	311	312	463	465	646	650	859	867	31
32	313	314	466	468	649	654	863	871	32
33	315	316	469	471	653	657	867	875	33
34	317	318	472	474	656	660	871	878	34
35	.00320	.00321	.00474	.00477	.00659	.00664	.00875	.00882	35
36	322	323	477	480	663	667	878	886	36
37	324	326	480	482	666	671	882	890	37
38	327	328	483	485	669	674	886	894	38
39	329	330	486	488	673	677	890	898	39
40	.00332	.00333	.00489	.00491	.00676	.00681	.00894	.00902	40
41	334	335	492	494	680	684	898	906	41
42	336	337	494	497	683	688	902	910	42
43	339	340	497	500	686	691	906	914	43
44	341	342	500	503	690	695	909	918	44
45	.00343	.00345	.00503	.00506	.00693	.00698	.00913	.00922	45
46	346	347	506	509	697	701	917	926	46
47	348	349	509	512	700	705	921	930	47
48	351	352	512	515	703	708	925	934	48
49	353	354	515	518	707	712	929	938	49
50	.00356	.00357	.00518	.00521	.00710	.00715	.00933	.00942	50
51	358	359	521	524	714	719	937	946	51
52	361	362	524	527	717	722	941	950	52
53	363	364	527	530	721	726	945	954	53
54	365	367	530	533	724	730	949	958	54
55	.00368	.00369	.00533	.00536	.00728	.00733	.00953	.00962	55
56	370	372	536	539	731	737	957	966	56
57	373	374	539	542	735	740	961	970	57
58	375	377	542	545	738	744	965	975	58
59	378	379	545	548	742	747	969	979	59
60	.00381	.00382	.00548	.00551	.00745	.00751	.00973	.00983	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	8° Vers.	8° Exsec.	9° Vers.	9° Exsec.	10° Vers.	10° Exsec.	11° Vers.	11° Exsec.	M.
0	.00973	.00983	.01231	.01247	.01519	.01543	.01837	.01872	0
1	977	987	236	251	524	548	843	877	1
2	981	991	240	256	529	553	848	883	2
3	985	995	245	261	534	558	854	889	3
4	989	999	249	265	539	564	860	895	4
5	.00994	.01004	.01254	.01270	.01545	.01569	.01865	.01901	5
6	998	008	259	275	550	574	871	906	6
7	.01002	012	263	279	555	579	876	912	7
8	006	016	268	284	560	585	882	918	8
9	010	020	272	289	565	590	888	924	9
10	.01014	.01024	.01277	.01294	.01570	.01595	.01893	.01930	10
11	018	029	282	298	575	601	899	936	11
12	022	033	286	303	580	606	904	941	12
13	027	037	291	308	586	611	910	947	13
14	031	041	296	313	591	616	916	953	14
15	.01035	.01046	.01300	.01317	.01596	.01622	.01921	.01959	15
16	039	050	305	322	601	627	927	965	16
17	043	054	310	327	606	633	933	971	17
18	047	059	314	332	611	638	939	977	18
19	052	063	319	337	617	643	944	983	19
20	.01056	.01067	.01324	.01342	.01622	.01649	.01950	.01989	20
21	060	071	329	346	627	654	956	995	21
22	064	076	333	351	632	659	961	.02001	22
23	069	080	338	356	638	665	967	007	23
24	073	084	343	361	643	670	973	013	24
25	.01077	.01089	.01348	.01366	.01648	.01676	.01979	.02019	25
26	081	093	352	371	653	681	984	025	26
27	086	097	357	376	659	687	990	031	27
28	090	102	362	381	664	692	996	037	28
29	094	106	367	386	669	698	.02002	043	29
30	.01098	.01111	.01371	.01391	.01675	.01703	.02008	.02049	30
31	103	115	376	395	680	709	013	055	31
32	107	119	381	400	685	714	019	061	32
33	111	124	386	405	690	720	025	067	33
34	116	128	391	410	696	725	031	073	34
35	.01120	.01133	.01396	.01415	.01701	.01731	.02037	.02079	35
36	124	137	400	420	706	736	042	085	36
37	129	142	405	425	712	742	048	091	37
38	133	146	410	430	717	747	054	097	38
39	137	151	415	435	723	753	060	103	39
40	.01142	.01155	.01420	.01440	.01728	.01758	.02066	.02110	40
41	146	160	425	445	733	764	072	116	41
42	151	164	430	450	739	769	078	122	42
43	155	169	435	455	744	775	084	128	43
44	159	173	439	460	750	781	090	134	44
45	.01164	.01178	.01444	.01466	.01755	.01786	.02095	.02140	45
46	168	182	449	471	760	792	101	146	46
47	173	187	454	476	766	798	107	153	47
48	177	191	459	481	771	803	113	159	48
49	182	196	464	486	777	809	119	165	49
50	.01186	.01200	.01469	.01491	.01782	.01815	.02125	.02171	50
51	191	205	474	496	788	820	131	178	51
52	195	209	479	501	793	826	137	184	52
53	200	214	484	506	799	832	143	190	53
54	204	219	489	512	804	837	149	196	54
55	.01209	.01223	.01494	.01517	.01810	.01843	.02155	.02203	55
56	213	228	499	522	815	849	161	209	56
57	218	233	504	527	821	854	167	215	57
58	222	237	509	532	826	860	173	221	58
59	227	242	514	537	832	866	179	228	59
60	.01231	.01247	.01519	.01543	.01837	.01872	.02185	.02234	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	12° Vers.	12° Exsec.	13° Vers.	13° Exsec.	14° Vers.	14° Exsec.	15° Vers.	15° Exsec.	M.
0	.02185	.02234	.02563	.02630	.02970	.03061	.03407	.03528	0
1	191	240	570	637	977	069	415	536	1
2	197	247	576	644	985	076	422	544	2
3	203	253	583	651	992	084	430	552	3
4	209	259	589	658	999	091	438	560	4
5	.02216	.02266	.02596	.02665	.03006	.03099	.03445	.03568	5
6	222	272	602	672	013	106	453	576	6
7	228	279	609	679	020	114	460	584	7
8	234	285	616	686	027	121	468	592	8
9	240	291	622	693	034	129	476	601	9
10	.02246	.02298	.02629	.02700	.03041	.03137	.03483	.03609	10
11	252	304	635	707	048	144	491	617	11
12	258	311	642	714	055	152	498	625	12
13	265	317	649	721	063	159	506	633	13
14	271	323	655	728	070	167	514	642	14
15	.02277	.02330	.02662	.02735	.03077	.03175	.03521	.03650	15
16	283	336	669	742	084	182	529	658	16
17	289	343	675	749	091	190	537	666	17
18	295	349	682	756	098	197	544	674	18
19	302	356	689	763	106	205	552	683	19
20	.02308	.02362	.02696	.02770	.03113	.03213	.03560	.03691	20
21	314	369	702	777	120	220	567	699	21
22	320	375	709	784	127	228	575	708	22
23	327	382	716	791	134	236	583	716	23
24	333	388	722	799	142	244	590	724	24
25	.02339	.02395	.02729	.02806	.03149	.03251	.03598	.03732	25
26	345	402	736	813	156	259	606	741	26
27	352	408	743	820	163	267	614	749	27
28	358	415	749	827	171	275	621	758	28
29	364	421	756	834	178	282	629	766	29
30	.02370	.02428	.02763	.02842	.03185	.03290	.03637	.03774	30
31	377	435	770	849	193	298	645	783	31
32	383	441	777	856	200	306	653	791	32
33	389	448	783	863	207	313	660	799	33
34	396	454	790	870	214	321	668	808	34
35	.02402	.02461	.02797	.02878	.03222	.03329	.03676	.03816	35
36	408	468	804	885	229	337	684	825	36
37	415	474	811	892	236	345	692	833	37
38	421	481	818	899	244	353	699	842	38
39	427	488	824	907	251	360	707	850	39
40	.02434	.02494	.02831	.02914	.03258	.03368	.03715	.03858	40
41	440	501	838	921	266	376	723	867	41
42	447	508	845	928	273	384	731	875	42
43	453	515	852	936	281	392	739	884	43
44	459	521	859	943	288	400	747	892	44
45	.02466	.02528	.02866	.02950	.03295	.03408	.03754	.03901	45
46	472	535	873	958	303	416	762	909	46
47	479	542	880	965	310	424	770	918	47
48	485	548	887	972	318	432	778	927	48
49	492	555	894	980	325	439	786	935	49
50	.02498	.02562	.02900	.02987	.03333	.03447	.03794	.03944	50
51	504	569	907	994	340	455	802	952	51
52	511	576	914	.03002	347	463	810	961	52
53	517	582	921	009	355	471	818	969	53
54	524	589	928	017	362	479	826	978	54
55	.02530	.02596	.02935	.03024	.03370	.03487	.03834	.03987	55
56	537	603	942	032	377	495	842	995	56
57	543	610	949	039	385	503	850	.04004	57
58	550	617	956	046	392	511	858	013	58
59	556	624	963	054	400	520	866	021	59
60	.02563	.02630	.02970	.03061	.03407	.03528	.03874	.04030	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—
(*Continued*)

M.	16° Vers.	16° Exsec.	17° Vers.	17° Exsec.	18° Vers.	18° Exsec.	19° Vers.	19° Exsec.	M.
0	.03874	.04030	.04370	.04569	.04894	.05146	.05448	.05762	0
1	882	039	378	578	903	156	458	773	1
2	890	047	387	588	912	166	467	783	2
3	898	056	395	597	921	176	477	794	3
4	906	065	404	606	930	186	486	805	4
5	.03914	.04073	.04412	.04616	.04939	.05196	.05496	.05815	5
6	922	082	421	625	948	206	505	826	6
7	930	091	429	635	957	216	515	836	7
8	938	100	438	644	967	226	524	847	8
9	946	108	446	653	976	236	534	858	9
10	.03954	.04117	.04455	.04663	.04985	.05246	.05543	.05869	10
11	963	126	464	672	994	256	553	879	11
12	971	135	472	682	.05003	266	562	890	12
13	979	144	481	691	012	276	572	901	13
14	987	152	489	700	021	286	582	911	14
15	.03995	.04161	.04498	.04710	.05030	.05297	.05591	.05922	15
16	.04003	170	507	719	039	307	601	933	16
17	011	179	515	729	048	317	610	944	17
18	019	188	524	738	057	327	620	955	18
19	028	197	533	748	067	337	630	965	19
20	.04036	.04206	.04541	.04757	.05076	.05347	.05639	.05976	20
21	044	214	550	767	085	357	649	987	21
22	052	223	559	776	094	367	658	998	22
23	060	232	567	786	103	378	668	.06009	23
24	069	241	576	795	112	388	678	020	24
25	.04077	.04250	.04585	.04805	.05122	.05398	.05687	.06030	25
26	085	259	593	815	131	408	697	041	26
27	093	268	602	824	140	418	707	052	27
28	102	277	611	834	149	429	716	063	28
29	110	286	620	843	158	439	726	074	29
30	.04118	.04295	.04628	.04853	.05168	.05449	.05736	.06085	30
31	126	304	637	863	177	460	746	096	31
32	135	313	646	872	186	470	755	107	32
33	143	322	655	882	195	480	765	118	33
34	151	331	663	891	205	490	775	129	34
35	.04159	.04340	.04672	.04901	.05214	.05501	.05785	.06140	35
36	168	349	681	911	223	511	794	151	36
37	176	358	690	920	232	521	804	162	37
38	184	367	699	930	242	532	814	173	38
39	193	376	707	940	251	542	824	184	39
40	.04201	.04385	.04716	.04950	.05260	.05552	.05833	.06195	40
41	209	394	725	959	270	563	843	206	41
42	218	403	734	969	279	573	853	217	42
43	226	413	743	979	288	584	863	228	43
44	234	422	752	989	298	594	873	239	44
45	.04243	.04431	.04760	.04998	.05307	.05604	.05882	.06250	45
46	251	440	769	.05008	316	615	892	261	46
47	260	449	778	018	326	625	902	272	47
48	268	458	787	028	335	636	912	283	48
49	276	468	796	038	344	646	922	295	49
50	.04285	.04477	.04805	.05047	.05354	.05657	.05932	.06306	50
51	293	486	814	057	363	667	942	317	51
52	302	495	823	067	373	678	951	328	52
53	310	504	832	077	382	688	961	339	53
54	319	514	841	087	391	699	971	350	54
55	.04327	.04523	.04850	.05097	.05401	.05709	.05981	.06362	55
56	336	532	858	107	410	720	991	373	56
57	344	541	867	116	420	730	.06001	384	57
58	353	551	876	126	429	741	011	395	58
59	361	560	885	136	439	751	021	407	59
60	.04370	.04569	.04894	.05146	.05448	.05762	.06031	.06418	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—
(*Continued*)

M	20° Vers.	20° Exsec.	21° Vers.	21° Exsec.	22° Vers.	22° Exsec.	23° Vers.	23° Exsec.	M.
0	.06031	.06418	.06642	.07114	.07282	.07853	.07950	.08636	0
1	041	429	652	126	293	866	961	649	1
2	051	440	663	138	303	879	972	663	2
3	061	452	673	150	314	892	984	676	3
4	071	463	684	162	325	904	995	690	4
5	.06081	.06474	.06694	.07174	.07336	.07917	.08006	.08703	5
6	091	486	705	186	347	930	018	717	6
7	101	497	715	199	358	943	029	730	7
8	111	508	726	211	369	955	041	744	8
9	121	520	736	223	380	968	052	757	9
10	.06131	.06531	.06747	.07235	.07391	.07981	.08064	.08771	10
11	141	542	757	247	402	994	075	784	11
12	151	554	768	259	413	.08006	086	798	12
13	161	565	778	271	424	019	098	811	13
14	171	577	789	283	435	032	109	825	14
15	.06181	.06588	.06799	.07295	.07446	.08045	.08121	.08839	15
16	191	600	810	307	457	058	132	852	16
17	201	611	820	320	468	071	144	866	17
18	211	623	831	332	479	084	155	880	18
19	221	634	841	344	490	097	167	893	19
20	.06231	.06645	.06852	.07356	.07501	.08109	.08178	.08907	20
21	241	657	863	368	512	122	190	920	21
22	252	668	873	380	523	135	201	934	22
23	262	680	884	393	534	148	213	948	23
24	272	691	894	405	545	161	225	962	24
25	.06282	.06703	.06905	.07417	.07556	.08174	.08236	.08975	25
26	292	715	916	429	568	187	248	989	26
27	302	726	926	442	579	200	259	.09003	27
28	312	738	937	454	590	213	271	017	28
29	323	749	948	466	601	226	282	030	29
30	.06333	.06761	.06958	.07479	.07612	.08239	.08294	.09044	30
31	343	773	969	491	623	252	306	058	31
32	353	784	980	503	634	265	317	072	32
33	363	796	990	516	645	278	329	086	33
34	374	807	.07001	528	657	291	340	099	34
35	.06384	.06819	.07012	.07540	.07668	.08305	.08352	.09113	35
36	394	831	022	553	679	318	364	127	36
37	404	842	033	565	690	331	375	141	37
38	415	854	044	578	701	344	387	155	38
39	425	866	055	590	713	357	399	169	39
40	.06435	.06878	.07065	.07602	.07724	.08370	.08410	.09183	40
41	445	889	076	615	735	383	422	197	41
42	456	901	087	627	746	397	434	211	42
43	466	913	098	640	757	410	445	224	43
44	476	925	108	652	769	423	457	238	44
45	.06486	.06936	.07119	.07665	.07780	.08436	.08469	.09252	45
46	497	948	130	677	791	449	481	266	46
47	507	960	141	690	802	463	492	280	47
48	517	972	151	702	814	476	504	294	48
49	528	984	162	715	825	489	516	308	49
50	.06538	.06995	.07173	.07727	.07836	.08503	.08528	.09323	50
51	548	.07007	184	740	848	516	539	337	51
52	559	019	195	752	859	529	551	351	52
53	569	031	206	765	870	542	563	365	53
54	580	043	216	778	881	556	575	379	54
55	.06590	.07055	.07227	.07790	.07893	.08569	.08586	.09393	55
56	600	067	238	803	904	582	598	407	56
57	611	079	249	816	915	596	610	421	57
58	621	091	260	828	927	609	622	435	58
59	632	103	271	841	938	623	634	449	59
60	.06642	.07114	.07282	.07853	.07950	.08636	.08645	.09464	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	24° Vers.	24° Exsec.	25° Vers.	25° Exsec.	26° Vers.	26° Exsec.	27° Vers.	27° Exsec.	M.
0	.08645	.09464	.09369	.10338	.10121	.11260	.10899	.12233	0
1	657	478	382	353	133	276	913	249	1
2	669	492	394	368	146	292	926	266	2
3	681	506	406	383	159	308	939	283	3
4	693	520	418	398	172	323	952	299	4
5	.08705	.09535	.09431	.10413	.10184	.11339	.10965	.12316	5
6	717	549	443	428	197	355	979	333	6
7	728	563	455	443	210	371	992	349	7
8	740	577	468	458	223	387	.11005	366	8
9	752	592	480	473	236	403	019	383	9
10	.08764	.09606	.09493	.10488	.10248	.11419	.11032	.12400	10
11	776	620	505	503	261	435	045	416	11
12	788	635	517	518	274	451	058	433	12
13	800	649	530	533	287	467	072	450	13
14	812	663	542	549	300	483	085	467	14
15	.08824	.09678	.09554	.10564	.10313	.11499	.11098	.12484	15
16	836	692	567	579	326	515	112	501	16
17	848	707	579	594	338	531	125	518	17
18	860	721	592	609	351	547	138	534	18
19	872	735	604	625	364	563	152	551	19
20	.08884	.09750	.09617	.10640	.10377	.11579	.11165	.12568	20
21	896	764	629	655	390	595	178	585	21
22	908	779	642	670	403	611	192	602	22
23	920	793	654	686	416	627	205	619	23
24	932	808	666	701	429	643	218	636	24
25	.08944	.09822	.09679	.10716	.10442	.11659	.11232	.12653	25
26	956	837	691	731	455	675	245	670	26
27	968	851	704	747	468	691	259	687	27
28	980	866	716	762	481	708	272	704	28
29	992	880	729	777	494	724	285	721	29
30	.09004	.09895	.09741	.10793	.10507	.11740	.11299	.12738	30
31	016	909	754	808	520	756	312	755	31
32	028	924	767	824	533	772	326	772	32
33	040	939	779	839	546	789	339	789	33
34	052	953	792	854	559	805	353	807	34
35	.09064	.09968	.09804	.10870	.10572	.11821	.11366	.12824	35
36	076	982	817	885	585	838	380	841	36
37	089	997	829	901	598	854	393	858	37
38	101	.10012	842	916	611	870	407	875	38
39	113	026	854	932	624	886	420	892	39
40	.09125	.10041	.09867	.10947	.10637	.11903	.11434	.12910	40
41	137	056	880	963	650	919	447	927	41
42	149	071	892	978	663	936	461	944	42
43	161	085	905	994	676	952	474	961	43
44	174	100	918	.11009	689	968	488	979	44
45	.09186	.10115	.09930	.11025	.10702	.11985	.11501	.12996	45
46	198	130	943	041	715	.12001	515	.13013	46
47	210	144	955	056	728	018	528	031	47
48	222	159	968	072	741	034	542	048	48
49	234	174	981	087	755	051	555	065	49
50	.09247	.10189	.09993	.11103	.10768	.12067	.11569	.13083	50
51	259	204	.10006	119	781	083	583	100	51
52	271	218	019	134	794	100	596	117	52
53	283	233	032	150	807	117	610	135	53
54	296	248	044	166	820	133	623	152	54
55	.09308	.10263	.10057	.11181	.10833	.12150	.11637	.13170	55
56	320	278	070	197	847	166	651	187	56
57	332	293	082	213	860	183	664	205	57
58	345	308	095	229	873	199	678	222	58
59	357	323	108	244	886	216	692	239	59
60	.09369	.10338	.10121	.11260	.10899	.12233	.11705	.13257	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	28° Vers.	28° Exsec.	29° Vers.	29° Exsec.	30° Vers.	30° Exsec.	31° Vers.	31° Exsec.	M.
0	.11705	.13257	.12538	.14335	.13397	.15470	.14283	.16663	0
1	719	275	552	354	412	489	298	684	1
2	733	292	566	372	427	509	313	704	2
3	746	310	580	391	441	528	328	725	3
4	760	327	594	409	456	548	343	745	4
5	.11774	.13345	.12609	.14428	.13470	.15567	.14358	.16766	5
6	787	362	623	446	485	587	373	786	6
7	801	380	637	465	499	606	388	806	7
8	815	398	651	483	514	626	403	827	8
9	828	415	665	502	529	645	418	848	9
10	.11842	.13433	.12679	.14521	.13543	.15665	.14433	.16868	10
11	856	451	694	539	558	684	449	889	11
12	870	468	708	558	573	704	464	909	12
13	883	486	722	576	587	724	479	930	13
14	897	504	736	595	602	743	494	950	14
15	.11911	.13521	.12750	.14614	.13616	.15763	.14509	.16971	15
16	925	539	765	632	631	782	524	992	16
17	938	557	779	651	646	802	539	.17012	17
18	952	575	793	670	660	822	554	033	18
19	966	593	807	689	675	841	569	054	19
20	.11980	.13610	.12822	.14707	.13690	.15861	.14584	.17075	20
21	994	628	836	726	705	881	599	095	21
22	.12007	646	850	745	719	901	615	116	22
23	021	664	864	764	734	920	630	137	23
24	035	682	879	782	749	940	645	158	24
25	.12049	.13700	.12893	.14801	.13763	.15960	.14660	.17178	25
26	063	718	907	820	778	980	675	199	26
27	077	735	921	839	793	.16000	690	220	27
28	091	753	936	858	808	019	706	241	28
29	104	771	950	877	822	039	721	262	29
30	.12118	.13789	.12964	.14896	.13837	.16059	.14736	.17283	30
31	132	807	979	914	852	079	751	304	31
32	146	825	993	933	867	099	766	325	32
33	160	843	.13007	952	881	119	782	346	33
34	174	861	022	971	896	139	797	367	34
35	.12188	.13879	.13036	.14990	.13911	.16159	.14812	.17388	35
36	202	897	051	.15009	926	179	827	409	36
37	216	915	065	028	941	199	843	430	37
38	230	934	079	047	955	219	858	451	38
39	244	952	094	066	970	239	873	472	39
40	.12257	.13970	.13108	.15085	.13985	.16259	.14888	.17493	40
41	271	988	122	105	.14000	279	904	514	41
42	285	.14006	137	124	015	299	919	535	42
43	299	024	151	143	030	319	934	556	43
44	313	042	166	162	044	339	949	577	44
45	.12327	.14061	.13180	.15181	.14059	.16359	.14965	.17598	45
46	341	079	195	200	074	380	980	620	46
47	355	097	209	219	089	400	995	641	47
48	369	115	223	239	104	420	.15011	662	48
49	383	134	238	258	119	440	026	683	49
50	.12397	.14152	.13252	.15277	.14134	.16460	.15041	.17704	50
51	411	170	267	296	149	481	057	726	51
52	425	188	281	315	164	501	072	747	52
53	439	207	296	335	179	521	087	768	53
54	454	225	310	354	194	541	103	790	54
55	.12468	.14243	.13325	.15373	.14208	.16562	.15118	.17811	55
56	482	262	339	393	223	582	134	832	56
57	496	280	354	412	238	602	149	854	57
58	510	299	368	431	253	623	164	875	58
59	524	317	383	451	268	643	180	896	59
60	.12538	.14335	.13397	.15470	.14283	.16663	.15195	.17918	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	32° Vers.	32° Exsec.	33° Vers.	33° Exsec.	34° Vers.	34° Exsec.	35° Vers.	35° Exsec.	M.
0	.15195	.17918	.16133	.19236	.17096	.20622	.18085	.22077	0
1	211	939	149	259	113	645	101	102	1
2	226	961	165	281	129	669	118	127	2
3	241	982	181	304	145	693	135	152	3
4	257	.18004	196	327	161	717	152	177	4
5	.15272	.18025	.16212	.19349	.17178	.20740	.18168	.22202	5
6	288	047	228	372	194	764	185	227	6
7	303	068	244	394	210	788	202	252	7
8	319	090	260	417	227	812	218	277	8
9	334	111	276	440	243	836	235	302	9
10	.15350	.18133	.16292	.19463	.17259	.20859	.18252	.22327	10
11	365	155	308	485	276	883	269	352	11
12	381	176	324	508	292	907	286	377	12
13	396	198	340	531	308	931	302	402	13
14	412	220	355	553	325	955	319	428	14
15	.15427	.18241	.16371	.19576	.17341	.20979	.18336	.22453	15
16	443	263	387	599	357	.21003	353	478	16
17	458	285	403	622	374	027	369	503	17
18	474	307	419	645	390	051	386	528	18
19	489	328	435	668	407	075	403	554	19
20	.15505	.18350	.16451	.19691	.17423	.21099	.18420	.22579	20
21	520	372	467	713	439	123	437	604	21
22	536	394	483	736	456	147	454	629	22
23	552	416	499	759	472	171	470	655	23
24	567	437	515	782	489	195	487	680	24
25	.15583	.18459	.16531	.19805	.17505	.21220	.18504	.22706	25
26	598	481	547	828	522	244	521	731	26
27	614	503	563	851	538	268	538	756	27
28	630	525	579	874	554	292	555	782	28
29	645	547	595	897	571	316	572	807	29
30	.15661	.18569	.16611	.19920	.17587	.21341	.13588	.22833	30
31	676	591	627	944	604	365	605	858	31
32	692	613	644	967	620	389	622	884	32
33	708	635	660	990	637	414	639	909	33
34	723	657	676	.20013	653	438	656	935	34
35	.15739	.18679	.16692	.20036	.17670	.21462	.18673	.22960	35
36	755	701	708	059	686	487	690	986	36
37	770	723	724	083	703	511	707	.23012	37
38	786	745	740	106	719	535	724	037	38
39	802	767	756	129	736	560	741	063	39
40	.15818	.18790	.16772	.20152	.17752	.21584	.18758	.23089	40
41	833	812	788	176	769	609	775	114	41
42	849	834	805	199	786	633	792	140	42
43	865	856	821	222	802	658	809	166	43
44	880	878	837	246	819	682	826	192	44
45	.15896	.18901	.16853	.20269	.17835	.21707	.18843	.23217	45
46	912	923	869	292	852	731	860	243	46
47	928	945	885	316	868	756	877	269	47
48	943	967	902	339	885	781	894	295	48
49	959	990	918	363	902	805	911	321	49
50	.15975	.19012	.16934	.20386	.17918	.21830	.18928	.23347	50
51	991	034	950	410	935	855	945	373	51
52	.16006	057	966	433	952	879	962	398	52
53	022	079	983	457	968	904	979	424	53
54	038	102	999	480	985	929	996	450	54
55	.16054	.19124	.17015	.20504	.18001	.21953	.19013	.23476	55
56	070	146	031	527	018	978	030	502	56
57	085	169	047	551	035	.22003	047	529	57
58	101	191	064	575	051	028	064	555	58
59	117	214	080	598	068	053	081	581	59
60	.16133	.19236	.17096	.20622	.18085	.22077	.19098	.23607	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—
(Continued)

M.	36° Vers.	36° Exsec.	37° Vers.	37° Exsec.	38° Vers.	38° Exsec.	39° Vers.	39° Exsec.	M.
0	.19098	.23607	.20136	.25214	.21199	.26902	.22285	.28676	0
1	115	633	154	241	217	931	304	706	1
2	133	659	171	269	235	960	322	737	2
3	150	685	189	296	253	988	340	767	3
4	167	711	207	324	271	.27017	359	797	4
5	.19184	.23738	.20224	.25351	.21289	.27046	.22377	.28828	5
6	201	764	242	379	306	075	395	858	6
7	218	790	259	406	324	104	414	889	7
8	235	816	277	434	342	133	432	919	8
9	252	843	294	462	360	162	450	950	9
10	.19270	.23869	.20312	.25489	.21378	.27191	.22469	.28980	10
11	287	895	329	517	396	221	487	.29011	11
12	304	922	347	545	414	250	506	042	12
13	321	948	365	572	432	279	524	072	13
14	338	975	382	600	450	308	542	103	14
15	.19356	.24001	.20400	.25628	.21468	.27337	.22561	.29133	15
16	373	028	417	656	486	366	579	164	16
17	390	054	435	683	504	396	598	195	17
18	407	081	453	711	522	425	616	226	18
19	424	107	470	739	540	454	634	256	19
20	.19442	.24134	.20488	.25767	.21558	.27483	.22653	.29287	20
21	459	160	506	795	576	513	671	318	21
22	476	187	523	823	595	542	690	349	22
23	493	213	541	851	613	572	708	380	23
24	511	240	559	879	631	601	727	411	24
25	.19528	.24267	.20576	.25907	.21649	.27630	.22745	.29442	25
26	545	293	594	935	667	660	764	473	26
27	562	320	612	963	685	689	782	504	27
28	580	347	629	991	703	719	801	535	28
29	597	373	647	.26019	721	748	819	566	29
30	.19614	.24400	.20665	.26047	.21739	.27778	.22838	.29597	30
31	632	427	682	075	757	807	856	628	31
32	649	454	700	104	775	837	875	659	32
33	666	481	718	132	794	867	893	690	33
34	684	508	736	160	812	896	912	721	34
35	.19701	.24534	.20753	.26188	.21830	.27926	.22930	.29752	35
36	718	561	771	216	848	956	949	784	36
37	736	588	789	245	866	985	967	815	37
38	753	615	807	273	884	.28015	986	846	38
39	770	642	824	301	902	045	.23004	877	39
40	.19788	.24669	.20842	.26330	.21921	.28075	.23023	.29909	40
41	805	696	860	358	939	105	041	940	41
42	822	723	878	387	957	134	060	971	42
43	840	750	895	415	975	164	079	.30003	43
44	857	777	913	443	993	194	097	034	44
45	.19875	.24804	.20931	.26472	.22012	.28224	.23116	.30066	45
46	892	832	949	500	030	254	134	097	46
47	909	859	967	529	048	284	153	129	47
48	927	886	984	557	066	314	172	160	48
49	944	913	.21002	586	084	344	190	192	49
50	.19962	.24940	.21020	.26615	.22103	.28374	.23209	.30223	50
51	979	967	038	643	121	404	228	255	51
52	997	995	056	672	139	434	246	287	52
53	.20014	.25022	074	701	157	464	265	318	53
54	032	049	092	729	176	495	283	350	54
55	.20049	.25077	.21109	.26758	.22194	.28525	.23302	.30382	55
56	066	104	127	787	212	555	321	413	56
57	084	131	145	815	231	585	339	445	57
58	101	159	163	844	249	615	358	477	58
59	119	186	181	873	267	646	377	509	59
60	.20136	.25214	.21199	.26902	.22285	.28676	.23396	.30541	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	40° Vers.	40° Exsec.	41° Vers.	41° Exsec.	42° Vers.	42° Exsec.	43° Vers.	43° Exsec.	M.
0	.23396	.30541	.24529	.32501	.25686	.34563	.26865	.36733	0
1	414	573	548	535	705	599	884	770	1
2	433	605	567	568	724	634	904	807	2
3	452	636	586	602	744	669	924	844	3
4	470	668	605	636	763	704	944	881	4
5	.23489	.30700	.24625	.32669	.25783	.34740	.26964	.36919	5
6	508	732	644	703	802	775	984	956	6
7	527	764	663	737	822	811	.27004	993	7
8	545	796	682	770	841	846	024	.37030	8
9	564	829	701	804	861	882	043	068	9
10	.23583	.30861	.24720	.32838	.25880	.34917	.27063	.37105	10
11	602	893	739	872	900	953	083	143	11
12	620	925	759	905	920	988	103	180	12
13	639	957	778	939	939	.35024	123	218	13
14	658	989	797	973	959	060	143	255	14
15	.23677	.31022	.24816	.33007	.25978	.35095	.27163	.37293	15
16	696	054	835	041	998	131	183	330	16
17	714	086	854	075	.26017	167	203	368	17
18	733	119	874	109	037	203	223	406	18
19	752	151	893	143	056	238	243	443	19
20	.23771	.31183	.24912	.33177	.26076	.35274	.27263	.37481	20
21	790	216	931	211	096	310	283	519	21
22	808	248	950	245	115	346	303	556	22
23	827	281	970	279	135	382	323	594	23
24	846	313	989	314	154	418	343	632	24
25	.23865	.31346	.25008	.33348	.26174	.35454	.27363	.37670	25
26	884	378	027	382	194	490	383	708	26
27	903	411	047	416	213	526	403	746	27
28	922	443	066	451	233	562	423	784	28
29	941	476	085	485	253	598	443	822	29
30	.23959	.31509	.25104	.33519	.26272	.35634	.27463	.37860	30
31	978	541	124	554	292	670	483	898	31
32	997	574	143	588	312	707	503	936	32
33	.24016	607	162	622	331	743	523	974	33
34	035	640	182	657	351	779	543	.38012	34
35	.24054	.31672	.25201	.33691	.26371	.35815	.27563	.38051	35
36	073	705	220	726	390	852	583	089	36
37	092	738	240	760	410	888	603	127	37
38	111	771	259	795	430	924	623	165	38
39	130	804	278	830	449	961	643	204	39
40	.24149	.31837	.25297	.33864	.26469	.35997	.27663	.38242	40
41	168	870	317	899	489	.36034	683	280	41
42	187	903	336	934	509	070	703	319	42
43	206	936	356	968	528	107	723	357	43
44	225	969	375	.34003	548	143	743	396	44
45	.24244	.32002	.25394	.34038	.26568	.36180	.27764	.38434	45
46	262	035	414	073	587	217	784	473	46
47	281	068	433	108	607	253	804	512	47
48	300	101	452	142	627	290	824	550	48
49	320	134	472	177	647	327	844	589	49
50	.24339	.32168	.25491	.34212	.26667	.36363	.27864	.38628	50
51	358	201	511	247	686	400	884	666	51
52	377	234	530	282	706	437	905	705	52
53	396	267	549	317	726	474	925	744	53
54	415	301	569	352	746	511	945	783	54
55	.24434	.32334	.25588	.34387	.26766	.36548	.27965	.38822	55
56	453	368	608	423	785	585	985	860	56
57	472	401	627	458	805	622	.28005	899	57
58	491	434	647	493	825	659	026	938	58
59	510	468	666	528	845	696	046	977	59
60	.24529	.32501	.25686	.34563	.26865	.36733	.28066	.39016	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	44°		45°		46°		47°		M.
	Vers.	Exsec.	Vers.	Exsec.	Vers.	Exsec.	Vers.	Exsec.	
0	.28066	.39016	.29289	.41421	.30534	.43956	.31800	.46628	0
1	086	055	310	463	555	999	821	674	1
2	106	095	330	504	576	.44042	843	719	2
3	127	134	351	545	597	086	864	765	3
4	147	173	372	586	618	129	885	811	4
5	.28167	.39212	.29392	.41627	.30639	.44173	.31907	.46857	5
6	187	251	413	669	660	217	928	903	6
7	208	291	433	710	681	260	949	949	7
8	228	330	454	752	702	304	971	995	8
9	248	369	475	793	723	347	992	.47041	9
10	.28268	.39409	.29495	.41835	.30744	.44391	.32013	.47087	10
11	289	448	516	876	765	435	035	134	11
12	309	487	537	918	786	479	056	180	12
13	329	527	557	959	807	523	077	226	13
14	350	566	578	.42001	828	567	099	272	14
15	.28370	.39606	.29599	.42042	.30849	.44610	.32120	.47319	15
16	390	646	619	084	870	654	141	365	16
17	410	685	640	126	891	698	163	411	17
18	431	725	661	168	912	742	184	458	18
19	451	764	681	209	933	787	205	504	19
20	.28471	.39804	.29702	.42251	.30954	.44831	.32227	.47551	20
21	492	844	723	293	975	875	248	598	21
22	512	884	743	335	996	919	270	644	22
23	532	924	764	377	.31017	963	291	691	23
24	553	963	785	419	038	.45007	312	738	24
25	.28573	.40003	.29805	.42461	.31059	.45052	.32334	.47784	25
26	593	043	826	503	080	096	355	831	26
27	614	083	847	545	101	141	377	878	27
28	634	123	868	587	122	185	398	925	28
29	655	163	888	630	143	229	420	972	29
30	.28675	.40203	.29909	.42672	.31165	.45274	.32441	.48019	30
31	695	243	930	714	186	319	462	066	31
32	716	283	951	756	207	363	484	113	32
33	736	324	971	799	228	408	505	160	33
34	757	364	992	841	249	452	527	207	34
35	.28777	.40404	.30013	.42883	.31270	.45497	.32548	.48254	35
36	797	444	034	926	291	542	570	301	36
37	818	485	054	968	312	587	591	349	37
38	838	525	075	.43011	334	631	613	396	38
39	859	565	096	053	355	676	634	443	39
40	.28879	.40606	.30117	.43096	.31376	.45721	.32656	.48491	40
41	900	646	138	139	397	766	677	538	41
42	920	687	158	181	418	811	699	586	42
43	941	727	179	224	439	856	720	633	43
44	961	768	200	267	461	901	742	681	44
45	.28981	.40808	.30221	.43309	.31482	.45946	.32763	.48728	45
46	.29002	849	242	352	503	992	785	776	46
47	022	890	263	395	524	.46037	806	824	47
48	043	930	283	438	545	082	828	871	48
49	063	971	304	481	566	127	849	919	49
50	.29084	.41012	.30325	.43524	.31588	.46173	.32871	.48967	50
51	104	053	346	567	609	218	893	.49015	51
52	125	093	367	610	630	263	914	063	52
53	145	134	388	653	651	309	936	111	53
54	166	175	409	696	673	354	957	159	54
55	.29187	.41216	.30430	.43739	.31694	.46400	.32979	.49207	55
56	207	257	451	783	715	445	.33001	255	56
57	228	298	471	826	736	491	022	303	57
58	248	339	492	869	758	537	044	351	58
59	269	380	513	912	779	582	065	399	59
60	.29289	.41421	.30534	.43956	.31800	.46628	.33087	.49448	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	48°		49°		50°		51°		M.
	Vers.	Exsec.	Vers.	Exsec.	Vers.	Exsec.	Vers.	Exsec.	
0	.33087	.49448	.34394	.52425	.35721	.55572	.37068	.58902	0
1	109	496	416	476	743	626	091	959	1
2	130	544	438	527	766	680	113	.59016	2
3	152	593	460	579	788	734	136	073	3
4	173	641	482	630	810	789	158	130	4
5	.33195	.49690	.34504	.52681	.35833	.55843	.37181	.59188	5
6	217	738	526	732	855	897	204	245	6
7	238	787	548	784	877	951	226	302	7
8	260	835	570	835	900	.56005	249	360	8
9	282	884	592	886	922	060	272	418	9
10	.33303	.49933	.34614	.52938	.35944	.56114	.37294	.59475	10
11	325	981	636	989	967	169	317	533	11
12	347	.50030	658	.53041	989	223	340	590	12
13	368	079	680	092	.36011	278	362	648	13
14	390	128	702	144	034	332	385	706	14
15	.33412	.50177	.34724	.53196	.36056	.56387	.37408	.59764	15
16	434	226	746	247	078	442	430	822	16
17	455	275	768	299	101	497	453	880	17
18	477	324	790	351	123	551	476	938	18
19	499	373	812	403	146	606	498	996	19
20	.33520	.50422	.34834	.53455	.36168	.56661	.37521	.60054	20
21	542	471	856	507	190	716	544	112	21
22	564	521	878	559	213	771	567	171	22
23	586	570	900	611	235	826	589	229	23
24	607	619	923	663	258	881	612	287	24
25	.33629	.50669	.34945	.53715	.36280	.56937	.37635	.60346	25
26	651	718	967	768	302	992	658	404	26
27	673	767	989	820	325	.57047	680	463	27
28	694	817	.35011	872	347	103	703	521	28
29	716	866	033	924	370	158	726	580	29
30	.33738	.50916	.35055	.53977	.36392	.57213	.37749	.60639	30
31	760	966	077	.54029	415	269	771	698	31
32	782	.51015	099	082	437	324	794	756	32
33	803	065	122	134	460	380	817	815	33
34	825	115	144	187	482	436	840	874	34
35	.33847	.51165	.35166	.54240	.36504	.57491	.37862	.60933	35
36	869	215	188	292	527	547	885	992	36
37	891	265	210	345	549	603	908	.61051	37
38	912	314	232	398	572	659	931	111	38
39	934	364	254	451	594	715	954	170	39
40	.33956	.51415	.35277	.54504	.36617	.57771	.37976	.61229	40
41	978	465	299	557	639	827	999	288	41
42	.34000	515	321	610	662	883	.38022	348	42
43	022	565	343	663	684	939	045	407	43
44	044	615	365	716	707	995	068	467	44
45	.34065	.51665	.35388	.54769	.36729	.58051	.38091	.61526	45
46	087	716	410	822	752	108	113	586	46
47	109	766	432	876	775	164	136	646	47
48	131	817	454	929	797	221	159	705	48
49	153	867	476	982	820	277	182	765	49
50	.34175	.51918	.35499	.55036	.36842	.58333	.38205	.61825	50
51	197	968	521	089	865	390	228	885	51
52	219	.52019	543	143	887	447	251	945	52
53	241	069	565	196	910	503	274	.62005	53
54	262	120	588	250	932	560	296	065	54
55	.34284	.52171	.35610	.55303	.36955	.58617	.38319	.62125	55
56	306	222	632	357	978	674	342	185	56
57	328	273	654	411	.37000	731	365	246	57
58	350	323	677	465	023	788	388	306	58
59	372	374	699	518	045	845	411	366	59
60	.34394	.52425	.35721	.55572	.37068	.58902	.38434	.62427	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	52° Vers.	52° Exsec.	53° Vers.	53° Exsec.	54° Vers.	54° Exsec.	55° Vers.	55° Exsec.	M.
0	.38434	.62427	.39818	.66164	.41221	.70130	.42642	.74345	0
1	457	487	842	228	245	198	666	417	1
2	480	548	865	292	269	267	690	490	2
3	503	609	888	357	292	335	714	562	3
4	526	669	911	421	316	403	738	635	4
5	.38549	.62730	.39935	.66486	.41339	.70472	.42762	.74708	5
6	571	791	958	550	363	540	785	781	6
7	594	852	981	615	386	609	809	854	7
8	617	913	.40005	679	410	677	833	927	8
9	640	974	028	744	433	746	857	.75000	9
10	.38663	.63035	.40051	.66809	.41457	.70815	.42881	.75073	10
11	686	096	074	873	481	884	905	146	11
12	709	157	098	938	504	953	929	219	12
13	732	218	121	.67003	528	.71022	953	293	13
14	755	279	144	068	551	091	976	366	14
15	.38778	.63341	.40168	.67133	.41575	.71160	.43000	.75440	15
16	801	402	191	198	599	229	024	513	16
17	824	464	214	264	622	298	048	587	17
18	847	525	237	329	646	368	072	661	18
19	870	587	261	394	670	437	096	734	19
20	.38893	.63648	.40284	.67460	.41693	.71506	.43120	.75808	20
21	916	710	307	525	717	576	144	882	21
22	939	772	331	591	740	646	168	956	22
23	962	834	354	656	764	715	192	.76031	23
24	985	895	378	722	788	785	216	105	24
25	.39009	.63957	.40401	.67788	.41811	.71855	.43240	.76179	25
26	032	.64019	424	853	835	925	264	253	26
27	055	081	448	919	859	995	287	328	27
28	078	144	471	985	882	.72065	311	402	28
29	101	206	494	.68051	906	135	335	477	29
30	.39124	.64268	.40518	.68117	.41930	.72205	.43359	.76552	30
31	147	330	541	183	953	275	383	626	31
32	170	393	564	250	977	346	407	701	32
33	193	455	588	316	.42001	416	431	776	33
34	216	518	611	382	024	487	455	851	34
35	.39239	.64580	.40635	.68449	.42048	.72557	.43479	.76926	35
36	262	643	658	515	072	628	503	.77001	36
37	286	705	682	582	096	698	527	077	37
38	309	768	705	648	119	769	551	152	38
39	332	831	728	715	143	840	575	227	39
40	.39355	.64894	.40752	.68782	.42167	.72911	.43599	.77303	40
41	378	957	775	848	190	982	623	378	41
42	401	.65020	799	915	214	.73053	647	454	42
43	424	083	822	982	238	124	671	530	43
44	447	146	846	.69049	262	195	695	606	44
45	.39471	.65209	.40869	.69116	.42285	.73267	.43720	.77681	45
46	494	272	892	183	309	338	744	757	46
47	517	335	916	250	333	409	768	833	47
48	540	399	939	318	357	481	792	910	48
49	563	462	963	385	381	552	816	986	49
50	.39586	.65526	.40986	.69452	.42404	.73624	.43840	.78062	50
51	610	589	.41010	520	428	696	864	138	51
52	633	653	033	587	452	768	888	215	52
53	656	717	057	655	476	840	912	291	53
54	679	780	080	723	499	911	936	368	54
55	.39702	.65844	.41104	.69790	.42523	.73983	.43960	.78445	55
56	726	908	127	858	547	.74056	984	521	56
57	749	972	151	926	571	128	.44008	598	57
58	772	.66036	174	994	595	200	032	675	58
59	795	100	198	.70062	619	272	057	752	59
60	.39818	.66164	.41221	.70130	.42642	.74345	.44081	.78829	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	56° Vers.	56° Exsec.	57° Vers.	57° Exsec.	58° Vers.	58° Exsec.	59° Vers.	59° Exsec.	M.
0	.44081	.78829	.45536	.83608	.47008	.88708	.48496	.94160	0
1	105	906	560	690	033	796	521	254	1
2	129	984	585	773	057	884	546	349	2
3	153	.79061	609	855	082	972	571	443	3
4	177	138	634	938	107	.89060	596	537	4
5	.44201	.79216	.45658	.84020	.47131	.89148	.48621	.94632	5
6	225	293	683	103	156	237	646	726	6
7	250	371	707	186	181	325	671	821	7
8	274	449	731	269	206	414	696	916	8
9	298	527	756	352	230	503	721	.95011	9
10	.44322	.79604	.45780	.84435	.47255	.89591	.48746	.95106	10
11	346	682	805	518	280	680	771	201	11
12	370	761	829	601	304	769	796	296	12
13	395	839	854	685	329	858	821	392	13
14	419	917	878	768	354	948	846	487	14
15	.44443	.79995	.45903	.84852	.47379	.90037	.48871	.95583	15
16	467	.80074	927	935	403	126	896	678	16
17	491	152	951	.85019	428	216	921	774	17
18	516	231	976	103	453	305	946	870	18
19	540	309	.46000	187	478	395	971	966	19
20	.44564	.80388	.46025	.85271	.47502	.90485	.48996	.96062	20
21	588	467	049	355	527	575	.49021	158	21
22	612	546	074	439	552	665	046	255	22
23	637	625	098	523	577	755	071	351	23
24	661	704	123	608	601	845	096	448	24
25	.44685	.80783	.46147	.85692	.47626	.90935	.49121	.96544	25
26	709	862	172	777	651	.91026	146	641	26
27	734	942	196	861	676	116	171	738	27
28	758	.81021	221	946	701	207	196	835	28
29	782	101	246	.86031	725	297	221	932	29
30	.44806	.81180	.46270	.86116	.47750	.91388	.49246	.97029	30
31	831	260	295	201	775	479	271	127	31
32	855	340	319	286	800	570	296	224	32
33	879	419	344	371	825	661	321	322	33
34	903	499	368	457	849	752	346	420	34
35	.44928	.81579	.46393	.86542	.47874	.91844	.49372	.97517	35
36	952	659	417	627	899	935	397	615	36
37	976	740	442	713	924	.92027	422	713	37
38	.45001	820	466	799	949	118	447	811	38
39	025	900	491	885	974	210	472	910	39
40	.45049	.81981	.46516	.86970	.47998	.92302	.49497	.98008	40
41	073	.82061	540	.87056	.48023	394	522	107	41
42	098	142	565	142	048	486	547	205	42
43	122	222	589	229	073	578	572	304	43
44	146	303	614	315	098	670	597	403	44
45	.45171	.82384	.46639	.87401	.48123	.92762	.49623	.98502	45
46	195	465	663	488	148	855	648	601	46
47	219	546	688	574	172	947	673	700	47
48	244	627	712	661	197	.93040	698	799	48
49	268	709	737	748	222	133	723	899	49
50	.45292	.82790	.46762	.87834	.48247	.93226	.49748	.98998	50
51	317	871	786	921	272	319	773	.99098	51
52	341	953	811	.88008	297	412	799	198	52
53	365	.83034	836	095	322	505	824	298	53
54	390	116	860	183	347	598	849	398	54
55	.45414	.83198	.46885	.88270	.48372	.93692	.49874	.99498	55
56	439	280	909	357	396	785	899	598	56
57	463	362	934	445	421	879	924	698	57
58	487	444	959	532	446	973	950	799	58
59	512	526	983	620	471	.94066	975	899	59
60	.45536	.83608	.47008	.88708	.48496	.94160	.50000	1.00000	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS
(*Continued*)

M.	60° Vers.	60° Exsec.	61° Vers.	61° Exsec.	62° Vers.	62° Exsec.	63° Vers.	63° Exsec.	M.
0	.50000	1.00000	.51519	1.06267	.53053	1.13005	.54601	1.20269	0
1	025	.00101	544	.06375	079	.13122	627	.20395	1
2	050	.00202	570	.06483	104	.13239	653	.20521	2
3	076	.00303	595	.06592	130	.13356	679	.20647	3
4	101	.00404	621	.06701	156	.13473	705	.20773	4
5	.50126	1.00505	.51646	1.06809	.53181	1.13590	.54731	1.20900	5
6	151	.00607	672	.06918	207	.13707	757	.21026	6
7	176	.00708	697	.07027	233	.13825	782	.21153	7
8	202	.00810	723	.07137	258	.13942	808	.21280	8
9	227	.00912	748	.07246	284	.14060	834	.21407	9
10	.50252	1.01014	.51774	1.07356	.53310	1.14178	.54860	1.21535	10
11	277	.01116	799	.07465	336	.14296	886	.21662	11
12	303	.01218	825	.07575	361	.14414	912	.21790	12
13	328	.01320	850	.07685	387	.14533	938	.21918	13
14	353	.01422	876	.07795	413	.14651	964	.22045	14
15	.50378	1.01525	.51901	1.07905	.53439	1.14770	.54990	1.22174	15
16	404	.01628	927	.08015	464	.14889	.55016	.22302	16
17	429	.01730	952	.08126	490	.15008	042	.22430	17
18	454	.01833	978	.08236	516	.15127	068	.22559	18
19	479	.01936	.52003	.08347	542	.15246	094	.22688	19
20	.50505	1.02039	.52029	1.08458	.53567	1.15366	.55120	1.22817	20
21	530	.02143	054	.08569	593	.15485	146	.22946	21
22	555	.02246	080	.08680	619	.15605	172	.23075	22
23	581	.02349	105	.08791	645	.15725	198	.23205	23
24	606	.02453	131	.08903	670	.15845	224	.23334	24
25	.50631	1.02557	.52156	1.09014	.53696	1.15965	.55250	1.23464	25
26	656	.02661	182	.09126	722	.16085	276	.23594	26
27	682	.02765	207	.09238	748	.16206	302	.23724	27
28	707	.02869	233	.09350	774	.16326	328	.23855	28
29	732	.02973	259	.09462	799	.16447	354	.23985	29
30	.50758	1.03077	.52284	1.09574	.53825	1.16568	.55380	1.24116	30
31	783	.03182	310	.09686	851	.16689	406	.24247	31
32	808	.03286	335	.09799	877	.16810	432	.24378	32
33	834	.03391	361	.09911	903	.16932	458	.24509	33
34	859	.03496	386	.10024	928	.17053	484	.24640	34
35	.50884	1.03601	.52412	1.10137	.53954	1.17175	.55510	1.24772	35
36	910	.03706	438	.10250	980	.17297	536	.24903	36
37	935	.03811	463	.10363	.54006	.17419	563	.25035	37
38	960	.03916	489	.10477	032	.17541	589	.25167	38
39	986	.04022	514	.10590	058	.17663	615	.25300	39
40	.51011	1.04128	.52540	1.10704	.54083	1.17786	.55641	1.25432	40
41	036	.04233	566	.10817	109	.17909	667	.25565	41
42	062	.04339	591	.10931	135	.18031	693	.25697	42
43	087	.04445	617	.11045	161	.18154	719	.25830	43
44	112	.04551	642	.11159	187	.18277	745	.25963	44
45	.51138	1.04657	.52668	1.11274	.54213	1.18401	.55771	1.26097	45
46	163	.04764	694	.11388	238	.18524	797	.26230	46
47	189	.04870	719	.11503	264	.18648	823	.26364	47
48	214	.04977	745	.11617	290	.18772	849	.26498	48
49	239	.05084	771	.11732	316	.18895	876	.26632	49
50	.51265	1.05191	.52796	1.11847	.54342	1.19019	.55902	1.26766	50
51	290	.05298	822	.11963	368	.19144	928	.26900	51
52	316	.05405	847	.12078	394	.19268	954	.27035	52
53	341	.05512	873	.12193	420	.19393	980	.27169	53
54	366	.05619	899	.12309	446	.19517	.56006	.27304	54
55	.51392	1.05727	.52924	1.12425	.54471	1.19642	.56032	1.27439	55
56	417	.05835	950	.12540	497	.19767	058	.27574	56
57	443	.05942	976	.12657	523	.19892	084	.27710	57
58	468	.06050	.53001	.12773	549	.20018	111	.27845	58
59	494	.06158	027	.12889	575	.20143	137	.27981	59
60	.51519	1.06267	.53053	1.13005	.54601	1.20269	.56163	1.28117	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	64° Vers.	64° Exsec.	65° Vers.	65° Exsec.	66° Vers.	66° Exsec.	67° Vers.	67° Exsec.	M.
0	.56163	1.28117	.57738	1.36620	.59326	1.45859	.60927	1.55930	0
1	189	.28253	765	.36768	353	.46020	954	.56106	1
2	215	.28390	791	.36916	379	.46181	980	.56282	2
3	241	.28526	817	.37064	406	.46342	.61007	.56458	3
4	267	.28663	844	.37212	433	.46504	034	.56634	4
5	.56294	1.28800	.57870	1.37361	.59459	1.46665	.61061	1.56811	5
6	320	.28937	896	.37509	486	.46827	088	.56988	6
7	346	.29074	923	.37658	512	.46989	114	.57165	7
8	372	.29211	949	.37808	539	.47152	141	.57342	8
9	398	.29349	976	.37957	566	.47314	168	.57520	9
10	.56425	1.29487	.58002	1.38107	.59592	1.47477	.61195	1.57698	10
11	451	.29625	028	.38256	619	.47640	222	.57876	11
12	477	.29763	055	.38406	645	.47804	248	.58054	12
13	503	.29901	081	.38556	672	.47967	275	.58233	13
14	529	.30040	108	.38707	699	.48131	302	.58412	14
15	.56555	1.30179	.58134	1.38857	.59725	1.48295	.61329	1.58591	15
16	582	.30318	160	.39008	752	.48459	356	.58771	16
17	608	.30457	187	.39159	779	.48624	383	.58950	17
18	634	.30596	213	.39311	805	.48789	409	.59130	18
19	660	.30735	240	.39462	832	.48954	436	.59311	19
20	.56687	1.30875	.58266	1.39614	.59859	1.49119	.61463	1.59491	20
21	713	.31015	293	.39766	885	.49284	490	.59672	21
22	739	.31155	319	.39918	912	.49450	517	.59853	22
23	765	.31295	345	.40070	938	.49616	544	.60035	23
24	791	.31436	372	.40222	965	.49782	570	.60217	24
25	.56818	1.31576	.58398	1.40375	.59992	1.49948	.61597	1.60399	25
26	844	.31717	425	.40528	.60018	.50115	624	.60581	26
27	870	.31858	451	.40681	045	.50282	651	.60763	27
28	896	.31999	478	.40835	072	.50449	678	.60946	28
29	923	.32140	504	.40988	098	.50617	705	.61129	29
30	.56949	1.32282	.58531	1.41142	.60125	1.50784	.61732	1.61313	30
31	975	.32424	557	.41296	152	.50952	759	.61496	31
32	.57001	.32566	584	.41450	178	.51120	785	.61680	32
33	028	.32708	610	.41605	205	.51289	812	.61864	33
34	054	.32850	637	.41760	232	.51457	839	.62049	34
35	.57080	1.32993	.58663	1.41914	.60259	1.51626	.61866	1.62234	35
36	106	.33135	690	.42070	285	.51795	893	.62419	36
37	133	.33278	716	.42225	312	.51965	920	.62604	37
38	159	.33422	743	.42380	339	.52134	947	.62790	38
39	185	.33565	769	.42536	365	.52304	974	.62976	39
40	.57212	1.33708	.58796	1.42692	.60392	1.52474	.62001	1.63162	40
41	238	.33852	822	.42848	419	.52645	027	.63348	41
42	264	.33996	849	.43005	445	.52815	054	.63535	42
43	291	.34140	875	.43162	472	.52986	081	.63722	43
44	317	.34284	902	.43318	499	.53157	108	.63909	44
45	.57343	1.34429	.58928	1.43476	.60526	1.53329	.62135	1.64097	45
46	369	.34573	955	.43633	552	.53500	162	.64285	46
47	396	.34718	981	.43790	579	.53672	189	.64473	47
48	422	.34863	.59008	.43948	606	.53845	216	.64662	48
49	448	.35009	034	.44106	633	.54017	243	.64851	49
50	.57475	1.35154	.59061	1.44264	.60659	1.54190	.62270	1.65040	50
51	501	.35300	087	.44423	686	.54363	297	.65229	51
52	527	.35446	114	.44582	713	.54536	324	.65419	52
53	554	.35592	140	.44741	740	.54709	351	.65609	53
54	580	.35738	167	.44900	766	.54883	378	.65799	54
55	.57606	1.35885	.59194	1.45059	.60793	1.55057	.62405	1.65989	55
56	633	.36031	220	.45219	820	.55231	431	.66180	56
57	659	.36178	247	.45378	847	.55405	458	.66371	57
58	685	.36325	273	.45539	873	.55580	485	.66563	58
59	712	.36473	300	.45699	900	.55755	512	.66755	59
60	.57738	1.36620	.59326	1.45859	.60927	1.55930	.62539	1.66947	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	68°		69°		70°		71°		M.
	Vers.	Exsec.	Vers.	Exsec.	Vers.	Exsec.	Vers.	Exsec.	
0	.62539	1.66947	.64163	1.79043	.65798	1.92380	.67443	2.07155	0
1	566	.67139	190	.79254	825	.92614	471	.07415	1
2	593	.67332	218	.79466	853	.92849	498	.07675	2
3	620	.67525	245	.79679	880	.93083	526	.07936	3
4	647	.67718	272	.79891	907	.93318	553	.08197	4
5	.62674	1.67911	.64299	1.80104	.65935	1.93554	.67581	2.08459	5
6	701	.68105	326	.80318	962	.93790	608	.08721	6
7	728	.68299	353	.80531	989	.94026	636	.08983	7
8	755	.68494	381	.80746	.66017	.94263	663	.09246	8
9	782	.68689	408	.80960	044	.94500	691	.09510	9
10	.62809	1.68884	.64435	1.81175	.66071	1.94737	.67718	2.09774	10
11	836	.69079	462	.81390	099	.94975	746	.10038	11
12	863	.69275	489	.81605	126	.95213	773	.10303	12
13	890	.69471	516	.81821	154	.95452	801	.10568	13
14	917	.69667	544	.82037	181	.95691	829	.10834	14
15	.62944	1.69864	.64571	1.82254	.66208	1.95931	.67856	2.11101	15
16	971	.70061	598	.82471	236	.96171	884	.11367	16
17	998	.70258	625	.82688	263	.96411	911	.11635	17
18	.63025	.70455	653	.82906	290	.96652	939	.11903	18
19	052	.70653	680	.83124	318	.96893	966	.12171	19
20	.63079	1.70851	.64707	1.83342	.66345	1.97135	.67994	2.12440	20
21	106	.71050	734	.83561	373	.97377	.68021	.12709	21
22	133	.71249	761	.83780	400	.97619	049	.12979	22
23	161	.71448	789	.83999	427	.97862	077	.13249	23
24	188	.71647	816	.84219	455	.98106	104	.13520	24
25	.63215	1.71847	.64843	1.84439	.66482	1.98349	.68132	2.13791	25
26	242	.72047	870	.84659	510	.98594	159	.14063	26
27	269	.72247	898	.84880	537	.98838	187	.14335	27
28	296	.72448	925	.85102	564	.99083	214	.14608	28
29	323	.72649	952	.85323	592	.99329	242	.14881	29
30	.63350	1.72850	.64979	1.85545	.66619	1.99574	.68270	2.15155	30
31	377	.73052	.65007	.85767	647	.99821	297	.15429	31
32	404	.73254	034	.85990	674	2.00067	325	.15704	32
33	431	.73456	061	.86213	702	.00315	352	.15979	33
34	458	.73659	088	.86437	729	.00562	380	.16255	34
35	.63485	1.73862	.65116	1.86661	.66756	2.00810	.68407	2.16531	35
36	512	.74065	143	.86885	784	.01059	435	.16808	36
37	539	.74269	170	.87109	811	.01308	463	.17085	37
38	566	.74473	197	.87334	839	.01557	490	.17363	38
39	594	.74677	225	.87560	866	.01807	518	.17641	39
40	.63621	1.74881	.65252	1.87785	.66894	2.02057	.68546	2.17920	40
41	648	.75086	279	.88011	921	.02308	573	.18199	41
42	675	.75292	306	.88238	949	.02559	601	.18479	42
43	702	.75497	334	.88465	976	.02810	628	.18759	43
44	729	.75703	361	.88692	.67003	.03062	656	.19040	44
45	.63756	1.75909	.65388	1.88920	.67031	2.03315	.68684	2.19322	45
46	783	.76116	416	.89148	058	.03568	711	.19604	46
47	810	.76323	443	.89376	086	.03821	739	.19886	47
48	838	.76530	470	.89605	113	.04075	767	.20169	48
49	865	.76737	497	.89834	141	.04329	794	.20453	49
50	.63892	1.76945	.65525	1.90063	.67168	2.04584	.68822	2.20737	50
51	919	.77154	552	.90293	196	.04839	849	.21021	51
52	946	.77362	579	.90524	223	.05094	877	.21306	52
53	973	.77571	607	.90754	251	.05350	905	.21592	53
54	.64000	.77780	634	.90986	278	.05607	932	.21878	54
55	.64027	1.77990	.65661	1.91217	.67306	2.05864	.68960	2.22165	55
56	055	.78200	689	.91449	333	.06121	988	.22452	56
57	082	.78410	716	.91681	361	.06379	.69015	.22740	57
58	109	.78621	743	.91914	388	.06637	043	.23028	58
59	136	.78832	771	.92147	416	.06896	071	.23317	59
60	.64163	1.79043	.65798	1.92380	.67443	2.07155	.69098	2.23607	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	72° Vers.	72° Exsec.	73° Vers.	73° Exsec.	74° Vers.	74° Exsec.	75° Vers.	75° Exsec.	M.
0	.69098	2.23607	.70763	2.42030	.72436	2.62796	.74118	2.86370	0
1	126	.23897	791	.42356	464	.63164	146	.86790	1
2	154	.24187	818	.42683	492	.63533	174	.87211	2
3	181	.24478	846	.43010	520	.63903	202	.87633	3
4	209	.24770	874	.43337	548	.64274	231	.88056	4
5	.69237	2.25062	.70902	2.43666	.72576	2.64645	.74259	2.88479	5
6	264	.25355	930	.43995	604	.65018	287	.88904	6
7	292	.25648	958	.44324	632	.65391	315	.89330	7
8	320	.25942	985	.44655	660	.65765	343	.89756	8
9	347	.26237	.71013	.44986	688	.66140	371	.90184	9
10	.69375	2.26531	.71041	2.45317	.72716	2.66515	.74399	2.90613	10
11	403	.26827	069	.45650	744	.66892	427	.91042	11
12	430	.27123	097	.45983	772	.67269	455	.91473	12
13	458	.27420	125	.46316	800	.67647	484	.91904	13
14	486	.27717	153	.46651	828	.68025	512	.92337	14
15	.69514	2.28015	.71180	2.46986	.72856	2.68405	.74540	2.92770	15
16	541	.28313	208	.47321	884	.68785	568	.93204	16
17	569	.28612	236	.47658	912	.69167	596	.93640	17
18	597	.28912	264	.47995	940	.69549	624	.94076	18
19	624	.29212	292	.48333	968	.69931	652	.94514	19
20	.69652	2.29512	.71320	2.48671	.72996	2.70315	.74680	2.94952	20
21	680	.29814	348	.49010	.73024	.70700	709	.95392	21
22	708	.30115	375	.49350	052	.71085	737	.95832	22
23	735	.30418	403	.49691	080	.71471	765	.96274	23
24	763	.30721	431	.50032	108	.71858	793	.96716	24
25	.69791	2.31024	.71459	2.50374	.73136	2.72246	.74821	2.97160	25
26	818	.31328	487	.50716	164	.72635	849	.97604	26
27	846	.31633	515	.51060	192	.73024	878	.98050	27
28	874	.31939	543	.51404	220	.73414	906	.98497	28
29	902	.32244	571	.51748	248	.73806	934	.98944	29
30	.69929	2.32551	.71598	2.52094	.73276	2.74198	.74962	2.99393	30
31	957	.32858	626	.52440	304	.74591	990	.99843	31
32	985	.33166	654	.52787	332	.74984	.75018	3.00293	32
33	.70013	.33474	682	.53134	360	.75379	046	.00745	33
34	040	.33783	710	.53482	388	.75775	075	.01198	34
35	.70068	2.34092	.71738	2.53831	.73416	2.76171	.75103	3.01652	35
36	096	.34403	766	.54181	444	.76568	131	.02107	36
37	124	.34713	794	.54531	472	.76966	159	.02563	37
38	151	.35025	822	.54883	500	.77365	187	.03020	38
39	179	.35336	850	.55234	529	.77765	216	.03479	39
40	.70207	2.35649	.71877	2.55587	.73557	2.78166	.75244	3.03938	40
41	235	.35962	905	.55940	585	.78568	272	.04398	41
42	263	.36276	933	.56294	613	.78970	300	.04860	42
43	290	.36590	961	.56649	641	.79374	328	.05322	43
44	318	.36905	989	.57005	669	.79778	356	.05786	44
45	.70346	2.37221	.72017	2.57361	.73697	2.80183	.75385	3.06251	45
46	374	.37537	045	.57718	725	.80589	413	.06717	46
47	401	.37854	073	.58076	753	.80996	441	.07184	47
48	429	.38171	101	.58434	781	.81404	469	.07652	48
49	457	.38489	129	.58794	809	.81813	497	.08121	49
50	.70485	2.38808	.72157	2.59154	.73837	2.82223	.75526	3.08591	50
51	513	.39128	185	.59514	865	.82633	554	.09063	51
52	540	.39448	213	.59876	893	.83045	582	.09535	52
53	568	.39768	241	.60238	921	.83457	610	.10009	53
54	596	.40089	269	.60601	950	.83871	638	.10484	54
55	.70624	2.40411	.72296	2.60965	.73978	2.84285	.75667	3.10960	55
56	652	.40734	324	.61330	.74006	.84700	695	.11437	56
57	679	.41057	352	.61695	034	.85116	723	.11915	57
58	707	.41381	380	.62061	062	.85533	751	.12394	58
59	735	.41705	408	.62428	090	.85951	780	.12875	59
60	.70763	2.42030	.72436	2.62796	.74118	2.86370	.75808	3.13357	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	76° Vers.	76° Exsec.	77° Vers.	77° Exsec.	78° Vers.	78° Exsec.	79° Vers.	79° Exsec.	M.
0	.75808	3.13357	.77505	3.44541	.79209	3.80973	.80919	4.24084	0
1	836	.13839	533	.45102	237	.81633	948	.24870	1
2	864	.14323	562	.45664	266	.82294	976	.25658	2
3	892	.14809	590	.46228	294	.82956	.81005	.26448	3
4	921	.15295	618	.46793	323	.83621	033	.27241	4
5	.75949	3.15782	.77647	3.47360	.79351	3.84288	.81062	4.28036	5
6	977	.16271	675	.47928	380	.84956	090	.28833	6
7	.76005	.16761	703	.48498	408	.85627	119	.29634	7
8	034	.17252	732	.49069	437	.86299	148	.30436	8
9	062	.17744	760	.49642	465	.86973	176	.31241	9
10	.76090	3.18238	.77788	3.50216	.79493	3.87649	.81205	4.32049	10
11	118	.18733	817	.50791	522	.88327	233	.32859	11
12	147	.19228	845	.51368	550	.89007	262	.33671	12
13	175	.19725	874	.51947	579	.89689	290	.34486	13
14	203	.20224	902	.52527	607	.90373	319	.35304	14
15	.76231	3.20723	.77930	3.53109	.79636	3.91058	.81348	4.36124	15
16	260	.21224	959	.53692	664	.91746	376	.36947	16
17	288	.21726	987	.54277	693	.92436	405	.37772	17
18	316	.22229	.78015	.54863	721	.93128	433	.38600	18
19	344	.22734	044	.55451	750	.93821	462	.39430	19
20	.76373	3.23239	.78072	3.56041	.79778	3.94517	.81491	4.40263	20
21	401	.23746	101	.56632	807	.95215	519	.41099	21
22	429	.24255	129	.57224	835	.95914	548	.41937	22
23	458	.24764	157	.57819	864	.96616	576	.42778	23
24	486	.25275	186	.58414	892	.97320	605	.43622	24
25	.76514	3.25787	.78214	3.59012	.79921	3.98025	.81633	4.44468	25
26	542	.26300	242	.59611	949	.98733	662	.45317	26
27	571	.26814	271	.60211	978	.99443	691	.46169	27
28	599	.27330	299	.60813	.80006	4.00155	719	.47023	28
29	627	.27847	328	.61417	035	.00869	748	.47881	29
30	.76655	3.28366	.78356	3.62023	.80063	4.01585	.81776	4.48740	30
31	684	.28885	384	.62630	092	.02303	805	.49603	31
32	712	.29406	413	.63238	120	.03024	834	.50468	32
33	740	.29929	441	.63849	149	.03746	862	.51337	33
34	769	.30452	470	.64461	177	.04471	891	.52208	34
35	.76797	3.30977	.78498	3.65074	.80206	4.05197	.81919	4.53081	35
36	825	.31503	526	.65690	234	.05926	948	.53958	36
37	854	.32031	555	.66307	263	.06657	977	.54837	37
38	882	.32560	583	.66925	291	.07390	.82005	.55720	38
39	910	.33090	612	.67545	320	.08125	034	.56605	39
40	.76938	3.33622	.78640	3.68167	.80348	4.08863	.82063	4.57493	40
41	967	.34154	669	.68791	377	.09602	091	.58383	41
42	995	.34689	697	.69417	405	.10344	120	.59277	42
43	.77023	.35224	725	.70044	434	.11088	148	.60174	43
44	052	.35761	754	.70673	462	.11835	177	.61073	44
45	.77080	3.36299	.78782	3.71303	.80491	4.12583	.82206	4.61976	45
46	108	.36839	811	.71935	519	.13334	234	.62881	46
47	137	.37380	839	.72569	548	.14087	263	.63790	47
48	165	.37923	868	.73205	577	.14842	292	.64701	48
49	193	.38466	896	.73843	605	.15599	320	.65616	49
50	.77222	3.39012	.78924	3.74482	.80634	4.16359	.82349	4.66533	50
51	250	.39558	953	.75123	662	.17121	377	.67454	51
52	278	.40106	981	.75766	691	.17886	406	.68377	52
53	307	.40656	.79010	.76411	719	.18652	435	.69304	53
54	335	.41206	038	.77057	748	.19421	463	.70234	54
55	.77363	3.41759	.79067	3.77705	.80776	4.20193	.82492	4.71166	55
56	392	.42312	095	.78355	805	.20966	521	.72102	56
57	420	.42867	123	.79007	833	.21742	549	.73041	57
58	448	.43424	152	.79661	862	.22521	578	.73983	58
59	477	.43982	180	.80316	891	.23301	607	.74929	59
60	.77505	3.44541	.79209	3.80973	.80919	4.24084	.82635	4.75877	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	80°		81°		82°		83°		M.
	Vers.	Exsec.	Vers.	Exsec.	Vers.	Exsec.	Vers.	Exsec.	
0	.82635	4.75877	.84357	5.39245	.86083	6.18530	.87813	7.20551	0
1	664	.76829	385	.40422	111	.20020	842	.22500	1
2	692	.77783	414	.41602	140	.21517	871	.24457	2
3	721	.78742	443	.42787	169	.23019	900	.26425	3
4	750	.79703	471	.43977	198	.24529	929	.28402	4
5	.82778	4.80667	.84500	5.45171	.86227	6.26044	.87957	7.30388	5
6	807	.81635	529	.46369	256	.27566	986	.32384	6
7	836	.82606	558	.47572	284	.29095	.88015	.34390	7
8	864	.83581	586	.48779	313	.30630	044	.36405	8
9	893	.84558	615	.49991	342	.32171	073	.38431	9
10	.82922	4.85539	.84644	5.51208	.86371	6.33719	.88102	7.40466	10
11	950	.86524	673	.52429	400	.35274	131	.42511	11
12	979	.87511	701	.53655	428	.36835	160	.44566	12
13	.83008	.88502	730	.54886	457	.38403	188	.46632	13
14	036	.89497	759	.56121	486	.39978	217	.48707	14
15	.83065	4.90495	.84788	5.57361	.86515	6.41560	.88246	7.50793	15
16	094	.91496	816	.58606	544	.43148	275	.52889	16
17	122	.92501	845	.59855	573	.44743	304	.54996	17
18	151	.93509	874	.61110	601	.46346	333	.57113	18
19	180	.94521	903	.62369	630	.47955	362	.59241	19
20	.83208	4.95536	.84931	5.63633	.86659	6.49571	.88391	7.61379	20
21	237	.96555	960	.64902	688	.51194	420	.63528	21
22	266	.97577	989	.66176	717	.52825	448	.65688	22
23	294	.98603	.85018	.67454	746	.54462	477	.67859	23
24	323	.99633	046	.68738	774	.56107	506	.70041	24
25	.83352	5.00666	.85075	5.70027	.86803	6.57759	.88535	7.72234	25
26	380	.01702	104	.71321	832	.59418	564	.74438	26
27	409	.02743	133	.72620	861	.61085	593	.76653	27
28	438	.03787	162	.73924	890	.62759	622	.78880	28
29	467	.04834	190	.75233	919	.64441	651	.81118	29
30	.83495	5.05886	.85219	5.76547	.86947	6.66130	.88680	7.83367	30
31	524	.06941	248	.77866	976	.67826	709	.85628	31
32	553	.08000	277	.79191	.87005	.69530	737	.87901	32
33	581	.09062	305	.80521	034	.71242	766	.90186	33
34	610	.10129	334	.81856	063	.72962	795	.92482	34
35	.83639	5.11199	.85363	5.83196	.87092	6.74689	.88824	7.94791	35
36	667	.12273	392	.84542	120	.76424	853	.97111	36
37	696	.13350	420	.85893	149	.78167	882	.99444	37
38	725	.14432	449	.87250	178	.79918	911	8.01788	38
39	754	.15517	478	.88612	207	.81677	940	.04146	39
40	.83782	5.16607	.85507	5.89979	.87236	6.83443	.88969	8.06515	40
41	811	.17700	536	.91352	265	.85218	998	.08897	41
42	840	.18797	564	.92731	294	.87001	.89027	.11292	42
43	868	.19898	593	.94115	322	.88792	055	.13699	43
44	897	.21004	622	.95505	351	.90592	084	.16120	44
45	.83926	5.22113	.85651	5.96900	.87380	6.92399	.89113	8.18553	45
46	954	.23226	680	.98301	409	.94216	142	.20999	46
47	983	.24343	708	.99708	438	.96040	171	.23459	47
48	.84012	.25464	737	6.01120	467	.97873	200	.25931	48
49	041	.26590	766	.02538	496	.99714	229	.28417	49
50	.84069	5.27719	.85795	6.03962	.87524	7.01565	.89258	8.30917	50
51	098	.28853	823	.05392	553	.03423	287	.33430	51
52	127	.29991	852	.06828	582	.05291	316	.35957	52
53	155	.31133	881	.08269	611	.07167	345	.38497	53
54	184	.32279	910	.09717	640	.09052	374	.41052	54
55	.84213	5.33429	.85939	6.11171	.87669	7.10946	.89403	8.43620	55
56	242	.34584	967	.12630	698	.12849	431	.46203	56
57	270	.35743	996	.14096	726	.14760	460	.48800	57
58	299	.36906	.86025	.15568	755	.16681	489	.51411	58
59	328	.38073	054	.17046	784	.18612	518	.54037	59
60	.84357	5.39245	.86083	6.18530	.87813	7.20551	.89547	8.56677	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—
(Continued)

M.	84° Vers.	84° Exsec.	85° Vers.	85° Exsec.	86° Vers.	86° Exsec.	M.
0	.89547	8.56677	.91284	10.47371	.93024	13.33559	0
1	576	.59332	313	.51199	053	.39547	1
2	605	.62002	342	.55052	082	.45586	2
3	634	.64687	371	.58932	111	.51676	3
4	663	.67387	400	.62837	140	.57817	4
5	.89692	8.70103	.91429	10.66769	.93169	13.64011	5
6	721	.72833	458	.70728	198	.70258	6
7	750	.75579	487	.74714	227	.76558	7
8	779	.78341	516	.78727	257	.82913	8
9	808	.81119	545	.82768	286	.89323	9
10	.89836	8.83912	.91574	10.86837	.93315	13.95788	10
11	865	.86722	603	.90934	344	14.02310	11
12	894	.89547	632	.95060	373	.08890	12
13	923	.92389	661	.99214	402	.15527	13
14	952	.95248	690	11.03397	431	.22223	14
15	.89981	8.98123	.91719	11.07610	.93460	14.28979	15
16	.90010	9.01015	748	.11852	489	.35795	16
17	039	.03923	777	.16125	518	.42672	17
18	068	.06849	806	.20427	547	.49611	18
19	097	.09792	835	.24761	576	.56613	19
20	.90126	9.12752	.91864	11.29125	.93605	14.63679	20
21	155	.15730	893	.33521	634	.70810	21
22	184	.18725	922	.37948	663	.78005	22
23	213	.21739	951	.42408	692	.85268	23
24	242	.24770	980	.46900	721	.92597	24
25	.90271	9.27819	.92009	11.51424	.93750	14.99995	25
26	300	.30887	038	.55981	779	15.07462	26
27	329	.33973	067	.60572	808	.14999	27
28	358	.37077	096	.65197	837	.22607	28
29	386	.40201	125	.69856	866	.30287	29
30	.90415	9.43343	.92154	11.74549	.93895	15.38041	30
31	444	.46505	183	.79278	924	.45869	31
32	473	.49685	212	.84042	953	.53772	32
33	502	.52886	241	.88841	982	.61751	33
34	531	.56106	270	.93677	.94011	.69808	34
35	.90560	9.59346	.92299	11.98549	.94040	15.77944	35
36	589	.62605	328	12.03458	069	.86159	36
37	618	.65885	357	.08404	098	.94456	37
38	647	.69186	386	.13388	127	16.02835	38
39	676	.72507	415	.18411	156	.11297	39
40	.90705	9.75849	.92444	12.23472	.94186	16.19843	40
41	734	.79212	473	.28572	215	.28476	41
42	763	.82596	502	.33712	244	.37196	42
43	792	.86001	531	.38891	273	.46005	43
44	821	.89428	560	.44112	302	.54903	44
45	.90850	9.92877	.92589	12.49373	.94331	16.63893	45
46	879	.96348	618	.54676	360	.72975	46
47	908	.99841	647	.60021	389	.82152	47
48	937	10.03356	676	.65408	418	.91424	48
49	966	.06894	705	.70838	447	17.00794	49
50	.90995	10.10455	.92734	12.76311	.94476	17.10262	50
51	.91024	.14039	763	.81829	505	.19830	51
52	053	.17646	792	.87391	534	.29500	52
53	082	.21277	821	.92999	563	.39274	53
54	111	.24932	850	.98651	592	.49153	54
55	.91140	10.28610	.92879	13.04350	.94621	17.59139	55
56	169	.32313	908	.10096	650	.69233	56
57	197	.36040	937	.15889	679	.79438	57
58	226	.39792	966	.21730	708	.89755	58
59	255	.43569	995	.27620	737	18.00185	59
60	.91284	10.47371	.93024	13.33559	.94766	18.10732	60

TABLE 22.—NATURAL VERSINES AND EXTERNAL SECANTS—(*Continued*)

M.	87° Vers	87° Exsec.	88° Vers.	88° Exsec.	89° Vers.	89° Exsec.	M.
0	.94766	18.10732	.96510	27.65371	.98255	56.29869	0
1	795	.21397	539	.89440	284	57.26975	1
2	825	.32182	568	28.13917	313	58.27431	2
3	854	.43088	597	.38812	342	59.31411	3
4	883	.54119	626	.64137	371	60.39105	4
5	.94912	18.65275	.96655	28.89903	.98400	61.50715	5
6	941	.76560	684	29.16120	429	62.66460	6
7	970	.87976	714	.42802	458	63.86572	7
8	999	.99524	743	.69960	487	65.11304	8
9	.95028	19.11207	772	.97607	517	66.40927	9
10	.95057	19.23028	.96801	30.25758	.98546	67.75736	10
11	086	.34989	830	.54425	575	69.16047	11
12	115	.47093	859	.83623	604	70.62205	12
13	144	.59341	888	31.13366	633	72.14583	13
14	173	.71737	917	.43671	662	73.73586	14
15	.95202	19.84283	.96946	31.74554	.98691	75.39655	15
16	231	96982	975	32.06030	720	77.13274	16
17	260	20.09838	.97004	.38118	749	78.94968	17
18	289	.22852	033	.70835	778	80.85315	18
19	318	.36027	062	33.04199	807	82.84947	19
20	.95347	20.49368	.97092	33.38232	.98836	84.94561	20
21	377	.62876	121	.72951	866	87.14924	21
22	406	.76555	150	34.08380	895	89.46886	22
23	435	.90409	179	.44539	924	91.91387	23
24	464	21.04440	208	.81452	953	94.49471	24
25	.95493	21.18653	.97237	35.19141	.98982	97.22303	25
26	522	.33050	266	.57633	.99011	100.11185	26
27	551	.47635	295	.96953	040	103.17574	27
28	580	.62413	324	36.37127	069	106.43114	28
29	609	.77386	353	.78185	098	109.89656	29
30	.95638	21.92559	.97382	37.20155	.99127	113.59301	30
31	667	22.07935	411	.63068	156	117.54440	31
32	696	.23520	440	38.06957	186	121.77803	32
33	725	.39316	470	.51855	215	126.32526	33
34	754	.55329	499	.97797	244	131.22229	34
35	.95783	22.71563	.97528	39.44820	.99273	136.51108	35
36	812	.88022	557	.92963	302	142.24061	36
37	841	23.04712	586	40.42266	331	148.46837	37
38	871	.21637	615	.92772	360	155.26228	38
39	900	.38802	644	41.44525	389	162.70325	39
40	.95929	23.56212	.97673	41.97571	.99418	170.88831	40
41	958	.73873	702	42.51961	447	179.93496	41
42	987	.91790	731	43.07746	476	189.98680	42
43	.96016	24.09969	760	.64980	505	201.22122	43
44	045	.28414	789	44.23719	535	213.85995	44
45	.96074	24.47134	.97819	44.84026	.99564	228.18385	45
46	103	.66132	848	45.45963	593	244.55402	46
47	132	.85417	877	46.09596	622	263.44269	47
48	161	25.04994	906	.74997	651	285.47948	48
49	190	.24869	935	47.42241	680	311.52297	49
50	.96219	25.45051	.97964	48.11406	.99709	342.77516	50
51	248	.65545	993	.82576	738	380.97230	51
52	277	.86360	.98022	49.55840	767	428.71873	52
53	307	26.07503	051	50.31290	796	490.10702	53
54	336	.28981	080	51.09027	825	571.95809	54
55	.96365	26.50804	.98109	51.89156	.99855	686.54960	55
56	394	.72978	138	52.71790	884	858.43689	56
57	423	.95512	168	53.57046	913	1144.91574	57
58	452	27.18417	197	54.45053	942	1717.87348	58
59	481	.41700	226	55.35946	971	3436.74682	59
60	.96510	27.65371	.98255	56.29869	1.00000	∞	60

TABLE 23.—MINUTES IN DECIMALS OF A DEGREE

′	0″	10″	15″	20″	30″	40″	45″	50″	′
0	.00000	.00278	.00417	.00556	.00833	.01111	.01250	.01389	0
1	.01667	.01944	.02083	.02222	.02500	.02778	.02917	.03055	1
2	.03333	.03611	.03750	.03889	.04167	.04444	.04583	.04722	2
3	.05000	.05278	.05417	.05556	.05833	.06111	.06250	.06389	3
4	.06667	.06944	.07083	.07222	.07500	.07778	.07917	.08056	4
5	.08333	.08611	.08750	.08889	.09167	.09444	.09583	.09722	5
6	.10000	.10278	.10417	.10556	.10833	.11111	.11250	.11389	6
7	.11667	.11944	.12083	.12222	.12500	.12778	.12917	.13056	7
8	.13333	.13611	.13750	.13889	.14167	.14444	.14583	.14722	8
9	.15000	.15278	.15417	.15556	.15833	.16111	.16250	.16389	9
10	.16667	.16944	.17083	.17222	.17500	.17778	.17917	.18056	10
11	.18333	.18611	.18750	.18889	.19167	.19444	.19583	.19722	11
12	.20000	.20278	.20417	.20556	.20833	.21111	.21250	.21389	12
13	.21667	.21944	.22083	.22222	.22500	.22778	.22917	.23056	13
14	.23333	.23611	.23750	.23889	.24167	.24444	.24583	.24722	14
15	.25000	.25278	.25417	.25556	.25833	.26111	.26250	.26389	15
16	.26667	.26944	.27083	.27222	.27500	.27778	.27917	.28056	16
17	.28333	.28611	.28750	.28889	.29167	.29444	.29583	.29722	17
18	.30000	.30278	.30417	.30556	.30833	.31111	.31250	.31389	18
19	.31667	.31944	.32083	.32222	.32500	.32778	.32917	.33056	19
20	.33333	.33611	.33750	.33889	.34167	.34444	.34583	.34722	20
21	.35000	.35278	.35417	.35556	.35833	.36111	.36250	.36389	21
22	.36667	.36944	.37083	.37222	.37500	.37778	.37917	.38056	22
23	.38333	.38611	.38750	.38889	.39167	.39444	.39583	.39722	23
24	.40000	.40278	.40417	.40556	.40833	.41111	.41250	.41389	24
25	.41667	.41944	.42083	.42222	.42500	.42778	.42917	.43056	25
26	.43333	.43611	.43750	.43889	.44167	.44444	.44583	.44722	26
27	.45000	.45278	.45417	.45556	.45833	.46111	.46250	.46389	27
28	.46667	.46944	.47083	.47222	.47500	.47778	.47917	.48056	28
29	.48333	.48611	.48750	.48889	.49167	.49444	.49583	.49722	29
30	.50000	.50278	.50417	.50556	.50833	.51111	.51250	.51389	30
31	.51667	.51944	.52083	.52222	.52500	.52778	.52917	.53056	31
32	.53333	.53611	.53750	.53889	.54167	.54444	.54583	.54722	32
33	.55000	.55278	.55417	.55556	.55833	.56111	.56250	.56389	33
34	.56667	.56944	.57083	.57222	.57500	.57778	.57917	.58056	34
35	.58333	.58611	.58750	.58889	.59167	.59444	.59583	.59722	35
36	.60000	.60278	.60417	.60556	.60833	.61111	.61250	.61389	36
37	.61667	.61944	.62083	.62222	.62500	.62778	.62917	.63056	37
38	.63333	.63611	.63750	.63889	.64167	.64444	.64583	.64722	38
39	.65000	.65278	.65417	.65556	.65833	.66111	.66250	.66389	39
40	.66667	.66944	.67083	.67222	.67500	.67778	.67917	.68056	40
41	.68333	.68611	.68750	.68889	.69167	.69444	.69583	.69722	41
42	.70000	.70278	.70417	.70556	.70833	.71111	.71250	.71389	42
43	.71667	.71944	.72083	.72222	.72500	.72778	.72917	.73056	43
44	.73333	.73611	.73750	.73889	.74167	.74444	.74583	.74722	44
45	.75000	.75278	.75417	.75556	.75833	.76111	.76250	.76389	45
46	.76667	.76944	.77083	.77222	.77500	.77778	.77917	.78056	46
47	.78333	.78611	.78750	.78889	.79167	.79444	.79583	.79722	47
48	.80000	.80278	.80417	.80556	.80833	.81111	.81250	.81389	48
49	.81667	.81944	.82083	.82222	.82500	.82778	.82917	.83056	49
50	.83333	.83611	.83750	.83889	.84167	.84444	.84583	.84722	50
51	.85000	.85278	.85417	.85556	.85833	.86111	.86250	.86389	51
52	.86667	.86944	.87083	.87222	.87500	.87778	.87917	.88056	52
53	.88333	.88611	.88750	.88889	.89167	.89444	.89583	.89722	53
54	.90000	.90278	.90417	.90556	.90833	.91111	.91250	.91389	54
55	.91667	.91944	.92083	.92222	.92500	.92778	.92917	.93056	55
56	.93333	.93611	.93750	.93889	.94167	.94444	.94583	.94722	56
57	.95000	.95278	.95417	.95556	.95833	.96111	.96250	.96389	57
58	.96667	.96944	.97083	.97222	.97500	.97778	.97917	.98056	58
59	.98333	.98611	.98750	.98889	.99167	.99444	.99583	.99722	59
′	0″	10″	15″	20″	30″	40″	45″	50″	′

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TABLE 24.—HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS

	0°		1°		2°		3°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23
2	100.00	0.06	99.97	1.80	99.87	3.55	99.72	5.28
4	100.00	0.12	99.97	1.86	99.87	3.60	99.71	5.34
6	100.00	0.17	99.96	1.92	99.87	3.66	99.71	5.40
8	100.00	0.23	99.96	1.98	99.86	3.72	99.70	5.46
10	100.00	0.29	99.96	2.04	99.86	3.78	99.69	5.52
12	100.00	0.35	99.96	2.09	99.85	3.84	99.69	5.57
14	100.00	0.41	99.95	2.15	99.85	3.90	99.68	5.63
16	100.00	0.47	99.95	2.21	99.84	3.95	99.68	5.69
18	100.00	0.52	99.95	2.27	99.84	4.01	99.67	5.75
20	100.00	0.58	99.95	2.33	99.83	4.07	99.66	5.80
22	100.00	0.64	99.94	2.38	99.83	4.13	99.66	5.86
24	100.00	0.70	99.94	2.44	99.82	4.18	99.65	5.92
26	99.99	0.76	99.94	2.50	99.82	4.24	99.64	5.98
28	99.99	0.81	99.93	2.56	99.81	4.30	99.63	6.04
30	99.99	0.87	99.93	2.62	99.81	4.36	99.63	6.09
32	99.99	0.93	99.93	2.67	99.80	4.42	99.62	6.15
34	99.99	0.99	99.93	2.73	99.80	4.48	99.62	6.21
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.33
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38
42	99.99	1.22	99.91	2.97	99.78	4.71	99.59	6.44
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61
50	99.98	1.45	99.90	3.20	99.76	4.94	99.56	6.67
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.78
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96
c = 0.75.	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05
c = 1.00.	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06
c = 1.25.	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08

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TABLE 24.—HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS—(*Continued*)

	4°		5°		6°		7°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21
6	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26
8	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94
32	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
c = 0.75.	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10
c = 1.00.	1.00	0.08	0.99	0.09	0.99	0.11	0.99	0.13
c = 1.25.	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16

TABLE 24.—HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS—(*Continued*)

	8°		9°		10°		11°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
c = 0.75.	0.74	0.11	0.74	0.12	0.74	0.14	0.73	0.15
c = 1.00.	0.99	0.15	0.99	0.16	0.98	0.18	0.98	0.20
c = 1.25.	1.23	0.18	1.23	0.21	1.23	0.23	1.22	0.25

TABLE 24.—HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS—(*Continued*)

	12°		13°		14°		15°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60
26	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
c = 0.75 .	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20
c = 1.00 .	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27
c = 1.25 .	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34

TABLE 24.—HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS—(*Continued*)

	16°		17°		18°		19°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33
26	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56
36	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09
60	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
c = 0.75.	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25
c = 1.00.	0.96	0.28	0.95	0.30	0.95	0.32	0.94	0.33
c = 1.25.	1.20	0.35	1.19	0.38	1.19	0.40	1.18	0.42

TABLE 24.—HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS—(*Continued*)

	20°		21°		22°		23°	
Minutes.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
c = 0.75.	0.70	0.26	0.70	0.27	0.69	0.29	0.69	0.30
c = 1.00.	0.94	0.35	0.93	0.37	0.92	0.38	0.92	0.40
c = 1.25.	1.17	0.44	1.16	0.46	1.15	0.48	1.15	0.50

TABLE 24.—HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS—(*Continued*)

	24°		25°		26°		27°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff Elev.	Hor. Dist.	Diff. Elev.
0	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92
30	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
c = 0.75	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35
c = 1.00	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46
c = 1.25	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58

TABLE 24.—HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS—(*Continued*)

	28°		29°		30°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	77.96	41.45	76.50	42.40	75.00	43.30
2	77.91	41.48	76.45	42.43	74.95	43.33
4	77.86	41.52	76.40	42.46	74.90	43.36
6	77.81	41.55	76.35	42.49	74.85	43.39
8	77.77	41.58	76.30	42.53	74.80	43.42
10	77.72	41.61	76.25	42.56	74.75	43.45
12	77.67	41.65	76.20	42.59	74.70	43.47
14	77.62	41.68	76.15	42.62	74.65	43.50
16	77.57	41.71	76.10	42.65	74.60	43.53
18	77.52	41.74	76.05	42.68	74.55	43.56
20	77.48	41.77	76.00	42.71	74.49	43.59
22	77.42	41.81	75.95	42.74	74.44	43.62
24	77.38	41.84	75.90	42.77	74.39	43.65
26	77.33	41.87	75.85	42.80	74.34	43.67
28	77.28	41.90	75.80	42.83	74.29	43.70
30	77.23	41.93	75.75	42.86	74.24	43.73
32	77.18	41.97	75.70	42.89	74.19	43.76
34	77.13	42.00	75.65	42.92	74.14	43.79
36	77.09	42.03	75.60	42.95	74.09	43.82
38	77.04	42.06	75.55	42.98	74.04	43.84
40	76.99	42.09	75.50	43.01	73.99	43.87
42	76.94	42.12	75.45	43.04	73.93	43.90
44	76.89	42.15	75.40	43.07	73.88	43.93
46	76.84	42.19	75.35	43.10	73.83	43.95
48	76.79	42.22	75.30	43.13	73.78	43.98
50	76.74	42.25	75.25	43.16	73.73	44.01
52	76.69	42.28	75.20	43.18	73.68	44.04
54	76.64	42.31	75.15	43.21	73.63	44.07
56	76.59	42.34	75.10	43.24	73.58	44.09
58	76.55	42.37	75.05	43.27	73.52	44.12
60	76.50	42.40	75.00	43.30	73.47	44.15
c = 0.75..	0.66	0.36	0.65	0.37	0.65	0.38
c = 1.00..	0.88	0.48	0.87	0.49	0.86	0.51
c = 1.25..	1.10	0.60	1.09	0.62	1.08	0.64

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
1	1	1	1.0000	1.0000	1.000000000	3.1416	0.7854
2	4	8	1.4142	1.2599	.500000000	6.2832	3.1416
3	9	27	1.7321	1.4422	.333333333	9.4248	7.0686
4	16	64	2.0000	1.5874	.250000000	12.5664	12.5664
5	25	125	2.2361	1.7100	.200000000	15.7080	19.635
6	36	216	2.4495	1.8171	.166666667	18.850	28.274
7	49	343	2.6458	1.9129	.142857143	21.991	38.485
8	64	512	2.8284	2.0000	.125000000	25.133	50.266
9	81	729	3.0000	2.0801	.111111111	28.274	63.617
10	100	1,000	3.1623	2.1544	.100000000	31.416	78.540
11	121	1,331	3.3166	2.2240	.090909091	34.558	95.033
12	144	1,728	3.4641	2.2894	.083333333	37.699	113.10
13	169	2,197	3.6056	2.3513	.076923077	40.841	132.73
14	196	2,744	3.7417	2.4101	.071428571	43.982	153.94
15	225	3,375	3.8730	2.4662	.066666667	47.124	176.71
16	256	4,096	4.0000	2.5198	.062500000	50.265	201.06
17	289	4,913	4.1231	2.5713	.058823529	53.407	226.98
18	324	5,832	4.2426	2.6207	.055555556	56.549	254.47
19	361	6,859	4.3589	2.6684	.052631579	59.690	283.53
20	400	8,000	4.4721	2.7144	.050000000	62.832	314.16
21	441	9,261	4.5826	2.7589	.047619048	65.973	346.36
22	484	10,648	4.6904	2.8020	.045454545	69.115	380.13
23	529	12,167	4.7958	2.8439	.043478261	72.257	415.48
24	576	13,824	4.8990	2.8845	.041666667	75.398	452.39
25	625	15,625	5.0000	2.9240	.040000000	78.540	490.87
26	676	17,576	5.0990	2.9625	.038461538	81.681	530.93
27	729	19,683	5.1962	3.0000	.037037037	84.823	572.56
28	784	21,952	5.2915	3.0366	.035714286	87.965	615.75
29	841	24,389	5.3852	3.0723	.034482759	91.106	660.52
30	900	27,000	5.4772	3.1072	.033333333	94.248	706.86
31	961	29,791	5.5678	3.1414	.032258065	97.389	754.77
32	1,024	32,768	5.6569	3.1748	.031250000	100.53	804.25
33	1,089	35,937	5.7446	3.2075	.030303030	103.67	855.30
34	1,156	39,304	5.8310	3.2396	.029411765	106.81	907.92
35	1,225	42,875	5.9161	3.2717	.028571429	109.96	962.11
36	1,296	46,656	6.0000	3.3019	.027777778	113.10	1,017.88
37	1,369	50,653	6.0828	3.3322	.027027027	116.24	1,075.21
38	1,444	54,872	6.1644	3.3620	.026315789	119.38	1,134.11
39	1,521	59,319	6.2450	3.3912	.025641026	122.52	1,194.59
40	1,600	64,000	6.3246	3.4200	.025000000	125.66	1,256.64
41	1,681	68,921	6.4031	3.4482	.024390244	128.81	1,320.25
42	1,764	74,088	6.4807	3.4760	.023809524	131.95	1,385.44
43	1,849	79,507	6.5574	3.5034	.023255814	135.09	1,452.20
44	1,936	85,184	6.6332	3.5303	.022727273	138.23	1,520.53
45	2,025	91,125	6.7082	3.5569	.022222222	141.37	1,590.43
46	2,116	97,336	6.7823	3.5830	.021739130	144.51	1,661.90
47	2,209	103,823	6.8557	3.6088	.021276600	147.65	1,734.94
48	2,304	110,592	6.9282	3.6342	.020833333	150.80	1,809.56
49	2,401	117,649	7.0000	3.6593	.020408163	153.94	1,885.74
50	2,500	125,000	7.0711	3.6840	.020000000	157.08	1,963.50
51	2,601	132,651	7.1414	3.7084	.019607843	160.22	2,042.82
52	2,704	140,608	7.2111	3.7325	.019230769	163.36	2,123.72
53	2,809	148,877	7.2801	3.7563	.018867925	166.50	2,206.18
54	2,916	157,464	7.3485	3.7798	.018518519	169.65	2,290.22
55	3,025	166,375	7.4162	3.8030	.018181818	172.79	2,375.83

From "Coal Miners' Pocketbook."

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum	Area
56	3,136	175,616	7.4833	3.8259	.017857143	175.93	2,463.01
57	3,249	185,193	7.5498	3.8485	.017543860	179.07	2,551.76
58	3,364	195,112	7.6158	3.8709	.017241379	182.21	2,642.08
59	3,481	205,379	7.6811	3.8930	.016949153	185.35	2,733.97
60	3,600	216,000	7.7460	3.9149	.016666667	188.50	2,827.43
61	3,721	226,981	7.8102	3.9365	.016393443	191.64	2,922.47
62	3,844	238,328	7.8740	3.9579	.016129032	194.78	3,019.07
63	3,969	250,047	7.9373	3.9791	.015873016	197.92	3,117.25
64	4,096	262,144	8.0000	4.0000	.015625000	201.06	3,216.99
65	4,225	274,625	8.0623	4.0207	.015384615	204.20	3,318.31
66	4,356	287,496	8.1240	4.0412	.015151515	207.34	3,421.19
67	4,489	300,763	8.1854	4.0615	.014925373	210.49	3,525.65
68	4,624	314,432	8.2462	4.0817	.014705882	213.63	3,631.68
69	4,761	328,509	8.3066	4.1016	.014492754	216.77	3,739.28
70	4,900	343,000	8.3666	4.1213	.014285714	219.91	3,848.45
71	5,041	357,911	8.4261	4.1408	.014084517	223.05	3,959.19
72	5,184	373,248	8.4853	4.1602	.013888889	226.19	4,071.50
73	5,329	389,017	8.5440	4.1793	.013698630	229.34	4,185.39
74	5,476	405,224	8.6023	4.1983	.013513514	232.48	4,300.84
75	5,625	421,875	8.6603	4.2172	.013333333	235.62	4,417.86
76	5,776	438,976	8.7178	4.2358	.013157895	238.76	4,536.46
77	5,929	456,533	8.7750	4.2543	.012987013	241.90	4,656.63
78	6,084	474,552	8.8318	4.2727	.012820513	245.04	4,778.36
79	6,241	493,039	8.8882	4.2908	.012658228	248.19	4,901.67
80	6,400	512,000	8.9443	4.3089	.012500000	251.33	5,026.55
81	6,561	531,441	9.0000	4.3267	.012345679	254.47	5,153.00
82	6,724	551,368	9.0554	4.3445	.012195122	257.61	5,281.02
83	6,889	571,787	9.1104	4.3621	.012048193	260.75	5,410.61
84	7,056	592,704	9.1652	4.3795	.011904762	263.89	5,541.77
85	7,225	614,125	9.2195	4.3968	.011764706	267.04	5,674.50
86	7,396	636,056	9.2736	4.4140	.011627907	270.18	5,808.80
87	7,569	658,503	9.3274	4.4310	.011494253	273.32	5,944.68
88	7,744	681,472	9.3808	4.4480	.011363636	276.46	6,082.12
89	7,921	704,969	9.4340	4.4647	.011235955	279.60	6,221.14
90	8,100	729,000	9.4868	4.4814	.011111111	282.74	6,361.73
91	8,281	753,571	9.5394	4.4979	.010989011	285.88	6,503.88
92	8,464	778,688	9.5917	4.5144	.010869565	289.03	6,647.61
93	8,649	804,357	9.6437	4.5307	.010752688	292.17	6,792.91
94	8,836	830,584	9.6954	4.5468	.010638298	295.31	6,939.78
95	9,025	857,375	9.7468	4.5629	.010526316	298.45	7,088.22
96	9,216	884,736	9.7980	4.5789	.010416667	301.59	7,238.23
97	9,409	912,673	9.8489	4.5947	.010309278	304.73	7,389.81
98	9,604	941,192	9.8995	4.6104	.010204082	307.88	7,542.96
99	9,801	970,299	9.9499	4.6261	.010101010	311.02	7,697.69
100	10,000	1,000,000	10.0000	4.6416	.010000000	314.16	7,853.98
101	10,201	1,030,301	10.0499	4.6570	.009900990	317.30	8,011.85
102	10,404	1,061,208	10.0995	4.6723	.009803922	320.44	8,171.28
103	10,609	1,092,727	10.1489	4.6875	.009708738	323.58	8,332.29
104	10,816	1,124,864	10.1980	4.7027	.009615385	326.73	8,494.87
105	11,025	1,157,625	10.2470	4.7177	.009523810	329.87	8,659.01
106	11,236	1,191,016	10.2956	4.7326	.009433962	333.01	8,824.73
107	11,449	1,225,043	10.3441	4.7475	.009345794	336.15	8,992.02
108	11,664	1,259,712	10.3923	4.7622	.009259259	339.29	9,160.88
109	11,881	1,295,029	10.4403	4.7769	.009174312	342.43	9,331.32
110	12,100	1,331,000	10.4881	4.7914	.009090909	345.58	9,503.32
111	12,321	1,367,631	10.5357	4.8059	.009009009	348.72	9,676.89
112	12,544	1,404,928	10.5830	4.8203	.008928571	351.86	9,852.03
113	12,769	1,442,897	10.6301	4.8346	.008849558	355.00	10,028.75
114	12,996	1,481,544	10.6771	4.8488	.008771930	358.14	10,207.03
115	13,225	1,520,875	10.7238	4.8629	.008695652	361.28	10,386.89
116	13,456	1,560,896	10.7703	4.8770	.008620690	364.42	10,568.32
117	13,689	1,601,613	10.8167	4.8910	.008547009	367.57	10,751.32
118	13,924	1,643,032	10.8628	4.9049	.008474576	370.71	10,935.88

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
119	14,161	1,685,159	10.9087	4.9187	.008403361	373.85	11,122.02
120	14,400	1,728,000	10.9545	4.9324	.008333333	376.99	11,309.73
121	14,641	1,771,561	11.0000	4.9461	.008264463	380.13	11,499.01
122	14,884	1,815,848	11.0454	4.9597	.008196721	383.27	11,689.87
123	15,129	1,860,867	11.0905	4.9732	.008130081	386.42	11,882.29
124	15,376	1,906,624	11.1355	4.9866	.008064516	389.56	12,076.28
125	15,625	1,953,125	11.1803	5.0000	.008000000	392.70	12,271.85
126	15,876	2,000,376	11.2250	5.0133	.007936508	395.84	12,468.98
127	16,129	2,048,383	11.2694	5.0265	.007874016	398.98	12,667.69
128	16,384	2,097,152	11.3137	5.0397	.007812500	402.12	12,867.96
129	16,641	2,146,689	11.3578	5.0528	.007751938	405.27	13,069.81
130	16,900	2,197,000	11.4018	5.0658	.007692308	408.41	13,273.23
131	17,161	2,248,091	11.4455	5.0788	.007633588	411.55	13,478.22
132	17,424	2,299,968	11.4891	5.0916	.007575758	414.69	13,684.78
133	17,689	2,352,637	11.5326	5.1045	.007518797	417.83	13,892.91
134	17,956	2,406,104	11.5758	5.1172	.007462687	420.97	14,102.61
135	18,225	2,460,375	11.6190	5.1299	.007407407	424.12	14,313.88
136	18,496	2,515,456	11.6619	5.1426	.007352941	427.26	14,526.72
137	18,769	2,571,353	11.7047	5.1551	.007299270	430.40	14,741.14
138	19,044	2,628,072	11.7473	5.1676	.007246377	433.54	14,957.12
139	19,321	2,685,619	11.7898	5.1801	.007194245	436.68	15,174.68
140	19,600	2,744,000	11.8322	5.1925	.007142857	439.82	15,393.80
141	19,881	2,803,221	11.8743	5.2048	.007092199	442.96	15,614.50
142	20,164	2,863,288	11.9164	5.2171	.007042254	446.11	15,836.77
143	20,449	2,924,207	11.9583	5.2293	.006993007	449.25	16,060.61
144	20,736	2,985,984	12.0000	5.2415	.006944444	452.39	16,286.02
145	21,025	3,048,625	12.0416	5.2536	.006896552	455.53	16,513.00
146	21,316	3,112,136	12.0830	5.2656	.006849315	458.67	16,741.55
147	21,609	3,176,523	12.1244	5.2776	.006802721	461.81	16,971.67
148	21,904	3,241,792	12.1655	5.2896	.006756757	464.96	17,203.36
149	22,201	3,307,949	12.2066	5.3015	.006711409	468.10	17,436.62
150	22,500	3,375,000	12.2474	5.3133	.006666667	471.24	17,671.46
151	22,801	3,442,951	12.2882	5.3251	.006622517	474.38	17,907.86
152	23,104	3,511,008	12.3288	5.3368	.006578947	477.52	18,145.84
153	23,409	3,581,577	12.3693	5.3485	.006535948	480.66	18,385.39
154	23,716	3,652,264	12.4097	5.3601	.006493506	483.81	18,626.50
155	24,025	3,723,875	12.4499	5.3717	.006451613	486.95	18,869.19
156	24,336	3,796,416	12.4900	5.3832	.006410256	490.09	19,113.45
157	24,649	3,869,893	12.5300	5.3947	.006369427	493.23	19,359.28
158	24,964	3,944,312	12.5698	5.4061	.006329114	496.37	19,606.68
159	25,281	4,019,679	12.6095	5.4175	.006289308	499.51	19,855.65
160	25,600	4,096,000	12.6491	5.4288	.006250000	502.65	20,106.19
161	25,921	4,173,281	12.6886	5.4401	.006211180	505.80	20,358.31
162	26,244	4,251,528	12.7279	5.4514	.006172840	508.94	20,611.99
163	26,569	4,330,747	12.7671	5.4626	.006134969	512.08	20,867.24
164	26,896	4,410,944	12.8062	5.4737	.006097561	515.22	21,124.07
165	27,225	4,492,125	12.8452	5.4848	.006060606	518.36	21,382.46
166	27,556	4,574,296	12.8841	5.4959	.006024096	521.50	21,642.43
167	27,889	4,657,463	12.9228	5.5069	.005988024	524.65	21,903.97
168	28,224	4,741,632	12.9615	5.5178	.005952381	527.79	22,167.08
169	28,561	4,826,809	13.0000	5.5288	.005917160	530.93	22,431.76
170	28,900	4,913,000	13.9384	5.5397	.005882353	534.07	22,698.01
171	29,241	5,000,211	13.0767	5.5505	.005847953	537.21	22,965.83
172	29,584	5,088,448	13.1149	5.5613	.005813953	540.35	23,235.22
173	29,929	5,177,717	13.1529	5.5721	.005780347	543.50	23,506.18
174	30,276	5,268,024	13.1909	5.5828	.005747126	546.64	23,778.71
175	30,625	5,359,375	13.2288	5.5934	.005714286	549.78	24,052.82
176	30,976	5,451,776	13.2665	5.6041	.005681818	552.92	24,328.49
177	31,329	5,545,233	13.3041	5.6147	.005649718	556.06	24,605.74
178	31,684	5,639,752	13.3417	5.6252	.005617978	559.20	24,884.56
179	32,041	5,735,339	13.3791	5.6357	.005586592	562.35	25,164.94
180	32,400	5,832,000	13.4164	5.6462	.005555556	565.49	25,446.90
181	32,761	5,929,741	13.4536	5.6567	.005524862	568.63	25,730.48

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
182	33,124	6,028,568	13.4907	5.6671	.005494505	571.77	26,015.53
183	33,489	6,128,487	13.5277	5.6774	.005464481	574.91	26,302.20
184	33,856	6,229,504	13.5647	5.6877	.005434783	578.05	26,590.44
185	34,225	6,331,625	13.6015	5.6980	.005405405	581.19	26,880.25
186	34,596	6,434,856	13.6382	5.7083	.005376344	584.34	27,171.63
187	34,969	6,539,203	13.6748	5.7185	.005347594	587.48	27,464.59
188	35,344	6,644,672	13.7113	5.7287	.005319149	590.62	27,759.11
189	35,721	6,751,269	13.7477	5.7388	.005291005	593.76	28,055.21
190	36,100	6,859,000	13.7840	5.7489	.005263158	596.90	28,352.87
191	36,481	6,967,871	13.8203	5.7590	.005235602	600.04	28,652.11
192	36,864	7,077,888	13.8564	5.7690	.005208333	603.19	28,952.92
193	37,249	7,189,017	13.8924	5.7790	.005181347	606.33	29,255.30
194	37,636	7,301,384	13.9284	5.7890	.005154639	609.47	29,559.25
195	38,025	7,414,875	13.9642	5.7989	.005128205	612.61	29,864.77
196	38,416	7,529,536	14.0000	5.8088	.005102041	615.75	30,171.86
197	38,809	7,645,373	14.0357	5.8186	.005076142	618.89	30,480.52
198	39,204	7,762,392	14.0712	5.8285	.005050505	622.04	30,790.75
199	39,601	7,880,599	14.1067	5.8383	.005025126	625.18	31,102.55
200	40,000	8,000,000	14.1421	5.8480	.005000000	628.32	31,415.93
201	40,401	8,120,601	14.1774	5.8578	.004975124	631.46	31,730.87
202	40,804	8,242,408	14.2127	5.8675	.004950495	634.60	32,047.39
203	41,209	8,365,427	14.2478	5.8771	.004926108	637.74	32,365.47
204	41,616	8,489,664	14.2829	5.8868	.004901961	640.88	32,685.13
205	42,025	8,615,125	14.3178	5.8964	.004878049	644.03	33,006.36
206	42,436	8,741,816	14.3527	5.9059	.004854369	647.17	33,329.16
207	42,849	8,869,743	14.3875	5.9155	.004830918	650.31	33,653.53
208	43,264	8,998,912	14.4222	5.9250	.004807692	653.45	33,979.47
209	43,681	9,129,329	14.4568	5.9345	.004784689	656.59	34,306.98
210	44,100	9,261,000	14.4914	5.9439	.004761905	659.73	34,636.06
211	44,521	9,393,931	14.5258	5.9533	.004739336	662.88	34,966.71
212	44,944	9,528,128	14.5602	5.9627	.004716981	666.02	35,298.94
213	45,369	9,663,597	14.5945	5.9721	.004694836	669.16	35,632.73
214	45,796	9,800,344	14.6287	5.9814	.004672897	672.30	35,968.09
215	46,225	9,938,375	14.6629	5.9907	.004651163	675.44	36,305.03
216	46,656	10,077,696	14.6969	6.0000	.004629630	678.58	36,643.54
217	47,089	10,218,313	14.7309	6.0092	.004608295	681.73	36,983.61
218	47,524	10,360,232	14.7648	6.0185	.004587156	684.87	37,325.26
219	47,961	10,503,459	14.7986	6.0277	.004566210	688.01	37,668.48
220	43,400	10,648,000	14.8324	6.0368	.004545455	691.15	38,013.27
221	48,841	10,793,861	14.8661	6.0459	.004524887	694.29	38,359.63
222	49,284	10,941,048	14.8997	6.0550	.004504505	697.43	38,707.56
223	49,729	11,089,567	14.9332	6.0641	.004484305	700.58	39,057.07
224	50,176	11,239,424	14.9666	6.0732	.004464286	703.72	39,408.14
225	50,625	11,390,625	15.0000	6.0822	.004444444	706.86	39,760.78
226	51,076	11,543,176	15.0333	6.0912	.004424779	710.00	40,115.00
227	51,529	11,697,083	15.0665	6.1002	.004405286	713.14	40,470.78
228	51,984	11,852,352	15.0997	6.1091	.004385965	716.28	40,828.14
229	52,441	12,008,989	15.1327	6.1180	.004366812	719.42	41,187.07
230	52,900	12,167,000	15.1658	6.1269	.004347826	722.57	41,547.56
231	53,361	12,326,391	15.1987	6.1358	.004329004	725.71	41,909.63
232	53,824	12,487,168	15.2315	6.1446	.004310345	728.85	42,273.27
233	54,289	12,649,337	15.2643	6.1534	.004291845	731.99	42,638.48
234	54,756	12,812,904	15.2971	6.1622	.004273504	735.13	43,005.26
235	55,225	12,977,875	15.3297	6.1710	.004255319	738.27	43,373.61
236	55,696	13,144,256	15.3623	6.1797	.004237288	741.42	43,743.54
237	56,169	13,312,053	15.3948	6.1885	.004219409	744.56	44,115.03
238	56,644	13,481,272	15.4272	6.1672	.004201681	747.70	44,488.09
239	57,121	13,651,919	15.4596	6.2058	.004184100	750.84	44,862.73
240	57,600	13,824,000	15.4919	6.2145	.004166667	753.98	45,238.93
241	58,081	13,997,521	15.5242	6.2231	.004149378	757.12	45,616.71
242	58,564	14,172,488	15.5563	6.2317	.004132231	760.27	45,996.06
243	59,049	14,348,907	15.5885	6.2403	.004115226	763.41	46,376.98
244	59,536	14,526,784	15.6205	6.2488	.004098361	766.55	46,759.47

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
245	60,025	14,706,125	15.6525	6.2573	.004081633	769.69	47,143.52
246	60,516	14,886,936	15.6844	6.2658	.004065041	772.83	47,529.16
247	61,009	15,069,223	15.7162	6.2743	.004048583	775.97	47,916.36
248	61,504	15,252,992	15.7480	6.2828	.004032258	779.11	48,305.13
249	62,001	15,438,249	15.7797	6.2912	.004016064	782.26	48,695.47
250	62,500	15,625,000	15.8114	6.2996	.004000000	785.40	49,087.39
251	63,001	15,813,251	15.8430	6.3080	.003984064	788.54	49,480.87
252	63,504	16,003,008	15.8745	6.3164	.003968254	791.68	49,875.92
253	64,009	16,194,277	15.9060	6.3247	.003952569	794.82	50,272.55
254	64,516	16,387,064	15.9374	6.3330	.003937008	797.96	50,670.75
255	65,025	16,581,375	15.9687	6.3413	.003921569	801.11	51,070.52
256	65,536	16,777,216	16.0000	6.3496	.003906250	804.25	51,471.85
257	66,049	16,974,593	16.0312	6.3579	.003891051	807.39	51,874.76
258	66,564	17,173,512	16.0624	6.3661	.003875969	810.53	52,279.24
259	67,081	17,373,979	16.0935	6.3743	.003861004	813.67	52,685.29
260	67,600	17,576,000	16.1245	6.3825	.003846154	816.81	53,092.92
261	68,121	17,779,581	16.1555	6.3907	.003831418	819.96	53,502.11
262	68,644	17,984,728	16.1864	6.3988	.003816794	823.10	53,912.87
263	69,169	18,191,447	16.2173	6.4070	.003802281	826.24	54,325.21
264	69,696	18,399,744	16.2481	6.4151	.003787879	829.38	54,739.11
265	70,225	18,609,625	16.2788	6.4232	.003773585	832.52	55,154.59
266	70,756	18,821,096	16.3095	6.4312	.003759398	835.66	55,571.63
267	71,289	19,034,163	16.3401	6.4393	.003745318	838.81	55,990.25
268	71,824	19,248,832	16.3707	6.4473	.003731343	841.95	56,410.44
269	72,361	19,465,109	16.4012	6.4553	.003717472	845.09	56,832.20
270	72,900	19,683,000	16.4317	6.4633	.003703704	848.23	57,255.53
271	73,441	19,902,511	16.4621	6.4713	.003690037	851.37	57,680.43
272	73,984	20,123,648	16.4924	6.4792	.003676471	854.51	58,106.90
273	74,529	20,346,417	16.5227	6.4872	.003663004	857.65	58,534.94
274	75,076	20,570,824	16.5529	6.4951	.003649635	860.80	58,964.55
275	75,625	20,796,875	16.5831	6.5030	.003636364	863.94	59,395.74
276	76,176	21,024,576	16.6132	6.5108	.003623188	867.08	59,828.49
277	76,729	21,253,933	16.6433	6.5187	.003610108	870.22	60,262.82
278	77,284	21,484,952	16.6783	6.5265	.003597122	873.36	60,698.71
279	77,841	21,717,639	16.7033	6.5343	.003584229	876.50	61,136.18
280	78,400	21,952,000	16.7332	6.5421	.003571429	879.65	61,575.22
281	78,961	22,188,041	16.7631	6.5499	.003558719	882.79	62,015.82
282	79,524	22,425,768	16.7929	6.5577	.003546099	885.93	62,458.00
283	80,089	22,665,187	16.8226	6.5654	.003533569	889.07	62,901.75
284	80,656	22,906,304	16.8523	6.5731	.003522127	892.21	63,347.07
285	81,225	23,149,125	16.8819	6.5808	.003508772	895.35	63,793.97
286	81,796	23,393,656	16.9115	6.5885	.003496503	898.50	64,242.43
287	82,369	23,639,903	16.9411	6.5962	.003484321	901.64	64,692.46
288	82,944	23,887,872	16.9706	6.6039	.003472222	904.78	65,144.07
289	83,521	24,137,569	17.0000	6.6115	.003460208	907.92	65,597.24
290	84,100	24,389 000	17.0294	6.6191	.003448276	911.06	66,051.99
291	84,681	24,642,171	17.0587	6.6267	.003436426	914.20	66,508.30
292	85,264	24,897,088	17.0880	6.6343	.003424658	917.35	66,966.19
293	85,849	25,153,757	17.1172	6.6419	.003412969	920.49	67,425.65
294	86,436	25,412,184	17.1464	6.6494	.003401361	923.63	67,886.68
295	87,025	25,672,375	17.1756	6.6569	.003389831	926.77	68,349.28
296	87,616	25,934,836	17.2047	6.6644	.003378378	929.91	68,813.45
297	88,209	26,198,073	17.2337	6.6719	.003367003	933.05	69,279.19
298	88,804	26,463,592	17.2627	6.6794	.003355705	936.19	69,746.50
299	89,401	26,730,899	17.2916	6.6869	.003344482	939.34	70,215.38
300	90,000	27,000,000	17.3205	6.6943	.003333333	942.48	70,685.83
301	90,601	27,270,901	17.3494	6.7018	.003322259	945.62	71,157.86
302	91,204	27,543,608	17.3781	6.7092	.003311258	948.76	71,631.45
303	91,809	27,818,127	17.4069	6.7166	.003300330	951.90	72,106.62
304	92,416	28,094,464	17.4356	6.7240	.003289474	955.04	72,583.36
305	93,025	28,372,625	17.4642	6.7313	.003278689	958.19	73,061.66
306	93,636	28,652,616	17.4929	6.7387	.003267974	961.33	73,541.54
307	94,249	28,934,443	17.5214	6.7460	.003257329	964.47	74,022.99

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
308	94,864	29,218,112	17.5499	6.7533	.003246753	967.61	74,506.01
309	95,481	29,503,629	17.5784	6.7606	.003236246	970.75	74,990.60
310	96,100	29,791,000	17.6068	6.7679	.003225806	973.89	75,476.76
311	96,721	30,080,231	17.6352	6.7752	.003215434	977.04	75,964.50
312	97,344	30,371,328	17.6635	6.7824	.003205128	980.18	76,453.80
313	97,969	30,664,297	17.6918	6.7897	.003194888	983.32	76,944.67
314	98,596	30,959,144	17.7200	6.7969	.003184713	986.46	77,437.12
315	99,225	31,255,875	17.7482	6.8041	.003174603	989.60	77,931.13
316	99,856	31,554,496	17.7764	6.8113	.003164557	992.74	78,426.72
317	100,489	31,855,013	17.8045	6.8185	.003154574	995.88	78,923.88
318	101,124	32,157,432	17.8326	6.8256	.003144654	999.03	79,422.60
319	101,761	32,461,759	17.8606	6.8328	.003134796	1,002.17	79,922.90
320	102,400	32,768,000	17.8885	6.8399	.003125000	1,005.31	80,424.77
321	103,041	33,076,161	17.9165	6.8470	.003115265	1,008.45	80,928.21
322	103,684	33,386,248	17.9444	6.8541	.003105590	1,011.59	81,433.22
323	104,329	33,698,267	17.9722	6.8612	.003095975	1,014.73	81,939.80
324	104,976	34,012,224	18.0000	6.8683	.003086420	1,017.88	82,447.96
325	105,625	34,328,125	18.0278	6.8753	.003076923	1,021.02	82,957.68
326	106,276	34,645,976	18.0555	6.8824	.003067485	1,024.16	83,468.98
327	106,929	34,965,783	18.0831	6.8894	.003058104	1,027.30	83,981.84
328	107,584	35,287,552	18.1108	6.8964	.003048780	1,030.44	84,496.28
329	108,241	35,611,289	18.1384	6.9034	.003039514	1,033.58	85,012.28
330	108,900	35,937,000	18.1659	6.9104	.003030303	1,036.73	85,529.86
331	109,561	36,264,691	18.1934	6.9174	.003021148	1,039.87	86,049.01
332	110,224	36,594,368	18.2209	6.9244	.003012048	1,043.01	86,569.73
333	110,889	36,926,037	18.2483	6.9313	.003003003	1,046.15	87,092.02
334	111,556	37,259,704	18.2757	6.9382	.002994012	1,049.29	87,615.88
335	112,225	37,595,375	18.3030	6.9451	.002985075	1,052.43	88,141.31
336	112,896	37,933,056	18.3303	6.9521	.002976190	1,055.58	88,668.31
337	113,569	38,272,753	18.3576	6.9589	.002967359	1,058.72	89,196.88
338	114,244	38,614,472	18.3848	6.9658	.002958580	1,061.86	89,727.03
339	114,921	38,958,219	18.4120	6.9727	.002949853	1,065.00	90,258.74
340	115,600	39,304,000	18.4391	6 9795	.002941176	1,068.14	90,792.03
341	116,281	39,651,821	18.4662	6.9864	.002932551	1,071.28	91,326.88
342	116,964	40,001,688	18.4932	6.9932	.002923977	1,074.42	91,863.31
343	117,649	40,353,607	18.5203	7.0000	.002915452	1,077.57	92,401.31
344	118,336	40,707,584	18.5472	7.0068	.002906977	1,080.71	92,940.88
345	119,025	41,063,625	18.5742	7.0136	.002898551	1,083.85	93,482.02
346	119,716	41,421,736	18.6011	7.0203	.002890173	1,086.99	94,024.73
347	120,409	41,781,923	18.6279	7.0271	.002881844	1,090.13	94,569.01
348	121,104	42,144,192	18.6548	7.0338	.002873563	1,093.27	95,114.86
349	121,801	42,508,549	18.6815	7.0406	.002865330	1,096.42	95,662.28
350	122,500	42,875,000	18.7083	7.0473	.002857143	1,099.56	96,211.28
351	123,201	43,243,551	18.7350	7.0540	.002849003	1,102.70	96,761.84
352	123,904	43,614,208	18.7617	7.0607	.002840909	1,105.84	97,313.97
353	124,609	43,986,977	18.7883	7.0674	.002832861	1,108.98	97,867.68
354	125,316	44,361,864	18.8149	7.0740	.002824859	1,112.12	98,422.96
355	126,025	44,738,875	18.8414	7.0807	.002816901	1,115.27	98,979.80
356	126,736	45,118,016	18.8680	7.0873	.002808989	1,118.41	99,538.22
357	127,449	45,499,293	18.8944	7.0940	.002801120	1,121.55	100,098.21
358	128,164	45,882,712	18.9209	7.1006	.002793296	1,124.69	100,659.77
359	128,881	46,268,279	18.9473	7.1072	.002785515	1,127.83	101,222.90
360	129,600	46,656,000	18.9737	7.1138	.002777778	1,130.97	101,787.60
361	130,321	47,045,881	19.0000	7.1204	.002770083	1,134.11	102,353.87
362	131,044	47,437,928	19.0263	7.1269	.002762431	1,137.26	102,921.72
363	131,769	47,832,147	19.0526	7.1335	.002754821	1,140.40	103,491.18
364	132,496	48,228,544	19.0788	7.1400	.002747253	1,143.54	104,062.12
365	133,225	48,627,125	19.1050	7.1466	.002739726	1,146.68	104,634.67
366	133,956	49,027,896	19.1311	7.1531	.002732240	1,149.82	105,208.80
367	134,689	49,430,863	19.1572	7.1596	.002724796	1,152.96	105,784.49
368	135,424	49,836,032	19.1833	7.1661	.002717391	1,156.11	106,361.76
369	136,161	50,243,409	19.2094	7.1726	.002710027	1,159.25	106,940.60
370	136,900	50,653,000	19.2354	7.1791	.002702703	1,162.39	107,521.01

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
371	137,641	51,064,811	19.2614	7.1855	.002695418	1,165.53	108,102.99
372	138,384	51,478,848	19.2873	7.1920	.002688172	1,168.67	108,686.54
373	139,129	51,895,117	19.3132	7.1984	.002680965	1,171.81	109,271.66
374	139,876	52,313,624	19.3391	7.2048	.002673797	1,174.96	109,858.35
375	140,625	52,734,375	19.3649	7.2112	.002666667	1,178.10	110,446.62
376	141,376	53,157,376	19.3907	7.2177	.002659574	1,181.24	111,036.45
377	142,129	53,582,633	19.4165	7.2240	.002652520	1,184.38	111,627.86
378	142,884	54,010,152	19.4422	7.2304	.002645503	1,187.52	112,220.83
379	143,641	54,439,939	19.4679	7.2368	.002638521	1,190.66	112,815.38
380	144,400	54,872,000	19.4936	7.2432	.002631579	1,193.81	113,411.49
381	145,161	55,306,341	19.5192	7.2495	.002624672	1,196.95	114,009.18
382	145,924	55,742,968	19.5448	7.2558	.002617801	1,200.09	114,608.44
383	146,689	56,181,887	19.5704	7.2622	.002610966	1,203.23	115,209.27
384	147,456	56,623,104	19.5959	7.2685	.002604167	1,206.37	115,811.67
385	148,225	57,066,625	19.6214	7.2748	.002597403	1,209.51	116,415.64
386	148,996	57,512,456	19.6469	7.2811	.002590674	1,212.65	117,021.18
387	149,769	57,960,603	19.6723	7.2874	.002583979	1,215.80	117,628.30
388	150,544	58,411,072	19.6977	7.2936	.002577320	1,218.94	118,236.98
389	151,321	58,863,869	19.7231	7.2999	.002570694	1,222.08	118,847.24
390	152,100	59,319,000	19.7484	7.3061	.002564103	1,225.22	119,459.06
391	152,881	59,776,471	19.7737	7.3124	.002557545	1,228.36	120,072.46
392	153,664	60,236,288	19.7990	7.3186	.002551020	1,231.50	120,687.42
393	154,449	60,698,457	19.8242	7.3248	.002544529	1,234.65	121,303.96
394	155,236	61,162,984	19.8494	7.3310	.002538071	1,237.79	121,922.07
395	156,025	61,629,875	19.8746	7.3372	.002531646	1,240.93	122,541.75
396	156,816	62,099,136	19.8997	7.3434	.002525253	1,244.07	123,163.00
397	157,609	62,570,773	19.9249	7.3496	.002518892	1,247.21	123,785.82
398	158,404	63,044,792	19.9499	7.3558	.002512563	1,250.35	124,410.21
399	159,201	63,521,199	19.9750	7.3619	.002506266	1,253.50	125,036.17
400	160,000	64,000,000	20.0000	7.3681	.002500000	1,256.64	125,663.71
401	160,801	64,481,201	20.0250	7.3742	.002493766	1,259.78	126,292.81
402	161,604	64,964,808	20.0499	7.3803	.002487562	1,262.92	126,923.48
403	162,409	65,450,827	20.0749	7.3864	.002481390	1,266.06	127,555.73
404	163,216	65,939,261	20.0998	7.3925	.002475248	1,269.20	128,189.55
405	164,025	66,430,125	20.1246	7.3986	.002469136	1.272.35	128,824.93
406	164,836	66,923,416	20.1494	7.4047	.002463054	1,275.49	129,461.89
407	165,649	67,419,143	20.1742	7.4108	.002457002	1,278.63	130,100.42
408	166,464	67,917,312	20.1990	7.4169	.002450980	1,281.77	130,740.52
409	167,281	68,417,929	20.2237	7.4229	.002444988	1,284.91	131,382.19
410	168,100	68,921,000	20.2485	7.4290	.002439024	1,288.05	132,025.43
411	168,921	69,426,531	20.2731	7.4350	.002433090	1,291.19	132,670.24
412	169,744	69,934,528	20.2978	7.4410	.002427184	1,294.34	133,316.63
413	170,569	70,444,997	20.3224	7.4470	.002421308	1,297.48	133,964.58
414	171,396	70,957,944	20.3470	7.4530	.002415459	1,300.62	134,614.10
415	172,225	71,473,375	20.3715	7.4590	.002409639	1,303.76	135,265.20
416	173,056	71,991,296	20.3961	7.4650	.002403846	1,306.90	135,917.86
417	173,889	72,511,713	20.4206	7.4710	.002398082	1,310.04	136,572.10
418	174,724	73,034,632	20.4450	7.4770	.002392344	1,313.19	137,227.91
419	175,561	73,560,059	20.4695	7.4829	.002386635	1,316.33	137,885.29
420	176,400	74,088,000	20.4939	7.4889	.002380952	1,319.47	138,544.24
421	177,241	74,618,461	20.5183	7.4948	.002375297	1,322.61	139,204.76
422	178,084	75,151,448	20.5426	7.5007	.002369668	1,325.75	139,866.85
423	178,929	75,686,967	20.5670	7.5067	.002364066	1,328.89	140,530.51
424	179,776	76,225,024	20.5913	7.5126	.002358491	1,332.04	141,195.74
425	180,625	76,765,625	20.6155	7.5185	.002352941	1,335.18	141,862.54
426	181,476	77,308,776	20.6398	7.5244	.002347418	1,338.32	142,530.92
427	182,329	77,854,483	20.6640	7.5302	.002341920	1,341.46	143,200.86
428	183,184	78,402,752	20.6882	7.5361	.002336449	1,344.60	143,872.38
429	184,041	78,953,589	20.7123	7.5420	.002331002	1,347.74	144,545.46
430	184,900	79,507,000	20.7364	7.5478	.002325581	1,350.88	145,220.12
431	185,761	80,062,991	20.7605	7.5537	.002320186	1,354.03	145,896.35
432	186,624	80,621,568	20.7846	7.5595	.002314815	1,357.17	146,574.15
433	187,489	81,182,737	20.8087	7.5654	.002309469	1,360.31	147,253.52

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
434	188,356	81,746,504	20.8327	7.5712	.002304147	1,363.45	147,934.46
435	189,225	82,312,875	20.8567	7.5770	.002298851	1,366.59	148,616.97
436	190,096	82,881,856	20.8806	7.5828	.002293578	1,369.73	149,301.05
437	190,969	83,453,453	20.9045	7.5886	.002288330	1,372.88	149,986.70
438	191,844	84,027,672	20.9284	7.5944	.002283105	1,376.02	150,673.93
439	192,721	84,604,519	20.9523	7.6001	.002277904	1,379.16	151,362.72
440	193,600	85,184,000	20.9762	7.6059	.002272727	1,382.30	152,053.08
441	194,481	85,766,121	21.0000	7.6117	.002267574	1,385.44	152,745.02
442	195,364	86,350,888	21.0238	7.6174	.002262443	1,388.58	153,438.53
443	196,249	86,938,307	21.0476	7.6232	.002257336	1,391.73	154,133.60
444	197,136	87,528,384	21.0713	7.6289	.002252252	1,394.87	154,830.25
445	198,025	88,121,125	21.0950	7.6346	.002247191	1,398.01	155,528.47
446	198,916	88,716,536	21.1187	7.6403	.002242152	1,401.15	156,228.26
447	199,809	89,314,623	21.1424	7.6460	.002237136	1,404.29	156,929.62
448	200,704	89,915,392	21.1660	7.6517	.002232143	1,407.43	157,632.55
449	201,601	90,518,849	21.1896	7.6574	.002227171	1,410.58	158,337.06
450	202,500	91,125,000	21.2132	7.6631	.002222222	1,413.72	159,043.13
451	203,401	91,733,851	21.2368	7.6688	.002217295	1,416.86	159,750.77
452	204,304	92,345,408	21.2603	7.6744	.002212389	1,420.00	160,459.99
453	205,209	92,959,677	21.2838	7.6801	.002207506	1,423.14	161,170.77
454	206,116	93,576,664	21.3073	7.6857	.002202643	1,426.28	161,883.13
455	207,025	94,196,375	21.3307	7.6914	.002197802	1,429.42	162,597.05
456	207,936	94,818,816	21.3542	7.6970	.002192982	1,432.57	163,312.55
457	208,849	95,443,993	21.3776	7.7026	.002188184	1,435.71	164,029.62
458	209,764	96,071,912	21.4009	7.7082	.002183406	1,438.85	164,748.26
459	210,681	96,702,579	21.4243	7.7188	.002178649	1,441.99	165,468.47
460	211,600	97,336,000	21.4476	7.7194	.002173913	1,445.13	166,190.25
461	212,521	97,972,181	21.4709	7.7250	.002169197	1,448.27	166,913.60
462	213,444	98,611,128	21.4942	7.7306	.002164502	1,451.42	167,638.53
463	214,369	99,252,847	21.5174	7.7362	.002159827	1,454.56	168,365.02
464	215,296	99,897,344	21.5407	7.7418	.002155172	1,457.70	169,093.08
465	216,225	100,544,625	21.5639	7.7473	.002150538	1,460.84	169,822.72
466	217,156	101,194,696	21.5870	7.7529	.002145923	1,463.98	170,553.92
467	218,089	101,847,563	21.6102	7.7584	.002141328	1,467.12	171,286.70
468	219,024	102,503,232	21.6333	7.7639	.002136752	1,470.27	172,021.05
469	219,961	103,161,709	21.6564	7.7695	.002132196	1,473.41	172,756.97
470	220,900	103,823,000	21.6795	7.7750	.002127660	1,476.55	173,494.45
471	221,841	104,487,111	21.7025	7.7805	.002123142	1,479.69	174,233.51
472	222,784	105,154,048	21.7256	7.7860	.002118644	1,482.83	174,974.14
473	223,729	105,823,817	21.7486	7.7915	.002114165	1,485.97	175,716.35
474	224,676	106,496,424	21.7715	7.7970	.002109705	1,489.11	176,460.12
475	225,625	107,171,875	21.7945	7.8025	.002105263	1,492.26	177,205.46
476	226,576	107,850,176	21.8174	7.8079	.002100840	1,495.40	177,952.37
477	227,529	108,531,333	21.8403	7.8134	.002096486	1,498.54	178,700.86
478	228,484	109,215,352	21.8632	7.8188	.002092050	1,501.68	179,450.91
479	229,441	109,902,239	21.8861	7.8243	.002087683	1,504.82	180,202.54
480	230,400	110,592,000	21.9089	7.8297	.002083333	1,507.96	180,955.74
481	231,361	111,284,641	21.9317	7.8352	.002079002	1,511.11	181,710.50
482	232,324	111,980,168	21.9545	7.8406	.002074689	1,514.25	182,466.84
483	233,289	112,678,587	21.9775	7.8460	.002070393	1,517.39	183,224.75
484	234,256	113,379,904	22.0000	7.8514	.002066116	1,520.53	183,984.23
485	235,225	114,084,125	22.0227	7.8568	.002061856	1,523.67	184,745.28
486	236,196	114,791,256	22.0454	7.8622	.002057613	1,526.81	185,507.90
487	237,169	115,501,303	22.0681	7.8676	.002053388	1,529.96	186,272.10
488	238,144	116,214,272	22.0907	7.8730	.002049180	1,533.10	187,037.86
489	239,121	116,930,169	22.1133	7.8784	.002044990	1,536.24	187,805.19
490	240,100	117,649,000	22.1359	7.8837	.002040816	1,539.38	188,574.10
491	241,081	118,370,771	22.1585	7.8891	.002036660	1,542.52	189,344.57
492	242,064	119,095,488	22.1811	7.8944	.002032520	1,545.66	190,116.62
493	243,049	119,823,157	22.2036	7.8998	.002028398	1,548.81	190,890.24
494	244,036	120,553,784	22.2261	7.9051	.002024291	1,551.95	191,665.43
495	245,025	121,287,875	22.2486	7.9105	.002020292	1,555.09	192,442.18
496	246,016	122,023,936	22.2711	7.9158	.002016129	1,558.23	193,220.51

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
497	247,009	122,763,473	22.2935	7.9211	.002012072	1,561.37	194,000.41
498	248,004	123,505,992	22.3159	7.9264	.002008032	1,564.51	194,781.89
499	249,001	124,251,499	22.3383	7.9317	.002004008	1,567.65	195,564.93
500	250,000	125,000,000	22.3607	7.9370	.002000000	1,570.80	196,349.54
501	251,001	125,751,501	22.3830	7.9423	.001996008	1,573.94	197,135.72
502	252,004	126,506,008	22.4054	7.9476	.001992032	1,577.08	197,923.48
503	253,009	127,263,527	22.4277	7.9528	.001988072	1,580.22	198,712.80
504	254,016	128,024,064	22.4499	7.9581	.001984127	1,583.36	199,503.70
505	255,025	128,787,625	22.4722	7.9634	.001980198	1,586.50	200,296.17
506	256,036	129,554,216	22.4944	7.9686	.001976285	1,589.65	201,090.20
507	257,049	130,323,843	22.5167	7.9739	.001972387	1,592.79	201,885.81
508	258,064	131,096,512	22.5389	7.9791	.001968504	1,595.93	202,682.99
509	259,081	131,872,229	22.5610	7.9843	.001964637	1,599.07	203,481.74
510	260,100	132,651,000	22.5832	7.9895	.001960785	1,602.21	204,282.06
511	261,121	133,432,831	22.6053	7.9948	.001956947	1,605.35	205,083.95
512	262,144	134,217,728	22.6274	8.0000	.001953125	1,608.50	205,887.42
513	263,169	135,005,697	22.6495	8.0052	.001949318	1,611.64	206,692.45
514	264,196	135,796,744	22.6716	8.0104	.001945525	1,614.78	207,499.05
515	265,225	136,590,875	22.6936	8.0156	.001941748	1,617.92	208,307.23
516	266,256	137,388,096	22.7156	8.0208	.001937984	1,621.06	209,116.97
517	267,289	138,188,413	22.7376	8.0260	.001934236	1,624.20	209,928.29
518	268,324	138,991,832	22.7596	8.0311	.001930502	1,627.34	210,741.18
519	269,361	139,798,359	22.7816	8.0363	.001926782	1,630.49	211,555.63
520	270,400	140,608,000	22.8035	8.0415	.001923077	1,633.63	212,371.66
521	271,441	141,420,761	22.8254	8.0466	.001919386	1,636.77	213,189.26
522	272,484	142,236,648	22.8473	8.0517	.001915709	1,639.91	214,008.43
523	273,529	143,055,667	22.8692	8.0569	.001912046	1,643.05	214,829.17
524	274,576	143,877,824	22.8910	8.0620	.001908397	1,646.19	215,651.49
525	275,625	144,703,125	22.9129	8.0671	.001904762	1,649.34	216,475.37
526	276,676	145,531,576	22.9347	8.0723	.001901141	1,652.48	217,300.82
527	277,729	146,363,183	22.9565	8.0774	.001897533	1,655.62	218,127.85
528	278,784	147,197,952	22.9783	8.0825	.001893939	1,658.76	218,956.44
529	279,841	148,035,889	23.0000	8.0876	.001890359	1,661.90	219,786.61
530	280,900	148,877,001	23.0217	8.0927	.001886792	1,665.04	220,618.34
531	281,961	149,721,291	23.0434	8.0978	.001883239	1,668.19	221,451.65
532	283,024	150,568,768	23.0651	8.1028	.001879699	1,671.33	222,286.53
533	284,089	151,419,437	23 0868	8.1079	.001876173	1,674.47	223,122.98
534	285,156	152,273,304	23.1084	8.1130	.001872659	1,677.61	223,961.00
535	286,225	153,130,375	23.1301	8.1180	.001869159	1,680.75	224,800.59
536	287,296	153,990,656	23.1517	8.1231	.001865672	1,683.89	225,641.75
537	288,369	154,854,153	23.1733	8.1281	.001862197	1,687.04	226,484.48
538	289,444	155,720,872	23.1948	8.1332	.001858736	1,690.18	227,328.79
539	290,521	156,590,819	23.2164	8.1382	.001855288	1,693.32	228,174.66
540	291,600	157,464,000	23.2379	8.1433	.001851852	1,696.46	229,022.10
541	292,681	158,340,421	23.2594	8.1483	.001848429	1,699.60	229,871.12
542	293,764	159,220,088	23.2809	8.1533	.001845018	1,702.74	230,721.71
543	294,849	160,103,007	23.3024	8.1583	.001841621	1,705.88	231,573.86
544	295,936	160,989,184	23.3238	8.1633	.001838235	1,709.03	232,427.59
545	297,025	161,878,625	23.3452	8.1683	.001834862	1,712.17	233,282.89
546	298,116	162,771,336	23.3666	8.1733	.001831502	1,715.31	234,139.76
547	299,209	163,667,323	23.3880	8.1783	.001828154	1,718.45	234,998.20
548	300,304	164,566,592	23.4094	8.1833	.001824818	1,721.59	235,858.21
549	301,401	165,469,149	23.4307	8.1882	.001821494	1,724.73	236,719.79
550	302,500	166,375,000	23.4521	8.1932	.001818182	1,727.88	237,582.94
551	303,601	167,284,151	23.4734	8.1982	.001814882	1,731.02	238,447.67
552	304,704	168,196,608	23.4947	8.2031	.001811594	1,734.16	239,313.96
553	305,809	169,112,377	23.5160	8.2081	.001808318	1,737.30	240,181.83
554	306,916	170,031,464	23.5372	8.2130	.001805054	1,740.44	241,051.26
555	308,025	170,953,875	23.5584	8.2180	.001801802	1,743.58	241,922.27
556	309,136	171,879,616	23.5797	8.2229	.001798561	1,746.73	242,794.85
557	310,249	172,808,693	23.6008	8.2278	.001795332	1,749.87	243,668.99
558	311,364	173,741,112	23.6220	8.2327	.001792115	1,753.01	244,544.71
559	312,481	174,676,879	23.6432	8.2377	.001788909	1,756.15	245,422.00

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
560	313,600	175,616,000	23.6643	8.2426	.001785714	1,759.29	246,300.86
561	314,721	176,558,481	23.6854	8.2475	.001782531	1,762.43	247,181.30
562	315,844	177,504,328	23.7065	8.2524	.001779359	1,765.58	248,063.30
563	316,969	178,453,547	23.7276	8.2573	.001776199	1,768.72	248,946.87
564	318,096	179,406,144	23.7487	8.2621	.001773050	1,771.86	249,832.01
565	319,225	180,362,125	23.7697	8.2670	.001769912	1,775.00	250,718.73
566	320,356	181,321,496	23.7908	8.2719	.001766784	1,778.14	251,607.01
567	321,489	182,284,263	23.8118	8.2768	.001763668	1,781.28	252,496.87
568	322,624	183,250,432	23.8328	8.2816	.001760563	1,784.42	253,388.30
569	323,761	184,220,009	23.8537	8.2865	.001757469	1,787.57	254,281.29
570	324,900	185,193,000	23.8747	8.2913	.001754386	1,790.71	255,175.86
571	326,041	186,169,411	23.8956	8.2962	.001751313	1,793.85	256,072.00
572	327,184	187,149,248	23.9165	8.3010	.001748252	1,796.99	256,969.71
573	328,329	188,132,517	23.9374	8.3059	.001745201	1,800.13	257,868.99
574	329,476	189,119,224	23.9583	8.3107	.001742164	1,803.27	258,769.85
575	330,625	190,109,375	23.9792	8.3155	.001739130	1,806.42	259,672.27
576	331,776	191,102,976	24.0000	8.3203	.001736111	1,809.56	260,576.26
577	332,929	192,100,033	24.0208	8.3251	.001733102	1,812.70	261,481.83
578	334,084	193,100,552	24.0416	8.3300	.001730104	1,815.84	262,388.96
579	335,241	194,104,539	24.0624	8.3348	.001727116	1,818.98	263,297.67
580	336,400	195,112,000	24.0832	8.3396	.001724138	1,822.12	264,207.94
581	337,561	196,122,941	24.1039	8.3443	.001721170	1,825.27	265,119.79
582	338,724	197,137,368	24.1247	8.3491	.001718213	1,828.41	266,033.21
583	339,889	198,155,287	24.1454	8.3539	.001715266	1,831.55	266,948.20
584	341,056	199,176,704	24.1661	8.3587	.001712329	1,834.69	267,864.76
585	342,225	200,201,625	24.1868	8.3634	.001709402	1,837.83	268,782.89
586	343,396	201,230,056	24.2074	8.3682	.001706485	1,840.97	269,702.59
587	344,569	202,262,003	24.2281	8.3730	.001703578	1,844.11	270,623.86
588	345,744	203,297,472	24.2487	8.3777	.001700680	1,847.26	271,546.70
589	346,921	204,336,469	24.2693	8.3825	.001697793	1,850.40	272,471.12
590	348,100	205,379,000	24.2899	8.3872	.001694915	1,853.54	273,397.10
591	349,281	206,425,071	24.3105	8.3919	.001692047	1,856.68	274,324.66
592	350,464	207,474,688	24.3311	8.3967	.001689189	1,859.82	275,253.78
593	351,649	208,527,857	24.3516	8.4014	.001686341	1,862.96	276,184.48
594	352,836	209,584,584	24.3721	8.4061	.001683502	1,866.11	277,116.75
595	354,025	210,644,875	24.3926	8.4108	.001680672	1,869.25	278,050.58
596	355,216	211,708,736	24.4131	8.4155	.001677852	1,872.39	278,985.99
597	356,409	212,776,173	24.4336	8.4202	.001675042	1,875.53	279,922.97
598	357,604	213,847,192	24.4540	8.4249	.001672241	1,878.67	280,861.52
599	358,801	214,921,799	24.4745	8.4296	.001669449	1,881.81	281,801.65
600	360,000	216,000,000	24.4949	8.4343	.001666667	1,884.96	282,743.34
601	361,201	217,081,801	24.5153	8.4390	.001663894	1,888.10	283,686.60
602	362,404	218,167,208	24.5357	8.4437	.001661130	1,891.24	284,631.44
603	363,609	219,256,227	24.5561	8.4484	.001658375	1,894.38	285,577.84
604	364,816	220,348,864	24.5764	8.4530	.001655629	1,897.52	286,525.82
605	366,025	221,445,125	24.5968	8.4577	.001652893	1,900.66	287,475.36
606	367,236	222,545,016	24.6171	8.4623	.001650165	1,903.81	288,426.48
607	368,449	223,648,543	24.6374	8.4670	.001647446	1,906.95	289,379.17
608	369,664	224,755,712	24.6577	8.4716	.001644737	1,910.09	290,333.43
609	370,881	225,866,529	24.6779	8.4763	.001642036	1,913.23	291,289.26
610	372,100	226,981,000	24.6982	8.4809	.001639344	1,916.37	292,246.66
611	373,321	228,099,131	24.7184	8.4856	.001636661	1,919.51	293,205.63
612	374,544	229,220,928	24.7386	8.4902	.001633987	1,922.65	294,166.17
613	375,769	230,346,397	24.7588	8.4948	.001631321	1,925.80	295,128.28
614	376,996	231,475,544	24.7790	8.4994	.001628664	1,928.94	296,091.97
615	378,225	232,608,375	24.7992	8.5040	.001626016	1,932.08	297,057.22
616	379,456	233,744,896	24.8193	8.5086	.001623377	1,935.22	298,024.05
617	380,689	234,885,113	24.8395	8.5132	.001620746	1,938.36	298,992.44
618	381,924	236,029,032	24.8596	8.5178	.001618123	1,941.50	299,962.41
619	383,161	237,176,659	24.8797	8.5224	.001615509	1,944.65	300,933.95
620	384,400	238,328,000	24.8998	8.5270	.001612903	1,947.79	301,907.05
621	385,641	239,483,061	24.9199	8.5316	.001610306	1,950.93	302,881.73
622	386,884	240,641,848	24.9399	8.5362	.001607717	1,954.07	303,857.98

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
623	388,129	241,804,367	24.9600	8.5408	.001605136	1,957.21	304,835.80
624	389,376	242,970,624	24.9800	8.5453	.001602564	1,960.35	305,815.20
625	390,625	214,140,625	25.0000	8.5499	.001600000	1,963.50	306,796.16
626	391,876	245,314,376	25.0200	8.5544	.001597444	1,966.64	307,778.69
627	393,129	246,491,883	25.0400	8.5589	.001594896	1,969.78	308,762.79
628	394,384	247,673,152	25.0599	8.5635	.001592357	1,972.92	309,748.47
629	395,641	248,858,189	25.0799	8.5681	.001589825	1,976.06	310,735.71
630	396,900	250,047,000	25.0998	8.5726	.001587302	1,979.20	311,724.53
631	398,161	251,239,591	25.1197	8.5772	.001584786	1,982.35	312,714.92
632	399,424	252,435,968	25.1396	8.5817	.001582278	1,985.49	313,706.88
633	400,689	253,636,137	25.1595	8.5862	.001579779	1,988.63	314.700.40
634	401,956	254,840,104	25.1794	8.5907	.001577287	1,991.77	315.695.50
635	403,225	256,047,875	25.1992	8.5952	.001574803	1,994.91	316.692.17
636	404,496	257,259,456	25.2190	8.5997	.001572327	1,998.05	317,690.42
637	405,769	258,474,853	25.2389	8.6043	.001569859	2,001.19	318,690.23
638	407,044	259,694,072	25.2587	8.6088	.001567398	2,004.34	319,691.61
639	408,321	260,917,119	25.2784	8.6132	.001564945	2,007.48	320,694.56
640	409,600	262,144,000	25.2982	8.6177	.001562500	2,010.62	321,699.09
641	410,881	263,374,721	25.3180	8.6222	.001560062	2,013.76	322.705.18
642	412,164	264,609,288	25.3377	8.6267	.001557632	2,016.90	323,712.85
643	413,449	265,847,707	25.3574	8.6312	.001555210	2,020.04	324,722.09
644	414,736	267,089,984	25.3772	8.6357	.001552795	2,023.19	325,732.89
645	416,125	268,336,125	25.3969	8.6401	.001550388	2,026.33	326,745.27
646	417,316	269,585,136	25.4165	8.6446	.001547988	2,029.47	327,759.22
647	418,609	270,840,023	25.4362	8.6490	.001545595	2,032.61	328,774.74
648	419,904	272,097,792	25.4558	8.6535	.001543210	2,035.75	329,791.83
649	421,201	273,359,449	25.4755	8.6579	.001540832	2,038.89	330,810.49
650	422,500	274,625,000	25.4951	8.6624	.001538462	2,042.04	331,830.72
651	423,801	275,894,451	25.5147	8.6668	.001536098	2,045.18	332,852.53
652	425,104	277,167,808	25.5343	8.6713	.001533742	2,048.32	333,875.90
653	426,409	278,445,077	25.5539	8.6757	.001531394	2,051.46	334,900.85
654	427,716	279,726,264	25.5734	8.6801	.001529052	2,054.60	335,927.36
655	429,025	281,011,375	25.5930	8.6845	.001526718	2,057.74	336,955.45
656	430,336	282,300,416	25.6125	8.6890	.001524390	2,060.88	337,985.10
657	431,639	283,593,393	25.6320	8.6934	.001522070	2,064.03	339,016.33
658	432,964	284,890,312	25.6515	8.6978	.001519751	2,067.17	340,049.13
659	434,281	286,191,179	25.6710	8.7022	.001517451	2,070.31	341,083.50
660	435,600	287,496,000	25.6905	8.7066	.001515152	2,073.45	342,119.44
661	436,921	288,804,781	25.7099	8.7110	.001512859	2,076.59	343,156.95
662	438,244	290,117,528	25.7294	8.7154	.001510574	2,079.73	341,196.03
663	439,569	291,434,247	25.7488	8.7198	.001508296	2,082.88	345,236.69
664	440,896	292,754,944	25.7682	8.7241	.001506024	2,086.02	346,278.91
665	442,225	294,079,625	25.7876	8.7285	.001503759	2,089.16	347,322.70
666	443,556	295,408,296	25.8070	8.7329	.001501502	2,092.30	348,368.07
667	444,899	296,740,963	25.8263	8.7373	.001499250	2,095.44	349,415.00
668	446,224	298,077,632	25.8457	8.7416	.001497006	2,098.58	350,463.51
669	447.561	299,418,309	25.8650	8.7460	.001494768	2,101.73	351,513.59
670	448,900	300,763,000	25.8844	8.7503	.001492537	2,104.87	352,565.24
671	450,241	302,111,711	25.9037	8.7547	.001490313	2,108.01	353,618.45
672	451,584	303,464,448	25.9230	8.7590	.001488095	2,111.15	354,673.24
673	452,929	304,821,217	25.9422	8.7634	.001485884	2,114.29	355,729.60
674	454,276	306,182,024	25.9615	8.7677	.001483680	2,117.43	356,787.54
675	455,625	307,546,875	25.9808	8.7721	.001481481	2,120.58	357,847.04
676	456,976	308,915,776	26.0000	8.7764	.001479290	2,123.72	358,908.11
677	458,329	310,288,733	26.0192	8.7807	.001477105	2,126.86	359,970.75
678	459,684	311,665,752	26.0384	8.7850	.001474926	2,130.00	361,034.97
679	461,041	313,046,839	26.0576	8.7893	.001472754	2,133.14	362,100.75
680	462,400	314,432,000	26.0768	8.7937	.001470588	2,136.28	363,168.11
681	463,761	315,821,241	26.0960	8.7980	.001468429	2,139.42	364,237.04
682	465,124	317,214,568	26.1151	8.8023	.001466276	2,142.57	365,307.54
683	466,489	318,611,987	26.1343	8.8066	.001464129	2,145.71	366,379.60
684	467,856	320,013,504	26.1534	8.8109	.001461988	2,148.85	367,453.24
685	469,225	321,419,125	26.1725	8.8152	.001459854	2,151.99	368,528.45

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
686	470,596	322,828,856	26.1916	8.8194	.001457726	2,155.13	369,605.28
687	471,969	324,242,703	26.2107	8.8237	.001455604	2,158.27	370,683.59
688	473,344	325,660,672	26.2298	8.8280	.001453488	2,161.42	371,763.51
689	474,721	327,082,769	26.2488	8.8323	.001451379	2,164.56	372,845.00
690	476,100	328,509,000	26.2679	8.8366	.001449275	2,167.70	373,928.07
691	477,481	329,939,371	26.2869	8.8408	.001447178	2,170.84	375,012.70
692	478,864	331,373,888	26.3059	8.8451	.001445087	2,173.98	376,098.91
693	480,249	332,812,557	26.3249	8.8493	.001443001	2,177.12	377,186.68
694	481,636	334,255,384	26.3439	8.8536	.001440922	2,180.27	378,276.03
695	483,025	335,702,375	26.3629	8.8578	.001438849	2,183.41	379,366.95
696	484,416	337,153,536	26.3818	8.8621	.001436782	2,186.55	380,459.44
697	485,809	338,608,873	26.4008	8.8663	.001434720	2,189.69	381,553.50
698	487,204	340,068,392	26.4197	8.8706	.001432665	2,192.83	382,649.13
699	488,601	341,532,099	26.4386	8.8748	.001430615	2,195.97	383,746.33
700	490,000	343,000,000	26.4575	8.8790	.001428571	2,199.11	384,845.10
701	491,401	344,472,101	26.4764	8.8833	.001426534	2,202.26	385,945.44
702	492,804	345,948,408	26.4953	8.8875	.001424501	2,205.40	387,047.36
703	494,209	347,428,927	26.5141	8.8917	.001422475	2,208.54	388,150.84
704	495,616	348,913,664	26.5330	8.8959	.001420455	2,211.68	389,255.90
705	497,025	350,402,625	26.5518	8.9001	.001418440	2,214.82	390,362.52
706	498,436	351,895,816	26.5707	8.9043	.001416431	2,217.96	391,470.72
707	499,849	353,393,243	26.5895	8.9085	.001414427	2,221.11	392,580.49
708	501,264	354,894,912	26.6083	8.9127	.001412429	2,224.25	393,691.82
709	502,681	356,400,829	26.6271	8.9169	.001410437	2,227.39	394,804.73
710	504,100	357,911,000	26.6458	8.9211	.001408451	2,230.53	395,919.21
711	505,521	359,425,431	26.6646	8.9253	.001406470	2,233.67	397,035.26
712	506,944	360,944,128	26.6833	8.9295	.001404494	2,236.81	398,152.89
713	508,369	362,467,097	26.7021	8.9337	.001402525	2,239.96	399,272.08
714	509,796	363,994,344	26.7208	8.9378	.001400560	2,243.10	400,392.84
715	511,225	365,525,875	26.7395	8.9420	.001398601	2,246.24	401,515.18
716	512,656	367,061,696	26.7582	8.9462	.001396648	2,249.38	402,639.08
717	514,089	368,601,813	26.7769	8.9503	.001394700	2,252.52	403,764.56
718	515,524	370,146,232	26.7955	8.9545	.001392758	2,255.66	404,891.60
719	516,961	371,694,959	26.8142	8.9587	.001390821	2,258.81	406,020.22
720	518,400	373,248,000	26.8328	8.9628	.001388889	2,261.95	407,150.41
721	519,841	374,805,361	26.8514	8.9670	.001386963	2,265.09	408,282.17
722	521,284	376,367,048	26.8701	8.9711	.001385042	2,268.23	409,415.50
723	522,729	377,933,067	26.8887	8.9752	.001383126	2,271.37	410,550.40
724	524,176	379,503,424	26.9072	8.9794	.001381215	2,274.51	411,686.87
725	525,625	381,078,125	26.9258	8.9835	.001379310	2,277.65	412,824.91
726	527,076	382,657,176	26.9444	8.9876	.001377410	2,280.80	413,964.52
727	528,529	384,240,583	26.9629	8.9918	.001375516	2,283.94	415,105.71
728	529,984	385,828,352	26.9815	8.9959	.001373626	2,287.08	416,248.46
729	531,441	387,420,489	27.0000	9.0000	.001371742	2,290.22	417,392.79
730	532,900	389,017,000	27.0185	9.0041	.001369863	2,293.36	418,538.68
731	534,361	390,617,891	27.0370	9.0082	.001367989	2,296.50	419,686.15
732	535,824	392,223,168	27.0555	9.0123	.001366120	2,299.65	420,835.19
733	537,289	393,832,837	27.0740	9.0164	.001364256	2,302.79	421,985.79
734	538,756	395,446,904	27.0924	9.0205	.001362398	2,305.93	423,137.97
735	540,225	397,065,375	27.1109	9.0246	.001360544	2,309.07	424,291.72
736	541,696	398,688,256	27.1293	9.0287	.001358696	2,312.21	425,447.04
737	543,169	400,315,553	27.1477	9.0328	.001356852	2,315.35	426,603.94
738	544,644	401,947,272	27.1662	9.0369	.001355014	2,318.50	427,762.40
739	546,121	403,583,419	27.1846	9.0410	.001353180	2,321.64	428,922.43
740	547,600	405,224,000	27.2029	9.0450	.001351351	2,324.78	430,084.03
741	549,081	406,869,021	27.2213	9.0491	.001349528	2,327.92	431,247.21
742	550,564	408,518,488	27.2397	9.0532	.001347709	2,331.06	432,411.95
743	552,049	410,172,407	27.2580	9.0572	.001345895	2,334.20	433,578.27
744	553,536	411,830,784	27.2764	9.0613	.001344086	2,337.34	434,746.16
745	555,025	413,493,625	27.2947	9.0654	.001342282	2,340.49	435,915.62
746	556,516	415,160,936	27.3130	9.0694	.001340483	2,343.63	437,086.64
747	558,009	416,832,723	27.3313	9.0735	.001338688	2,346.77	438,259.24
748	559,504	418,508,992	27.3496	9.0775	.001336898	2,349.91	439,433.41

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
749	561,001	420,189,749	27.3679	9.0816	.001335113	2,353.05	440,609.16
750	562,500	421,875,000	27.3861	9.0856	.001333333	2,356.19	441,786.47
751	564,001	423,564,751	27.4044	9.0896	.001331558	2,359.34	442,965.35
752	565,504	425,259,008	27.4226	9.0937	.001329787	2,362.48	444,145.80
753	567,009	426,957,777	27.4408	9.0977	.001328021	2,365.62	445,327.83
754	568,516	428,661,064	27.4591	9.1017	.001326260	2,368.76	446,511.42
755	570,025	430,368,875	27.4773	9.1057	.001324503	2,371.90	447,696.59
756	571,536	432,081,216	27.4955	9.1098	.001322751	2,375.04	448,883.32
757	573,049	433,798,093	27.5136	9.1138	.001321004	2,378.19	450,071.63
758	574,564	435,519,512	27.5318	9.1178	.001319261	2,381.33	451,261.51
759	576,081	437,245,479	27.5500	9.1218	.001317523	2,384.47	452,452.96
760	577,600	438,976,000	27.5681	9.1258	.001315789	2,387.61	453,645.98
761	579,121	440,711,081	27.5862	9.1298	.001314060	2,390.75	454,840.57
762	580,644	442,450,728	27.6043	9.1338	.001312336	2,393.89	456,036.73
763	582,169	444,194,947	27.6225	9.1378	.001310616	2,397.04	457,234.46
764	583,696	445,943,744	27.6405	9.1418	.001308901	2,400.18	458,433.77
765	585,225	447,697,125	27.6586	9.1458	.001307190	2,403.32	459,634.64
766	586,756	449,455,096	27.6767	9.1498	.001305483	2,406.46	460,837.08
767	588,289	451,217,663	27.6948	9.1537	.001303781	2,409.60	462,041.10
768	589,824	452,984,832	27.7128	9.1577	.001302083	2,412.74	463,246.69
769	591,361	454,756,609	27.7308	9.1617	.001300390	2,415.88	464,453.84
770	592,900	456,533,000	27.7489	9.1657	.001298701	2,419.03	465,662.57
771	594,441	458,314,011	27.7669	9.1696	.001297017	2,422.17	466,872.87
772	595,984	460,099,648	27.7849	9.1736	.001295337	2,425.31	468,084.74
773	597,529	461,889,917	27.8029	9.1775	.001293661	2,428.45	469,298.18
774	599,076	463,684,824	27.8209	9.1815	.001291990	2,431.59	470,513.19
775	600,625	465,484,375	27.8388	9.1855	.001290323	2,434.73	471,729.77
776	602,176	467,288,576	27.8568	9.1894	.001288660	2,437.88	472,947.92
777	603,729	469,097,433	27.8747	9.1933	.001287001	2,441.02	474,167.65
778	605,284	470,910,952	27.8927	9.1973	.001285347	2,444.16	475,388.94
779	606,841	472,729,139	27.9106	9.2012	.001283697	2,447.30	476,611.81
780	608,400	474,552,000	27.9285	9.2052	.001282051	2,450.44	477,836.24
781	609,961	476,379,541	27.9464	9.2091	.001280410	2,453.58	479,062.25
782	611,524	478,211,768	27.9643	9.2130	.001278772	2,456.73	480,289.83
783	613,089	480,048,687	27.9821	9.2170	.001277139	2,459.87	481,518.97
784	614,656	481,890,304	28.0000	9.2209	.001275510	2,463.01	482,749.69
785	616,225	483,736,625	28.0179	9.2248	.001273885	2,466.15	483,981.98
786	617,796	485,587,656	28.0357	9.2287	.001272265	2,469.29	485,215.84
787	619,369	487,443,403	28.0535	9.2326	.001270648	2,472.43	486,451.28
788	620,944	489,303,872	28.0713	9.2365	.001269036	2,475.58	487,688.28
789	622,521	491,169,069	28.0891	9.2404	.001267427	2,478.72	488,926.85
790	624,100	493,039,000	28.1069	9.2443	.001265823	2,481.86	490,166.99
791	625,681	494,913,671	28.1247	9.2482	.001264223	2,485.00	491,408.71
792	627,264	496,793,088	28.1425	9.2521	.001262626	2,488.14	492,651.99
793	628,849	498,677,257	28.1603	9.2560	.001261034	2,491.28	493,896.85
794	630,436	500,566,184	28.1780	9.2599	.001259446	2,494.42	495,143.28
795	632,025	502,459,875	28.1957	9.2638	.001257862	2,497.57	496,391.27
796	633,616	504,358,336	28.2135	9.2677	.001256281	2,500.71	497,640.84
797	635,209	506,261,573	28.2312	9.2716	.001254705	2,503.85	498,891.98
798	636,804	508,169,592	28.2489	9.2754	.001253133	2,506.99	500,144.69
799	638,401	510,082,399	28.2666	9.2793	.001251364	2,510.13	501,398.97
800	640,000	512,000,000	28.2843	9.2832	.001250000	2,513.27	502,654.82
801	641,601	513,922,401	28.3019	9.2870	.001248439	2,516.42	503,912.25
802	643,204	515,849,608	28.3196	9.2909	.001246883	2,519.56	505,171.24
803	644,809	517,781,627	28.3373	9.2948	.001245330	2,522.70	506,431.80
804	646,416	519,718,464	28.3549	9.2986	.001243781	2,525.84	507,693.94
805	648,025	521,660,125	28.3725	9.3025	.001242236	2,528.98	508,957.64
806	649,636	523,606,616	28.3901	9.3063	.001240695	2,532.12	510,222.92
807	651,249	525,557,943	28.4077	9.3102	.001239157	2,535.27	511,489.77
808	652,864	527,514,112	28.4253	9.3140	.001237624	2,538.41	512,758.19
809	654,481	529,475,129	28.4429	9.3179	.001236094	2,541.55	514,028.18
810	656,100	531,441,000	28.4605	9.3217	.001234568	2,544.69	515,299.74
811	657,721	533,411,731	28.4781	9.3255	.001233046	2,547.83	516,572.87

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
812	659,344	535,387,328	28.4956	9.3294	.001231527	2,550.97	517,847.57
813	660,969	537,367,797	28.5132	9.3332	.001230012	2,554.11	519,123.84
814	662,596	539,353,144	28.5307	9.3370	.001228501	2,557.26	520,401.68
815	664,225	541,343,375	28.5482	9.3408	.001226994	2,560.40	521,681.10
816	665,856	543,338,496	28.5657	9.3447	.001225490	2,563.54	522,962.08
817	667,489	545,338,513	28.5832	9.3485	.001223990	2,566.68	524,244.63
818	669,124	547,343,432	28.6007	9.3523	.001222494	2,569.82	525,528.76
819	670,761	549,353,259	28.6182	9.3561	.001221001	2,572.96	526,814.46
820	672,400	551,368,000	28.6356	9.3599	.001219512	2,576.11	528,101.73
821	674,041	553,387,661	28.6531	9.3637	.001218027	2,579.25	529,390.56
822	675,584	555,412,248	28.6705	9.3675	.001216545	2,582.39	530,680.97
823	677,329	557,441,767	28.6880	9.3713	.001215067	2,585.53	531,972.95
824	678,976	559,476,224	28.7054	9.3751	.001213592	2,588.67	533,266.50
825	680,625	561,515,625	28.7228	9.3789	.001212121	2,591.81	534,561.62
826	682,276	563,559,976	28.7402	9.3827	.001210654	2,594.96	535,858.32
827	683,929	565,609,283	28.7576	9.3865	.001209190	2,598.10	537,156.58
828	685,584	567,663,552	28.7750	9.3902	.001207729	2,601.24	538,456.41
829	687,241	569,722,789	28.7924	9.3940	.001206273	2,604.38	539,757.82
830	688,900	571,787,000	28.8097	9.3978	.001204819	2,607.52	541,060.79
831	690,561	573,856,191	28.8271	9.4016	.001203369	2,610.66	542,365.34
832	692,224	575,930,368	28.8444	9.4053	.001201923	2,613.81	543 671.46
833	693,889	578,009,537	28.8617	9.4091	.001200480	2,616.95	544,979.15
834	695,556	580,093,704	28.8791	9.4129	.001199041	2,620.09	546,288.40
835	697,225	582,182,875	28.8964	9.4166	.001197605	2,623.23	547,599.23
836	698,896	584,277,056	28.9137	9.4204	.001196172	2,626.37	548,911.63
837	700,569	586,376,253	28.9310	9.4241	.001194743	2,629.51	550,225.61
838	702,244	588,480,472	28.9482	9.4279	.001193317	2,632.65	551,541.15
839	703,921	590,589,719	28.9655	9.4316	.001191895	2,635.80	552,858.26
840	705,600	592,704,000	28.9828	9.4354	.001190476	2,638.94	554,176.94
841	707,281	594,823,321	29.0000	9.4391	.001189061	2,642.08	555,497.20
842	708,964	596,947,688	29.0172	9.4429	.001187648	2,645.22	556,819.02
843	710,649	599,077,107	29.0345	9.4466	.001186240	2,648.36	558,142.42
844	712,336	601,211,584	29.0517	9.4503	.001184834	2,651.50	559,467.39
845	714,025	603,351,125	29.0689	9.4541	.001183432	2,654.65	560,793.92
846	715,716	605,495,736	29.0861	9.4578	.001182033	2,657.79	562,122.03
847	717,409	607,645,423	29.1033	9.4615	.001180638	2,660.93	563,451.71
848	719,104	609,800,192	29.1204	9.4652	.001179245	2,664.07	564,782.96
849	720,801	611,960,049	29.1376	9.4690	.001177856	2,667.21	566,115.78
850	722,500	614,125,000	29.1548	9.4727	.001176471	2,670.35	567,450.17
851	724,201	616,295,051	29.1719	9.4764	.001175088	2,673.50	568,786.14
852	725,904	618,470,208	29.1890	9.4801	.001173709	2,676.64	570,123.67
853	727,609	620,650,477	29.2062	9.4838	.001172333	2,679.78	571,462.77
854	729,316	622,835,864	29.2233	9.4875	.001170960	2,682.92	572,803.45
855	731,025	625,026,375	29.2404	9.4912	.001169591	2,686.06	574,145.69
856	732,736	627,222,016	29.2575	9.4949	.001168224	2,689.20	575,489.51
857	734,449	629,422,793	29.2746	9.4986	.001166861	2,692.34	576,834.90
858	736,164	631,628,712	29.2916	9.5023	.001165501	2,695.49	578,181.85
859	737,881	633,839,779	29.3087	9.5060	.001164144	2,698.63	579,530.38
860	739,600	636,056,000	29.3258	9.5097	.001162791	2,701.77	580,880.48
861	741,321	638,277,381	29.3428	9.5135	.001161440	2,704.91	582,232.15
862	743,044	640,503,928	29.3598	9.5171	.001160093	2,708.05	583,585.39
863	744,769	642,735,647	29.3769	9.5207	.001158749	2,711.19	584,940.20
864	746,496	644,972,544	29.3939	9.5244	.001157407	2,714.34	586,296.59
865	748,225	647,214,625	29.4109	9.5281	.001156069	2,717.48	587,654.54
866	749,956	649,461,896	29.4279	9.5317	.001154734	2,720.62	589,014.07
867	751,689	651,714,363	29.4449	9.5354	.001153403	2,723.76	590,375.16
868	753,424	653,972,032	29.4618	9.5391	.001152074	2,726.90	591,737.83
869	755,161	656,234,909	29.4788	9.5427	.001150748	2,730.04	593,102.06
870	756,900	658,503,000	29.4958	9.5464	.001149425	2,733.19	594,467.87
871	758,641	660,776,311	29.5127	9.5501	.001148106	2,736.33	595,835.25
872	760,384	663,054,848	29.5296	9.5537	.001146789	2,739.47	597,204.20
873	762,129	665,338,617	29.5466	9.5574	.001145475	2,742.61	598,574.72
874	763,876	667,627,624	29.5635	9.5610	.001144165	2,745.75	599,946.81

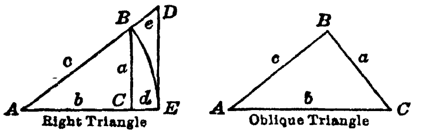
TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
875	765,625	669,921,875	29.5804	9.5647	.001142857	2,748.89	601,320.47
876	767,376	672,221,376	29.5973	9.5683	.001141553	2,752.04	602,695.70
877	769,129	674,526,133	29.6142	9.5719	.001140251	2,755.18	604,072.50
878	770,884	676,836,152	29.6311	9.5756	.001138952	2,758.32	605,450.88
879	772,641	679,151.439	29.6479	9.5792	.001137656	2,761.46	606,830.82
880	774,400	681,472,000	29.6648	9.5828	.001136364	2,764.60	608,212.34
881	776,161	683,797.841	29.6816	9.5865	.001135074	2,767.74	609,595.42
882	777,924	686,128,968	29.6985	9.5901	.001133787	2,770.88	610,980.08
883	779,689	688,465,387	29.7153	9.5937	.001132503	2,774.03	612,366.31
884	781,456	690,807,104	29.7321	9.5973	.001131222	2,777.17	613,754.11
885	783,225	693,154,125	29.7489	9.6010	.001129944	2,780.31	615,143.48
886	784,996	695,506,456	29.7658	9.6046	.001128668	2,783.45	616,534.42
887	786,769	697,864,103	29.7825	9.6082	.001127396	2,786.59	617,926.93
888	788.544	700.227,072	29.7993	9.6118	.001126126	2,789.73	619,321.01
889	790,321	702,595,369	29.8161	9.6154	.001124859	2,792.88	620,716.66
890	792,100	704,969,000	29.8329	9.6190	.001123596	2,796.02	622,113.89
891	793,881	707,347,971	29.8496	9.6226	.001122334	2,799.16	623,512.68
892	795,664	707,932,288	29.8664	9.6262	.001121076	2,802.30	624,913.04
893	797,449	712,121,957	29.8831	9.6298	.001119821	2,805.44	626,314.98
894	799,236	714,516,984	29.8998	9.6334	.001118568	2,808.58	627,718.49
895	801,025	716,917,375	29.9166	9.6370	.001117818	2,811.73	629,123.56
896	802,816	719,323,136	29.9333	9.6406	.001116071	2,814.87	630,530.21
897	804,609	721,734,273	29.9500	9.6442	.001114827	2,818.01	631,938.43
898	806,404	724,150,792	29.9666	9.6477	.001113586	2,821.15	633,348.22
899	808,201	726,572,699	29.9833	9.6513	.001112347	2,824.29	634,759.58
900	810,000	729,000,000	30.0000	9.6549	.001111111	2,827.43	636,172.51
901	811,801	731,432,701	30.0167	9.6585	.001109878	2,830.58	637,587.01
902	813,604	733,870,808	30.0333	9.6620	.001108647	2,833.72	639.003.09
903	815,409	736,314,327	30.0500	9.6656	.001107420	2,836.86	640,420.73
904	817,216	738,763,264	30.0666	9.6692	.001106195	2,840.00	641,839.95
905	819,025	741,217,625	30.0832	9.6727	.001104972	2,843.14	643,260.73
906	820,836	743,677,416	30.0998	9.6763	.001103753	2,846.28	644,683.09
907	822,649	746,142,643	30.1164	9.6799	.001102536	2,849.42	646,107.01
908	824,464	748,613,312	30.1330	9.6834	.001101322	2,852.57	647,532.51
909	826,281	751,089,429	30.1496	9.6870	.001100110	2,855.71	648,959.58
910	828,100	753,571,000	30.1662	9.6905	.001098901	2,858.85	650,388.22
911	829,921	756,058,031	30.1828	9.6941	.001091695	2,861.99	651,818.43
912	831,744	758,550,825	30.1993	9.6976	.001096491	2,865.13	653,250.21
913	833,569	761,048,497	30.2159	9.7012	.001095290	2,868.27	654,683.56
914	835,396	763,551,944	30.2324	9.7047	.001094092	2,871.42	656,118.48
915	837,225	766,060,875	30.2490	9.7082	.001092896	2,874.56	657,554.98
916	839,056	768,575,296	30.2655	9.7118	.001091703	2,877.70	658,993.04
917	840,889	771,095,213	30.2820	9.7153	.001090513	2,880.84	660,432.68
918	842,724	773,620,632	30.2985	9.7188	.001089325	2,883.98	661,873.88
919	844,561	776,151,559	30.3150	9.7224	.001088139	2,887.12	663,316.66
920	846,400	778,688,000	30.3315	9.7259	.001086957	2,890.27	664,761.01
921	848,241	781,229,961	30.3480	9.7294	.001085776	2,893.41	666,206.92
922	850,084	783,777,448	30.3645	9.7329	.001084599	2,896.55	667,654.41
923	851,929	786,330,467	30.3809	9.7364	.001083423	2,899.69	669,103.47
924	853,776	788,889,024	30.3974	9.7400	.001082251	2,902.83	670,554.10
925	855,625	791,453,125	30.4138	9.7435	.001081081	2,905.97	672.006.30
926	857,476	794,022,776	30.4302	9.7470	.001079914	2,909.11	673,460.08
927	859,329	796,597,983	30.4467	9.7505	.001078749	2,912.26	674,915.42
928	861,184	799,178,752	30.4631	9.7540	.001077586	2,915.40	676,372.33
929	863,041	801.765,089	30.4795	9.7575	.001076426	2,918.54	677,830.82
930	864,900	804,357,000	30.4959	9.7610	.001075269	2,921.68	679,290.87
931	866,761	806,954,491	30.5123	9.7645	.001074114	2,924.82	680,752.50
932	868,624	809,557,568	30.5287	9.7680	.001072961	2,927.96	682,215.69
933	870,489	812,166,237	30.5450	9.7715	.001071811	2,931.11	683,680.46
934	872,356	814,780,504	30.5614	9.7750	.001070664	2,934.25	685,146.80
935	874,225	817,400,375	30.5778	9.7785	.001069519	2,937.39	686,614.71
936	876,096	820,025,856	30.5941	9.7829	.001068376	2,940.53	688,084.19
937	877,969	822,656,953	30.6105	9.7854	.001067236	2,943.67	689,555.24

TABLE 25.—SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS—(*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
938	879,844	825,293,672	30.6268	9.7889	.001066098	2,946.81	691,027.86
939	881,721	827,936,019	30.6431	9.7924	.001064963	2,949.96	692.502.05
940	883,600	830,584,000	30.6594	9.7959	.001063830	2,953.10	693,977.82
941	885,481	833,237,621	30.6757	9.7993	.001062699	2,956.24	695,455.15
942	887,364	835,896,888	30.6920	9.8028	.001061571	2,959.38	696,934.06
943	889,249	838,561,807	30.7083	9.8063	.001060445	2,962.52	698,414.53
944	891,136	841,232,384	30.7246	9.8097	.001059322	2,965.66	699,896.58
945	893,025	843,908,625	30.7409	9.8132	.001058201	2,968.81	701,380.19
946	894,916	846,590,536	30.7571	9.8167	.001057082	2,971.95	702,865.38
947	896,808	849,278,123	30.7734	9.8201	.001055966	2,975.09	704,352.14
948	898,704	851,971,392	30.7896	9.8236	.001054852	2,978.23	705,840.47
949	900,601	854,670,349	30.8058	9.8270	.001053741	2,981.37	707,330.37
950	902,500	857,375,000	30.8221	9.8305	.001052632	2,984.51	708,821.84
951	904,401	860,085,351	30.8383	9.8339	.001051525	2,987.65	710,314.88
952	906,304	862,801,408	30.8545	9.8374	.001050420	2,990.80	711,809.50
953	908,209	865,523,177	30.8707	9.8408	.001049318	2,993.94	713,305.68
954	910,116	868,250,664	30.8869	9.8443	.001048218	2,997.08	714,803.43
955	912,025	870,983,875	30.9031	9.8477	.001047120	3,000.22	716,302.76
956	913,936	873,722,816	30.9192	9.8511	.001046025	3,003.36	717,803.66
957	915,849	876,467,493	30.9354	9.8546	.001044932	3,006.50	719,306.12
958	917,764	879,217,912	30.9516	9.8580	.001043841	3,009.65	720,810.16
959	919,681	881,974,079	30.9677	9.8614	.001042753	3,012.79	722,315.77
960	921,600	884,736,000	30.9839	9.8648	.001041667	3,015.93	723,822.95
961	923,521	887,503,681	31.0000	9.8683	.001040583	3,019.07	725,331.70
962	925,444	890,277,128	31.0161	9.8717	.001039501	3,022.21	726,842.02
963	927,369	893,056,347	31.0322	9.8751	.001038422	3,025.35	728,353.91
964	929,296	895,841,344	31.0483	9.8785	.001037344	3,028.50	729,867.37
965	931,225	898,632,125	31.0644	9.8819	.001036269	3,031.64	731,382.40
966	933,156	901,428,696	31.0805	9.8854	.001035197	3,034.78	732,899.01
967	935,089	904,231,063	31.0966	9.8888	.001034126	3,037.92	734,417.18
968	937,024	907,039,232	31.1127	9.8922	.001033058	3,041.06	735,936.93
969	938,961	909,853,209	31.1288	9.8956	.001031992	3,044.20	737,458.24
970	940,900	912,673,000	31.1448	9.8990	.001030928	3,047.34	738,981.13
971	942,841	915,498,611	31.1609	9.9024	.001029866	3,050.49	740,505.59
972	944,784	918,330,048	31.1769	9.9058	.001028807	3,053.63	742,031.62
973	946,729	921,167,317	31.1929	9.9092	.001027749	3,056.77	743,559.22
974	948,676	924,010,424	31.2090	9.9126	.001026694	3,059.91	745,088.39
975	950,625	926,859,375	31.2250	9.9160	.001025641	3,063.05	746,619.13
976	952,576	929,714,176	31.2410	9.9194	.001024590	3,066.19	748,151.44
977	954,529	932,574,833	31.2570	9.9228	.001023541	3,069.34	749,685.32
978	956,484	935,441,352	31.2730	9.9261	.001022495	3,072.48	751,220.78
979	958,441	938,313,739	31.2890	9.9295	.001021450	3,075.62	752,757.80
980	960,400	941,192,000	31.3050	9.9329	.001020408	3,078.76	754,296.40
981	962,361	944,076,141	31.3209	9.9363	.001019168	3,081.90	755,836.56
982	964,324	946,966,168	31.3369	9.9396	.001018330	3,085.04	757,378.30
983	966,289	949,862,087	31.3528	9.9430	.001017294	3,088.19	758,921.61
984	968,256	952,763,904	31.3688	9.9464	.001016260	3,091.33	760,466.48
985	970,225	955,671,625	31.3847	9.9497	.001015228	3,094.47	762,012.93
986	972,196	958,585,256	31.4006	9.9531	.001014199	3,097.61	763,560.95
987	974,169	961,504,803	31.4166	9.9565	.001013171	3,100.75	765,110.54
988	976,144	964,430,272	31.4325	9.9598	.001012146	3,103.89	766,661.70
989	978,121	967,361,669	31.4484	9.9632	.001011122	3,107.04	768,214.44
990	980,100	970,299,000	31.4643	9.9666	.001010101	3,110.18	769,768.74
991	982,081	973,242,271	31.4802	9.9699	.001009082	3,113.32	771,324.61
992	984,064	976,191,488	31.4960	9.9733	.001008065	3,116.46	772,882.06
993	986,049	979,146,657	31.5119	9.9766	.001007049	3,119.60	774,441.07
994	988,036	982,107,784	31.5278	9.9800	.001006036	3,122.74	776,001.66
995	990,025	985,074,875	31.5436	9.9833	.001005025	3,125.88	777,563.82
996	992,016	988,047,936	31.5595	9.9866	.001004016	3,129.03	779,127.54
997	994,009	991,026,973	31.5753	9.9900	.001003009	3,132.17	780,692.84
998	996,004	994,011,992	31.5911	9.9933	.001002004	3,135.31	782,259.71
999	998,001	997,002,999	31.6070	9.9967	.001001001	3,138.45	783,828.15
1000	1,000,000	1,000,000,000	31.6228	10.0000	.001000000	3,141.59	785,398.16

Table 26.—Trigonometric Formulas



Right Triangles

$$\sin A = \frac{a}{c} = \cos B \qquad \sec A = \frac{c}{b} = \operatorname{cosec} B$$

$$\cos A = \frac{b}{c} = \sin B \qquad \operatorname{cosec} A = \frac{c}{a} = \sec B$$

$$\tan A = \frac{a}{b} = \cot B \qquad \operatorname{vers} A = \frac{c-b}{c} = \frac{d}{c}$$

$$\cot A = \frac{b}{a} = \tan B \qquad \operatorname{exsec} A = \frac{e}{c}$$

$$a = c \sin A = c \cos B = b \tan A = b \cot B = \sqrt{c^2 - b^2}$$

$$b = c \cos A = c \sin B = a \cot A = a \tan B = \sqrt{c^2 - a^2}$$

$$c = \frac{a}{\sin A} = \frac{a}{\cos B} = \frac{b}{\sin B} = \frac{b}{\cos A} = \frac{d}{\operatorname{vers} A} = \frac{e}{\operatorname{exsec} A} = \sqrt{a^2 + b^2}$$

$$d = c \operatorname{vers} A \qquad e = c \operatorname{exsec} A$$

Oblique Triangles

Given	Sought	Formulas
A, B, a	b, c	$b = \dfrac{a}{\sin A} \cdot \sin B \qquad c = \dfrac{a}{\sin A} \cdot \sin (A + B)$
A, a, b	B, c	$\sin B = \dfrac{\sin A}{a} \cdot b \qquad c = \dfrac{a}{\sin A} \cdot \sin C$
C, a, b	$\frac{1}{2}(A + B)$	$\frac{1}{2}(A + B) = 90° - \frac{1}{2}C$
	$\frac{1}{2}(A - B)$	$\tan \frac{1}{2}(A - B) = \dfrac{a - b}{a + b} \cdot \tan \frac{1}{2}(A + B)$
a, b, c	A	If $s = \frac{1}{2}(a + b + c)$, $\sin \frac{1}{2}A = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$
		$\cos \frac{1}{2}A = \sqrt{\dfrac{s(s-a)}{bc}}$, $\tan \frac{1}{2}A = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$
		$\sin A = 2\dfrac{\sqrt{s(s-a)(s-b)(s-c)}}{bc}$
		$\operatorname{vers} A = \dfrac{2(s-b)(s-c)}{bc}$
A, b, c	a	$a^2 = b^2 + c^2 - 2bc \cos A$
B, a, c	b	$b^2 = a^2 + c^2 - 2ac \cos B$
C, a, b	c	$c^2 = a^2 + b^2 - 2ab \cos C$

TABLE 26.—TRIGONOMETRIC FORMULAS
General Formulas—(*Continued*)

Given	*Area of Triangle*
a, b, c	$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$
C, a, b	$\text{Area} = \frac{1}{2}ab \sin C$
A, B, C, a	$\text{Area} = \dfrac{a^2 \sin B \sin C}{2 \sin A}$

$$\sin A = \sqrt{1 - \cos^2 A} = 2 \sin \tfrac{1}{2}A \cos \tfrac{1}{2}A = \sqrt{\frac{1 - \cos 2A}{2}}.$$

$$\cos A = \sqrt{1 - \sin^2 A} = \cos^2 \tfrac{1}{2}A - \sin^2 \tfrac{1}{2}A = 1 - 2 \sin^2 \tfrac{1}{2}A$$
$$= 2 \cos^2 \tfrac{1}{2}A - 1 = \sqrt{\frac{1 + \cos 2A}{2}}.$$

$$\text{vers } A = 1 - \cos A = 2 \sin^2 \tfrac{1}{2}A.$$

$$\tan A = \frac{1}{\cot A} = \frac{\sin A}{\cos A} = \sqrt{\sec^2 A - 1} = \frac{1 - \cos 2A}{\sin 2A}.$$

$$\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} = \sqrt{\text{cosec}^2 A - 1} = \frac{1 + \cos 2A}{\sin 2A}.$$

$$\sec A = \frac{1}{\cos A} = \sqrt{1 + \tan^2 A}.$$

$$\text{exsec } A = \sec A - 1 = \tan A \tan \tfrac{1}{2}A.$$

$$\text{cosec } A = \frac{1}{\sin A} = \sqrt{1 + \cot^2 A}.$$

$$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A.$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B.$$
$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B).$$
$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B).$$
$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B).$$
$$\cos A - \cos B = -2 \sin \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B).$$

Trig. Function	*Angles*							
	0°	**30°**	**45°**	**60°**	**90°**	**180°**	**270°**	**360°**
sine............	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cosine..........	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0	1
tangent.........	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	−∞	0

TABLE 27.—USEFUL NUMBERS AND FORMULAS

Ratio of circumference to diameter = π = 3.14159265.

Circumference of circle = $2\pi r$.

1 radian = $\frac{180}{\pi}$ degrees = 57.29578 degrees.

1 degree = $\frac{\pi}{180}$ radians = 0.0174533 radian.

Length of arc = radius $\times$ central angle (in radians) = $\frac{\pi r}{180}$ $\times$ central angle (in degrees).

Acceleration due to gravity = 32.2 (approx.).

1 meter = 39.37 inches = 3.2808 feet.

1 foot = 0.3048 meter = 12 inches.

1 square yard = 9 square feet = 144 square inches.

1 kilometer = 1,000 meters = 0.62137 mile.

1 mile = 5,280 feet = 1,760 yards = 1.60935 kilometers.

1 acre = 43,560 square feet = 4,840 square yards.

Area of:

Trapezoid: $\frac{1}{2}$(sum of parallel sides) $\times$ (perpendicular height).

Circle: πr^2, where r = radius.

Sector of circle: Arc $\times$ $\frac{1}{2}$ radius = $\frac{1}{2}r^2$ $\times$ central angle (in radians) = $0.0087266r^2$ $\times$ central angle (in degrees).

Segment of circle: $\frac{r^2}{2}\left(\frac{\pi\theta}{180} - \sin\theta\right)$, where θ is central angle in degrees.

Ellipse: πab = 0.78540 $\times$ long diameter $\times$ short diameter.

Parabola: Base $\times$ $\frac{2}{3}$ perpendicular height.

Volume of:

Prism or cylinder: Area base $\times$ perpendicular height.

Pyramid or cone: Area base $\times$ 1 perpendicular height.

Frustum of pyramid or cone: (Sum of end areas, A_1 and A_2, + square root of their products) $\times$ $\frac{1}{3}$ perpendicular height h $= (A_1 + A_2 + \sqrt{A_1A_2})\frac{h}{3}$.

Sphere: $\frac{4}{3}\pi r^3$.

Weight of

Cement = 94 lb. per cubic foot.

Concrete = 150 lb. per cubic foot.

Earth = 100 lb. per cubic foot.

Sand = 110 lb. per cubic foot.

Steel = 490 lb. per cubic foot.

Water = 62.5 lb. per cubic foot.

TABLE 28.—DEFLECTIONS FROM THE P.I. TO THE 10-POINT CIRCULAR CURVE.

Vernier reads zero on first tangent.

Pts.	Δ = 3° 00′	Diff. per min.	Δ = 3° 30′	Diff. per min.	Δ = 4° 00′	Diff. per min.	Δ = 4° 30′	Diff. per min.	Δ = 5° 00′	Diff. per min.	Δ = 5° 30′	Diff. per min.	Δ = 6° 00′	Diff. per min.	Δ = 6° 30′	Diff. per min.	Δ = 7° 00′	Diff. per min.
1	0° 02′	+0.03′	0° 03′	+0.00′	0° 03′	+0.00′	0° 03′	+0.03′	0° 04′	+0.00′	0° 04′	+0 00′	0° 04′	+0.03′	0° 05′	+0.00′	0° 05′	+0 03′
2	0 11	+0.13	0 15	+0.03	0 16	+0.07	0 18	+0.03	0 19	+0 07	0 21	+0.07	0 23	+0.10	0 26	+0 07	0 28	+0.07
3	0 38	+0.30	0 47	+0.27	0 55	+0.17	1 00	+0.20	1 06	+0.23	1 13	+0.23	1 20	+0 23	1 27	+0.23	1 34	+0.20
4	2 19	+1.07	2 51	+0.63	3 10	+0.80	3 34	+0.80	3 58	+0.80	4 22	+0.70	4 43	+0.83	5 08	+0.77	5 31	+0 90
5	87 30	−0.50	87 15	−0.50	88 00	−0.50	87 45	−0.50	87 30	−0.50	87 15	−0.50	87 00	−0.50	86 45	−0.50	86 30	−0.50
6	174 41	−2.07	173 39	−1.63	172 50	−1.80	171 56	−1.80	171 02	−1.80	170 08	−1.70	169 17	−1.83	168 22	−1.77	167 29	−1.90
7	176 22	−1.30	175 43	−1.27	175 05	−1.17	174 30	−1 20	173 54	−1.23	173 17	−1.23	172 40	−1.23	172 03	−1.23	171 26	
8	176 49	−1.13	176 15	−1.03	175 44	−1.07	175 12	−1.03	174 41	−1.07	174 09	−1.07	173 37	−1.10	173 04	−1.07	172 32	−1.07
9	176 58	−1.03	176 27	−1.00	175 57	−1.00	175 27	−1.03	174 56	−1.00	174 26	−1.00	173 56	−1.03	173 25	−1.00	172 55	−1.03
10	177 00	−1.00	176 30	−1.00	176 00	−1.00	175 30	−1.00	175 00	−1 00	174 30	−1.00	174 00	−1.00	173 30	−1.00	173 00	−1.00

Pts.	Δ = 7° 30′	Diff. per min.	Δ = 8° 00′	Diff. per min.	Δ = 8° 30′	Diff. per min.	Δ = 9° 00′	Diff. per min.	Δ = 9° 30′	Diff. per min.	Δ = 10° 00′	Diff. per min.	Δ = 10° 30′	Diff. per min.	Δ = 11° 00′	Diff. per min.	Δ = 11° 30′	Diff. per min.
1	0° 06′	+0.00′	0° 06′	+0.00′	0° 06′	+0.03′	0° 07′	+0.00′	0° 07′	+0.03′	0° 08′	+0.00′	0° 08′	+0.00′	0° 08′	+0.03′	0° 09′	+0 00′
2	0 30	+0.07	0 32	+0.07	0 34	+0.07	0 36	+0.07	0 38	+0.07	0 40	+0.07	0 42	+0.07	0 44	+0.07	0 46	+0.07
3	1 40	+0.23	1 47	+0.23	1 54	+0.23	2 01	+0.20	2 07	+0.23	2 14	+0 23	2 21	+0.20	2 27	+0.23	2 34	+0.23
4	5 58	+0.70	6 19	+0.77	6 42	+0.70	7 03	+0 83	7 28	+0.70	7 49	+0.77	8 12	+0.73	8 34	+0.73	8 56	+0.73
5	86 15	−0.50	86 00	−0.50	85 45	−0.50	85 30	−0.50	85 15	−0.50	85 00	−0.50	84 45	−0.50	84 30	−0.50	84 15	−0.50
6	166 32	−1.70	165 41	−1.77	164 48	−1.70	163 57	−1.83	163 02	−1.70	162 11	−1.77	161 18	−1.73	160 26	−1.73	159 34	−1.73
7	170 49	−1.23	170 12	—	169 36	−1.23	168 59	−1.20	168 23	−1.23	167 46	−1.23	167 09	−1.20	166 33	−1.23	165 56	−1.23
8	172 00	−1.07	171 28	−1.07	170 56	−1.07	170 24	−1.07	169 52	−1.07	169 20	−1.07	168 48	−1.07	168 16	−1 07	167 44	−1.07
9	172 24	−1.00	171 54	−1.00	171 24	−1.03	170 53	−1.00	170 23	−1.03	169 52	−1 00	169 22	−1 00	168 52	−1.03	168 21	−1.00
10	172 30	−1.00	172 00	−1.00	171 30	−1.00	171 00	−1.00	170 30	−1.00	170 00	−1.00	169 30	−1.00	169 00	−1 00	168 30	−1.00

TABLE 28.—DEFLECTIONS FROM THE P.I. TO THE 10-POINT CIRCULAR CURVE—(*Continued*).
Vernier reads zero on first tangent.

Pts.	Δ = 12° 00′	Diff. per min.	Δ = 12° 30′	Diff. per min.	Δ = 13° 00′	Diff. per min.	Δ = 14° 00′	Diff. per min.	Δ = 15° 00′	Diff. per min.	Δ = 16° 00′	Diff. per min.	Δ = 17° 00′	Diff. per min.	Δ = 18° 00′	Diff. per min.	Δ = 19° 00′	Diff. per min.
1	0° 09′	+0.00′	0° 09′	+0.03′	0° 10′	+0.00′	0° 10′	+0.02′	0° 11′	+0.02′	0° 12′	+0.02′	0° 13′	+0.00′	0° 13′	+0.02′	0° 14′	+0.02′
2	0 48	+0.07	0 50	+0.03	0 51	+0.07	0 55	+0.07	0 59	+0.07	1 03	+0.07	1 07	+0.07	1 11	+0.07	1 15	+0.07
3	2 41	+0.20	2 47	+0.20	2 53	+0.22	3 06	+0.22	3 19	+0.22	3 32	+0.20	3 44	+0.22	3 57	+0.22	4 10	+0.20
4	9 18	+0.73	9 40	+0.70	10 01	+0.72	10 44	+0.68	11 25	+0.68	12 05	+0.67	12 46	+0.65	13 25	+0.65	14 04	+0.62
5	84 00	−0.50	83 45	−0.50	83 30	−0.50	83 00	−0.50	82 30	−0.50	82 00	−0.50	81 30	−0.50	81 00	−0.50	80 30	−0.50
6	158 42	−1.73	157 50	−1.70	156 59	−1.72	155 16	−1.68	153 35	−1.68	151 54	−1.67	150 14	−1.65	148 35	−1.65	146 56	−1.62
7	165 19	−1.20	164 43	−1.20	164 07	−1.22	162 54	−1.22	161 41	−1.22	160 28	−1.20	159 16	−1.22	158 03	−1.22	156 50	−1.20
8	167 12	−1.07	166 40	−1.03	166 09	−1.07	165 05	−1.07	164 01	−1.07	162 57	−1.07	161 53	−1.07	160 49	−1.07	159 45	−1.07
9	167 51	−1.00	167 21	−1.03	166 50	−1.00	165 50	−1.02	164 49	−1.02	163 43	−1.02	162 47	−1.00	161 47	−1.02	160 46	−1.02
10	168 00	−1.00	167 30	−1.00	167 00	−1.00	166 00	−1.00	165 00	−1.00	164 00	−1.00	163 00	−1.00	162 00	−1.00	161 00	−1.00

Pts.	Δ = 20° 00′	Diff. per min.	Δ = 21° 00′	Diff. per min.	Δ = 22° 00′	Diff. per min.	Δ = 23° 00′	Diff. per min.	Δ = 24° 00′	Diff. per min.	Δ = 25° 00′	Diff. per min.	Δ = 26° 00′	Diff. per min.	Δ = 27° 00′	Diff. per min.	Δ = 28° 00′	Diff. per min.
1	0° 15′	+0.02′	0° 16′	+0.00′	0° 16′	+0.02′	0° 17′	+0.02′	0° 18′	+0.00′	0° 18′	+0.02′	0° 19′	+0.02′	0° 20′	+0.00′	0° 20′	+0.02′
2	1 19	+0.05	1 22	+0.07	1 26	+0.07	1 30	+0.05	1 33	+0.07	1 37	+0.07	1 41	+0.07	1 45	+0.05	1 48	+0.07
3	4 22	+0.20	4 34	+0.20	4 46	+0.20	4 58	+0.20	5 10	+0.20	5 22	+0.18	5 33	+0.18	5 44	+0.20	5 56	+0.18
4	14 41	+0.60	15 17	+0.60	15 53	+0.57	16 27	+0.55	17 00	+0.55	17 33	+0.52	18 04	+0.52	18 35	+0.48	19 04	+0.48
5	80 00	−0.50	79 30	−0.50	79 00	−0.50	78 30	−0.50	78 00	−0.50	77 30	−0.50	77 00	−0.50	76 30	−0.50	76 00	−0.50
6	145 19	−1.60	143 43	−1.60	142 07	−1.57	140 33	−1.55	139 00	−1.55	137 27	−1.52	135 56	−1.52	134 25	−1.48	132 56	−1.48
7	155 38	−1.20	154 26	−1.20	153 14	−1.20	152 02	−1.20	150 50	−1.20	149 38	−1.18	148 27	−1.18	147 16	−1.20	146 04	−1.18
8	158 41	−1.05	157 38	−1.07	156 34	−1.07	155 30	−1.07	154 26	−1.06	153 23	−1.07	152 19	−1.07	151 15	−1.05	150 12	−1.07
9	159 45	−1.02	158 44	−1.00	157 44	−1.02	156 43	−1.02	155 42	−1.00	154 42	−1.02	153 41	−1.02	152 40	−1.00	151 40	−1.02
10	160 00	−1.00	159 00	−1.00	158 00	−1.00	157 00	−1.00	156 00	−1.00	155 00	−1.00	154 00	−1.00	153 00	−1.00	152 00	−1.00

TABLE 28.—DEFLECTIONS FROM THE P.I. TO THE 10-POINT CIRCULAR CURVE—(*Continued*).
Vernier reads zero on first tangent.

Pts.	Δ = 29° 00′	Diff. per min.	Δ = 30° 00′	Diff. per min.	Δ = 31° 00′	Diff. per min.	Δ = 32° 00′	Diff. per min.	Δ = 33° 00	Diff. per min.	Δ = 34° 00′	Diff. per min.	Δ = 35° 00′	Diff. per min.	Δ = 36° 00′	Diff. per min.	Δ = 37° 00′	Diff. per min.
1	0° 21′	+0.02′	0° 22′	+0.02′	0° 23′	+0.00′	0° 23′	+0.02′	0° 24′	+0.00′	0° 24′	+0.02′	0° 25′	+0.02′	0° 26′	+0.00′	0° 26′	+0.02′
2	1 52	+0.05	1 55	+0.07	1 59	+0.05	2 02	+0.07	2 06	+0.05	2 09	+0.07	2 13	+0.05	2 16	+0.05	2 19	+0.05
3	6 07	+0.18	6 18	+0.18	6 29	+0.17	6 39	+0.17	6 49	+0.18	7 00	+0.17	7 10	+0.17	7 20	+0.17	7 30	+0.15
4	19 33	+0.45	20 00	+0.43	20 26	+0.43	20 52	+0.42	21 17	+0.38	21 40	+0.38	22 03	+0.35	22 24	+0.33	22 44	+0.33
5	75 30	−0.50	75 00	−0.50	74 30	−0.50	74 00	−0.50	73 30	−0.50	73 00	−0.50	72 30	−0.50	72 00	−0.50	71 30	−0.50
6	131 27	−1.45	130 00	−1.43	128 34	−1.43	127 08	−1.42	125 43	−1.38	124 20	−1.38	122 57	−1.35	121 36	−1.33	120 16	−1.33
7	144 53	−1.18	143 42	−1.18	142 31	−1.17	141 21	−1.17	140 11	−1.18	139 00	−1.17	137 50	−1.17	136 40	−1.17	135 30	−1.15
8	149 08	−1.05	148 05	−1.07	147 01	−1.05	145 58	−1.07	144 54	−1.05	143 51	−1.07	142 47	−1.05	141 44	−1.05	140 41	−1.05
9	150 39	−1.02	149 38	−1.02	148 37	−1.00	147 37	−1.02	146 36	−1.00	145 36	−1.02	144 35	−1.02	143 34	−1.00	142 34	−1.02
10	151 00	−1.00	150 00	−1.00	149 00	−1.00	148 00	−1.00	147 00	−1.00	146 00	−1.00	145 00	−1.00	144 00	−1.00	143 00	−1.00

Pts.	Δ = 38° 00′	Diff. per min.	Δ = 39° 00′	Diff. per min.	Δ = 40° 00′	Diff. per min.	Δ = 41° 00′	Diff. per min.	Δ = 42° 00′	Diff. per min.	Δ = 43° 00′	Diff. per min.	Δ = 44° 00′	Diff. per min.	Δ = 45° 00′	Diff. per min.	Δ = 46° 00′	Diff. per min.
1	0° 27′	+0.02′	0° 28′	+0.02′	0° 29′	+0.00′	0° 29′	+0.02′	0° 30′	+0.00′	0° 30′	+0.02′	0° 31′	+0.02′	0° 32′	+0.00′	0° 32′	+0.02′
2	2 22	+0.07	2 26	+0.05	2 29	+0.05	2 32	+0.05	2 35	+0.05	2 38	+0.05	2 41	+0.05	2 44	+0.05	2 47	+0.05
3	7 39	+0.17	7 49	+0.15	7 58	+0.15	8 07	+0.15	8 16	+0.15	8 25	+0.13	8 33	+0.13	8 41	+0.13	8 49	+0.13
4	23 04	+0.32	23 23	+0.30	23 41	+0.28	23 58	+0.27	24 14	+0.25	24 29	+0.23	24 43	+0.23	24 57	+0.22	25 10	+0.20
5	71 00	−0.50	70 30	−0.50	70 00	−0.50	69 30	−0.50	69 00	−0.50	68 30	−0.50	68 00	−0.50	67 30	−0.50	67 00	−0.50
6	118 56	−1.32	117 37	−1.30	116 19	−1.28	115 02	−1.27	113 46	−1.25	112 31	−1.23	111 17	−1.23	110 03	−1.22	108 50	−1.20
7	134 21	−1.17	133 11	−1.15	132 02	−1.15	130 53	−1.15	129 44	−1.15	128 35	−1.13	127 27	−1.13	126 19	−1.13	125 11	−1.13
8	139 38	−1.07	138 34	−1.05	137 31	−1.05	136 28	−1.05	135 25	−1.05	134 22	−1.05	133 19	−1.05	132 16	−1.05	131 13	−1.05
9	141 33	−1.02	140 32	−1.02	139 31	−1.00	138 31	−1.02	137 30	−1.00	136 30	−1.02	135 29	−1.02	134 28	−1.00	133 28	−1.02
10	142 00	−1.00	141 00	−1.00	140 00	−1.00	139 00	−1.00	138 00	−1.00	137 00	−1.00	136 00	−1.00	135 00	−1.00	134 00	−1.00

TABLE 28.—DEFLECTIONS FROM THE P.I. TO THE 10-POINT CIRCULAR CURVE—(*Continued*).
Vernier reads zero on first tangent.

Pts.	Δ = 47° 00′	Diff. per min.	Δ = 48° 00′	Diff. per min.	Δ = 49° 00′	Diff. per min.	Δ = 50° 00′	Diff. per min.	Δ = 51° 00′	Diff. per min.	Δ = 52° 00′	Diff. per min.	Δ = 53° 00′	Diff per min.	Δ = 54° 00′	Diff. per min.	Δ = 55° 00′	Diff. per min.
1	0° 33′	+0.00′	0° 33′	+0 02′	0° 34′	+0 00′	0° 34′	+0 02′	0° 35′	+0 02′	0° 36′	+0.00′	0° 36′	+0.02′	0° 37′	+0 00′	0° 37′	+0.02′
2	2 50	+0.05	2 53	+0.03	2 55	+0.05	2 58	+0.05	3 01	+0 05	3 04	+0 03	3 06	+0 05	3 09	+0 03	3 11	+0 05
3	8 57	+0.13	9 05	+0.12	9 12	+0.12	9 19	+0.12	9 26	+0.12	9 33	+0 12	9 40	+0.10	9 46	+0 12	9 53	+0.10
4	25 22	+0.18	25 33	+0.17	25 43	+0.17	25 53	+0.15	26 02	+0.13	26 10	+0 13	26 18	+0 12	26 25	+0.10	26 31	+0 08
5	66 30	−0.50	66 00	−0.50	65 30	−0.50	65 00	−0.50	64 30	−0.50	64 00	−0.50	63 30	−0.50	63 00	−0 50	62 30	−0.50
6	107 38	−1.18	106 27	−1.17	105 17	−1 17	104 07	−1.15	102 58	−1.13	101 50	−1.13	100 42	−1 12	99 35	−1 10	98 29	−1 08
7	124 03	−1.13	122 55	−1.12	121 48	−1.12	120 41	−1 12	119 34	−1 12	118 27	−1 12	117 20	−1 10	116 14	−1 12	115 07	−1 10
8	130 10	−1.05	129 07	−1.03	128 05	−1.05	127 02	−1.05	125 59	−1 05	124 56	−1 03	123 54	−1 05	122 51	−1 03	121 49	−1 05
9	132 27	−1.00	131 27	−1.02	130 26	−1.02	129 25	−1.00	128 25	−1 02	127 24	−1 00	126 24	−1.02	125 23	−1.00	124 23	−1.02
10	133 00	−1.00	132 00	−1.00	131 00	−1.00	130 00	−1 00	129 00	−1.00	128 00	−1.00	127 00	−1.00	126 00	−1.00	125 00	−1.00

Pts.	Δ = 56° 00′	Diff. per min.	Δ = 57° 00′	Diff. per min.	Δ = 58° 00′	Diff. per min.	Δ = 59° 00	Diff. per min.	Δ = 60° 00′	Diff. per min.	Δ = 61° 00′	Diff. per min.	Δ = 62° 00′	Diff. per min.	Δ = 63° 00′	Diff. per min.	Δ = 64° 00′	Diff. per min.
1	0° 38′	+0.00′	0° 33′	+0.02′	0° 39′	+0.00′	0° 39′	+0 02′	0° 40′	+0 00′	0° 40′	+0.02′	0° 41′	+0.00′	0° 41′	+0.02′	0° 42′	+0 00′
2	3 14	+0.03	3 16	+0.03	3 18	+0 05	3 21	+0 03	3 23	+0 03	3 25	+0.03	3 27	+0.03	3 29	+0 05	3 32	+0 03
3	9 59	+0.08	10 04	+0.10	10 10	+0 08	10 15	+0 08	10 20	+0.08	10 25	+0 08	10 30	+0 07	10 34	+0.08	10 39	+0.07
4	26 36	+0.08	26 41	+0.07	26 45	+0.07	26 49	+0.05	26 52	+0 05	26 55	+0.03	26 57	+0.02	26 58	+0.02	26 59	+0.02
5	62 00	−0.50	61 30	−0.50	61 00	−0.50	60 30	−0.50	60 00	−0 50	59 30	−0.50	59 00	−0.50	58 30	−0.50	58 00	−0.50
6	97 24	−1.08	96 19	−1.07	95 15	−1.07	94 11	−1.05	93 08	−1 05	92 05	−1.03	91 03	−1 02	90 02	−1.02	89 01	−1.02
7	114 01	−1.08	112 56	−1.10	111 50	−1.08	110 45	−1.08	109 40	−1 08	108 35	−1.08	107 30	−1 07	106 26	−1.08	105 21	−1 07
8	120 46	−1.03	119 44	−1.03	118 42	−1 05	117 39	−1.03	116 37	−1.03	115 35	−1.03	114 33	−1.03	113 31	−1.05	112 28	−1.03
9	123 22	−1.00	122 22	−1 02	121 21	−1.00	120 21	−1.02	119 20	−1.00	118 20	−1.02	117 19	−1.00	116 19	−1.02	115 18	−1.00
10	124 00	−1.00	123 00	−1.00	122 00	−1.00	121 00	−1.00	120 00	−1.00	119 00	−1.00	118 00	−1.00	117 00	−1.00	116 00	−1.00

TABLE 28.—DEFLECTIONS FROM THE P.I. TO THE 10-POINT CIRCULAR CURVE—(*Continued*).
Vernier reads zero on first tangent.

Pts.	Δ = 65° 00′	Diff. per min.	Δ = 66° 00′	Diff. per min.	Δ = 67° 00′	Diff. per min.	Δ = 68° 00′	Diff. per min.	Δ = 69° 00′	Diff. per min.	Δ = 70° 00′	Diff. per min.	Δ = 71° 00′	Diff. per min.	Δ = 72° 00′	Diff. per min.	Δ = 73° 00′	Diff. per min.
1	0° 42′	+0.02′	0° 43′	+0.00′	0° 43′	+0.00′	0° 44′	+0.00′	0° 44′	+0.00′	0° 44′	+0.02′	0° 45′	+0.00′	0° 45′	+0.00′	0° 45′	+0.02′
2	3 34	+0.02	3 35	+0.03	3 37	+0.03	3 39	+0.03	3 41	+0.02	3 42	+0.03	3 44	+0.03	3 46	+0.02	3 47	+0.03
3	10 43	+0.05	10 46	+0.07	10 50	+0.05	10 53	+0.07	10 57	+0.05	11 00	+0.05	11 03	+0.03	11 05	+0.05	11 08	+0.03
4	27 00	−0.02	26 59	−0.00	26 59	−0.02	26 58	−0.03	26 56	−0.03	26 54	−0.03	26 52	−0.05	26 49	−0.07	26 45	−0.07
5	57 30	−0.50	57 00	−0.50	56 30	−0.50	56 00	−0.50	55 30	−0.50	55 00	−0.50	54 30	−0.50	54 00	−0.50	53 30	−0.50
6	88 00	−0.98	87 01	−1.00	86 01	−0.98	85 02	−0.97	84 04	−0.97	83 06	−0.97	82 08	−0.95	81 11	−0.93	80 15	−0.93
7	104 17	−1.05	103 14	−1.07	102 10	−1.05	101 07	−1.07	100 03	−1.05	99 00	−1.05	97 57	−1.03	96 55	−1.05	95 52	−1.03
8	111 26	−1.02	110 25	−1.03	109 23	−1.03	108 21	−1.03	107 19	−1.02	106 18	−1.03	105 16	−1.03	104 14	−1.02	103 13	−1.03
9	114 18	−1.02	113 17	−1.00	112 17	−1.02	111 16	−1.00	110 16	−1.00	109 16	−1.02	108 15	−1.00	107 15	−1.00	106 15	−1.02
10	115 00	−1.00	114 00	−1.00	113 00	−1.00	112 00	−1.00	111 00	−1.00	110 00	−1.00	109 00	−1.00	108 00	−1.00	107 00	−1.00

Pts.	Δ = 74° 00′	Diff. per min.	Δ = 75° 00′	Diff. per min.	Δ = 76° 00′	Diff. per min	Δ = 77° 00′	Diff. per min.	Δ = 78° 00′	Diff. per min.	Δ = 79° 00′	Diff. per min.	Δ = 80° 00′	Diff. per min.	Δ = 81° 00′	Diff. per min.	Δ = 82° 00′	Diff. per min.
1	0° 46′	+0.00′	0° 46′	+0.02′	0° 47′	+0.00′	0° 47′	+0.00′	0° 47′	+0.00′	0° 47′	+0.02′	0° 48′	+0.00′	0° 48′	+0.00′	0° 48′	+0.02′
2	3 49	+0.02	3 50	+0.02	3 51	+0.02	3 52	+0.03	3 54	+0.02	3 55	+0.02	3 56	+0.02	3 57	+0.02	3 58	+0.02
3	11 10	+0.03	11 12	+0.03	11 14	+0.02	11 15	+0.02	11 16	+0.03	11 18	+0.02	11 19	+0.00	11 19	+0.02	11 20	+0.00
4	26 41	−0.07	26 37	−0.08	26 32	−0.07	26 28	−0.10	26 22	−0.08	26 17	−0.12	26 10	−0.10	26 04	−0.12	25 57	−0.12
5	53 00	−0.50	52 30	−0.50	52 00	−0.50	51 30	−0.50	51 00	−0.50	50 30	−0.50	50 00	−0.50	49 30	−0.50	49 00	−0.50
6	79 19	−0.93	78 23	−0.92	77 28	−0.93	76 32	−0.90	75 38	−0.92	74 43	−0.88	73 50	−0.90	72 56	−0.88	72 03	−0.88
7	94 50	−1.03	93 48	−1.03	92 46	−1.02	91 45	−1.02	90 44	−1.03	89 42	−1.02	88 41	−1.00	87 41	−1.02	86 40	−1.00
8	102 11	−1.02	101 10	−1.02	100 09	−1.02	99 08	−1.03	98 06	−1.02	97 05	−1.02	96 04	−1.02	95 03	−1.02	94 02	−1.02
9	105 14	−1.00	104 14	−1.02	103 13	−1.00	102 13	−1.00	101 13	−1.00	100 13	−1.02	99 12	−1.00	98 12	−1.00	97 12	−1.02
10	106 00	−1.00	105 00	−1.00	104 00	−1.00	103 00	−1.00	102 00	−1.00	101 00	−1.00	100 00	−1.00	99 00	−1.00	98 00	−1.00

TABLE 28.—DEFLECTIONS FROM THE P.I. TO THE 10-POINT CIRCULAR CURVE—(*Continued*).
Vernier reads zero on first tangent.

Pts.	Δ = 83° 00′	Diff. per min.	Δ = 84° 00′	Diff. per min.	Δ = 85° 00′	Diff. per min.	Δ = 86° 00′	Diff. per min.	Δ = 87° 00′	Diff. per min.	Δ = 88° 00′	Diff. per min.	Δ = 89° 00′	Diff. per min.	Δ = 90° 00′	Diff. per min.	Δ = 91° 00′	Diff. per min.
1	0° 49′	+0.00′	0° 49′	+0.00′	0° 49′	+0.00′	0° 49′	+0.02′	0° 50′	+0.00′	0° 50′	+0.00′	0° 50′	+0.00′	0° 50′	+0.00′	0° 50′	+0.00′
2	3 59	+0.00	3 59	+0.02	4 00	+0.02	4 01	+0.00	4 01	+0.02	4 02	+0.02	4 03	+0.00	4 03	+0.00	4 03	+0.02
3	11 20	+0.00	11 20	+0.00	11 20	−0.00	11 20	−0.00	11 20	−0.02	11 19	−0.02	11 18	−0.02	11 17	−0.02	11 16	−0.02
4	25 50	−0.12	25 43	−0.13	25 35	−0.13	25 27	−0.15	25 18	−0.13	25 10	−0.15	25 01	−0.15	24 52	−0.17	24 42	−0.17
5	48 30	−0.50	48 00	−0.50	47 30	−0.50	47 00	−0.50	46 30	−0.50	46 00	−0.50	45 30	−0.50	45 00	−0.50	44 30	−0.50
6	71 10	−0.88	70 17	−0.87	69 25	−0.87	68 33	−0.85	67 42	−0.87	66 50	−0.85	65 59	−0.85	65 08	−0.83	64 18	−0.83
7	85 40	−1.00	84 40	−1.00	83 40	−1.00	82 40	−1.00	81 40	−0.98	80 41	−0.98	79 42	−0.98	78 43	−0.98	77 44	−0.98
8	93 01	−1.00	92 01	−1.02	91 00	−1.02	89 59	−1.00	88 59	−1.02	87 58	−1.02	86 57	−1.00	85 57	−1.00	84 57	−1.02
9	96 11	−1.00	95 11	−1.00	94 11	−1.00	93 11	−1.02	92 10	−1.00	91 10	−1.00	90 10	−1.00	89 10	−1.00	88 10	−1.02
10	97 00	−1.00	96 00	−1.00	95 00	−1.00	94 00	−1.00	93 00	−1.00	92 00	−1.00	91 00	−1.00	90 00	−1.00	89 00	−1.00

Pts.	Δ = 92° 00′	Diff. per min.	Δ = 93° 00′	Diff. per min.	Δ = 94° 00′	Diff. per min.	Δ = 95° 00′	Diff. per min.	Δ = 96° 00′	Diff. per min.	Δ = 97° 00′	Diff. per min.	Δ = 98° 00′	Diff. per min.	Δ = 99° 00′	Diff. per min.	Δ = 100° 00′	Diff. per min.
1	0° 51′	+0.00′	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′
2	4 04	+0.00	4 04	+0.00	4 04	+0.00	4 04	+0.00	4 04	+0.00	4 04	−0.00	4 04	−0.00	4 04	−0.00	4 04	−0.02
3	11 15	−0.03	11 13	−0.02	11 12	−0.03	11 10	−0.03	11 03	−0.05	11 05	−0.03	11 03	−0.03	11 01	−0.05	10 58	−0.05
4	24 32	−0.17	24 22	−0.17	24 12	−0.18	24 01	−0.18	23 50	−0.17	23 40	−0.20	23 28	−0.18	23 17	−0.20	23 05	−0.20
5	44 00	−0.50	43 30	−0.50	43 00	−0.50	42 30	−0.50	42 00	−0.50	41 30	−0.50	41 00	−0.50	40 30	−0.50	40 00	−0.50
6	63 28	−0.83	62 38	−0.83	61 48	−0.82	60 59	−0.82	60 10	−0.83	59 20	−0.80	58 32	−0.82	57 43	−0.80	56 55	−0.80
7	76 45	−0.97	75 47	−0.98	74 48	−0.97	73 50	−0.97	72 52	−0.95	71 55	−0.97	70 57	−0.97	69 59	−0.95	69 02	−0.95
8	83 56	−1.00	82 56	−1.00	81 56	−1.00	80 56	−1.00	79 56	−1.00	78 56	−1.00	77 56	−1.00	76 56	−1.00	75 56	−0.98
9	87 09	−1.00	86 09	−1.00	85 09	−1.00	84 09	−1.00	83 09	−1.00	82 09	−1.00	81 09	−1.00	80 09	−1.00	79 09	−1.00
10	88 00	−1.00	87 00	−1.00	86 00	−1.00	85 00	−1.00	84 00	−1.00	83 00	−1.00	82 00	−1.00	81 00	−1.00	80 00	−1.00

TABLE 28.—DEFLECTIONS FROM THE P.I. TO THE 10-POINT CIRCULAR CURVE—(*Continued*).
Vernier reads zero on first tangent.

Pts.	Δ = 101° 00′	Diff. per min.	Δ = 102° 00′	Diff. per min.	Δ = 103° 00′	Diff. per min.	Δ = 104° 00′	Diff. per min.	Δ = 105° 00′	Diff. per min.	Δ = 106° 00′	Diff. per min.	Δ = 107° 00′	Diff. per min.	Δ = 108° 00′	Diff. per min.	Δ = 109° 00′	Diff. per min.
1	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′	0° 51′	0.00′
2	4 03	−0.00	4 03	−0.02	4 02	−0.00	4 02	−0.02	4 01	−0.00	4 01	−0.02	4 00	−0.02	3 59	−0.02	3 58	−0.02
3	10 55	−0.05	10 52	−0.07	10 48	−0.05	10 45	−0.07	10 41	−0.05	10 38	−0.07	10 34	−0.07	10 30	−0.08	10 25	−0.07
4	22 53	−0.20	22 41	−0.20	22 29	−0.22	22 16	−0.22	22 03	−0.22	21 50	−0.22	21 37	−0.22	21 24	−0.23	21 10	−0.22
5	39 30	−0.50	39 00	−0.50	38 30	−0.50	38 00	−0.50	37 30	−0.50	37 00	−0.50	36 30	−0.50	36 00	−0.50	35 30	−0.50
6	56 07	−0.80	55 19	−0.80	54 31	−0.78	53 44	−0.78	52 57	−0.78	52 10	−0.78	51 23	−0.78	50 36	−0.77	49 50	−0.78
7	68 05	−0.95	67 08	−0.93	66 12	−0.95	65 15	−0.93	64 19	−0.95	63 22	−0.93	62 26	−0.93	61 30	−0.92	60 35	−0.93
8	74 57	−1.00	73 57	−0.98	72 58	−1.00	71 58	−0.98	70 59	−1.00	69 59	−0.98	69 00	−1.00	68 00	−0.97	67 02	−0.98
9	78 09	−1.00	77 09	−1.00	76 09	−1.00	75 09	−1.00	74 09	−1.00	73 09	−1.00	72 09	−1.00	71 09	−1.00	70 09	−1.00
10	79 00	−1.00	78 00	−1.00	77 00	−1.00	76 00	−1.00	75 00	−1.00	74 00	−1.00	73 00	−1.00	72 00	−1.00	71 00	−1.00

Pts.	Δ = 110° 00′	Diff. per min.	Δ = 111° 00′	Diff. per min.	Δ = 112° 00′	Diff. per min.	Δ = 113° 00′	Diff. per min.	Δ = 114° 00′	Diff. per min.	Δ = 115° 00′	Diff. per min.	Δ = 116° 00′	Diff. per min.	Δ = 117° 00′	Diff. per min.	Δ = 118° 00′	Diff. per min.
1	0° 51′	−0.00′	0° 51′	−0.00′	0° 51′	−0.00′	0° 51′	−0.00′	0° 51′	−0.01′	0° 50′	−0.00′	0° 50′	−0.00′	0° 50′	−0.00′	0° 50′	−0.00′
2	3 57	−0.02	3 56	−0.02	3 55	−0.02	3 54	−0.02	3 53	−0.03	3 51	−0.02	3 50	−0.02	3 49	−0.03	3 47	−0.02
3	10 21	−0.08	10 16	−0.07	10 12	−0.08	10 07	−0.08	10 02	−0.08	9 57	−0.10	9 51	−0.08	9 46	−0.10	9 40	−0.10
4	20 57	−0.25	20 42	−0.23	20 28	−0.23	20 14	−0.23	20 00	−0.25	19 45	−0.25	19 30	−0.23	19 16	−0.25	19 01	−0.27
5	35 00	−0.50	34 30	−0.50	34 00	−0.50	33 30	−0.50	33 00	−0.50	32 30	−0.50	32 30	−0.50	31 30	−0.50	31 00	−0.50
6	49 03	−0.75	48 18	−0.77	47 32	−0.77	46 46	−0.77	46 00	−0.75	45 15	−0.75	44 30	−0.77	43 44	−0.75	42 59	−0.73
7	59 39	−0.92	58 44	−0.93	57 48	−0.92	56 53	−0.92	55 58	−0.92	55 03	−0.90	54 09	−0.92	53 14	−0.90	52 20	−0.90
8	66 03	−0.98	65 04	−0.98	64 05	−0.98	63 06	−0.98	62 07	−0.97	61 09	−0.98	60 10	−0.98	59 11	−0.97	58 13	−0.98
9	69 09	−1.00	68 09	−1.00	67 09	−1.00	66 09	−1.00	65 09	−0.98	64 10	−1.00	63 10	−1.00	62 10	−1.00	61 10	−1.00
10	70 00	−1.00	69 00	−1.00	68 00	−1.00	67 00	−1.00	66 00	−1.00	65 00	−1.00	64 00	−1.00	63 00	−1.00	62 00	−1.00

TABLE 28.—DEFLECTIONS FROM THE P.I. TO THE 10-POINT CIRCULAR CURVE—(*Continued*).
Vernier reads zero on first tangent.

Pts.	Δ = 119° 00′	Diff. per min.	Δ = 120° 00′	Diff. per min.	Δ = 121° 00′	Diff. per min.	Δ = 122° 00′	Diff. per min.	Δ = 123° 00′	Diff. per min.	Δ = 124° 00′	Diff. per min.	Δ = 125° 00′	Diff. per min.	Δ = 126° 00′	Diff. per min.	Δ = 127° 00′	Diff. per min.
1	0° 50′	−0.02′	0° 49′	−0.00′	0° 49′	−0.00′	0° 49′	−0.02′	0° 48′	−0.00′	0° 48′	−0.00′	0° 48′	−0.00′	0° 48′	−0.02′	0° 47′	−0.00′
2	3 46	−0.03	3 44	−0.03	3 42	−0.03	3 40	−0.02	3 39	−0.03	3 37	−0.03	3 35	−0.03	3 33	−0.05	3 30	−0.03
3	9 34	−0.10	9 28	−0.08	9 23	−0.12	9 16	−0.10	9 10	−0.10	9 01	−0.12	8 57	−0.12	8 50	−0.12	8 43	−0.12
4	18 45	−0.25	18 30	−0.25	18 15	−0.27	17 59	−0.27	17 43	−0.27	17 27	−0.27	17 11	−0.27	16 55	−0.27	16 39	−0.28
5	30 30	−0.50	30 00	−0.50	29 30	−0.50	29 00	−0.50	28 30	−0.50	28 00	−0.50	27 30	−0.50	27 00	−0.50	26 30	−0.50
6	42 15	−0.75	41 30	−0.75	40 45	−0.73	40 01	−0.73	39 17	−0.73	38 33	−0.73	37 49	−0.73	37 05	−0.73	36 21	−0.72
7	51 26	−0.90	50 32	−0.92	49 37	−0.88	48 44	−0.90	47 50	−0.90	46 56	−0.88	46 03	−0.88	45 10	−0.88	44 17	−0.88
8	57 14	−0.97	56 16	−0.97	55 18	−0.97	54 20	−0.98	53 21	−0.97	52 23	−0.97	51 25	−0.97	50 27	−0.95	49 30	−0.97
9	60 10	−0.98	59 11	−1.00	58 11	−1.00	57 11	−0.98	56 12	−1.00	55 12	−1.00	54 12	−1.00	53 12	−0.98	52 13	−1.00
10	61 00	−1.00	60 00	−1.00	59 00	−1.00	58 00	−1.00	57 00	−1.00	56 00	−1.00	55 00	−1.00	54 00	−1.00	53 00	−1.00

Pts.	Δ = 128° 00′	Diff. per min.	Δ = 129° 00′	Diff. per min.	Δ = 130° 00′	Diff. per min.	Δ = 131° 00′	Diff. per min.	Δ = 132° 00′	Diff. per min.	Δ = 133° 00′	Diff. per min.	Δ = 134° 00′	Diff. per min.	Δ = 135° 00′	Diff. per min.	Δ = 136° 00′	Diff. per min.
1	0° 47′	−0.02′	0° 46′	−0.00′	0° 46′	−0.00′	0° 46′	−0.02′	0° 45′	−0.00′	0° 45′	−0.02′	0° 44′	−0.00′	0° 44′	−0.02′	0° 43′	
2	3 28	−0.03	3 26	−0.03	3 24	−0.05	3 21	−0.03	3 19	−0.05	3 16	−0.03	3 14	−0.05	3 11	−0.05	3 08	
3	8 36	−0.10	8 30	−0.13	8 22	−0.12	8 15	−0.13	8 07	−0.12	8 00	−0.13	7 52	−0.13	7 44	−0.13	7 36	
4	16 22	−0.27	16 06	−0.28	15 49	−0.28	15 32	−0.27	15 16	−0.28	14 59	−0.30	14 41	−0.28	14 24	−0.28	14 07	
5	26 00	−0.50	25 30	−0.50	25 00	−0.50	24 30	−0.50	24 00	−0.50	23 30	−0.50	23 00	−0.50	22 30	−0.50	22 00	
6	35 38	−0.73	34 54	−0.72	34 11	−0.72	33 28	−0.73	32 44	−0.72	32 01	−0.70	31 19	−0.72	30 36	−0.72	29 53	
7	43 24	−0.90	42 30	−0.87	41 38	−0.88	40 45	−0.87	39 53	−0.88	39 00	−0.87	38 08	−0.87	37 16	−0.87	36 24	
8	48 32	−0.97	47 34	−0.97	46 36	−0.95	45 39	−0.97	44 41	−0.95	43 44	−0.97	42 46	−0.95	41 49	−0.95	40 52	
9	51 13	−0.98	50 14	−1.00	49 14	−1.00	48 14	−0.98	47 15	−1.00	46 15	−0.98	45 16	−1.00	44 16	−0.98	43 17	
10	52 00	−1.00	51 00	−1.00	50 00	−1.00	49 00	−1.00	48 00	−1.00	47 00	−1.00	46 00	−1.00	45 00	−1.00	44 00	

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